Visual Predictive Control for Manipulators with Catadioptric Camera

Guillaume Allibert, Estelle Courtial and Youssoufi Touré

Abstract— **This paper deals with Image Based Visual Servoing (IBSV) by a Visual Predictive Control (VPC) approach. Based on Nonlinear Model Predictive Control (NMPC), the visual servoing problem is formulated into a nonlinear constrained minimization problem in the image plane. A global model describing the behavior of the robotic system equipped with the camera is used to predict the evolution of the visual feature on a future horizon. The main interest of this method is the capability to easily take into account different constraints like mechanical limitations and/or visibility contraints. Simulation experiments are performed on a planar manipulator with an omnidirectional camera. Comparisons with the classical control law based on the interaction matrix highlight the efficiency and the robustness of the proposed approach, especially in difficult initial configurations and large displacements.**

I. INTRODUCTION

Visual servoing has become an attractive strategy for the motion control of autonomous manipulators and mobile robots. The visual servoing principle is to control the movement of a robotic system from a current pose to a desired pose. Visual control law design depends on several parameters such as the camera configuration (eye-to-hand, eye-in-hand or stereovision), the kind of camera (perspective or catadioptric) and the control scheme. The fundamental classification of visual servoing distinguishes different approaches depending on the design of the control scheme: image-based control (2D), position-based control (3D) and a hybrid approach (2D/dt, 2D $\frac{1}{2}$). Further details about visual servoing can be found in [6],[7].

The principle of Image-Based Visual Servoing (IBVS) is to minimize image error between the desired image and the current image from the camera. In the classical IBVS approach, an interaction matrix converts image errors into Cartesian errors. This interaction matrix depends on the visual feature considered ($(u_p\ v_p)$ for a point), on the intrinsic camera parameters and on the depth, i.e. the distance of the considered point w.r.t the camera frame.

While the main interest of IBVS is its robustness to modeling errors, like robot and camera calibration errors or/and image measurement errors, several drawbacks should be mentioned:

- For the computation of the interaction matrix, an approximate value of the depth at the final desired position is generally used. This choice involves a non optimal trajectory motion between the initial and desired position : the trajectory and the visibility of the features are not controlled [8].
- A weak point of 2D visual servoing concerns the visibility constraint. If some targets get out the camera field of view during convergence, the value of the current features can no longer be computed, which leads to the interruption of the control algorithm.
- Mechanical constraints such as joint limits, actuator limitations, non holonomic constraints, can not explicitly be taken into account in the IBVS design either.
- IBVS is known to be satisfactory when the error between the initial position and the final one is small. For large displacements or rotations, the camera motion may involve visibility loss of some features which may be disastrous for the control law.
- Singularities of the interaction matrix can also appear due to the number of visual features and their configurations. In this case, the synthesis of the control law is not possible [8].

Numerous works have investigated these critical issues. Different approaches proposed have partially studied these issues: path planning of the features in the image plane [12], switching control [9] and zoom ajustement [11] to ensure visibility constraints, different choice of features [8], visual servoing based on optimization [1],[10], etc.

Visual Predictive Control, an alternative approach of IBVS based on NMPC, has been presented in [3], [13]. The task of 2D visual servoing is written as a constrained minimization of a cost function, over a prediction horizon, in the image plane. The proposed approach is a global method, where the constrained optimal path of visual features is computed implicitly by minimization along a prediction horizon. One of the main interests of the VPC strategy is the capability to explicitly take into account constraints in the control design. In [2], the point stabilization of a mobile robot is considered. The visual information is given by a catadioptric camera embedded in the mobile robot (eye-in-hand). A real time application shows the efficiency and the robustness of the VPC approach. In [3], trajectory tracking in the image plane of a mobile robot is addressed. The flatness property of the process model is used to reduce the computational time, the real challenge in this kind of application. Obstacle avoidance is ensured thanks to visibility constraint handling. In [14], the visual servoing of a manipulator in an eye-to-hand configuration is studied with a perspective camera. A linear model obtained by linearizing and decoupling technique based on the inverse dynamic model has been considered. It allows the reduction of the computational time. In [13], the NMPC strategy has been addressed for 3D visual servoing tasks. However, no constraints are taken into account in this work and the optimization solution is explicit.

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In this paper, the VPC strategy is applied to a nonlinear robotic system subject to mechanical constraints and visibility constraints. The visual servoing task is tested on a planar manipulator arm equipped with an omnidirectional camera. A comparison with the classical control law based on the interaction matrix is carried out and shows the efficiency of the proposed method. Difficult configurations are also considered such as large motion and rotation, the Chaumette Conundrum, etc.
The paper is organized as follows. In section II, the principle of NMPC is briefly recalled. The VPC strategy, the extension to the visual servoing task, is developed in section III. In Section IV, simulation experiments highlight the efficiency of the VPC. Finally conclusions and future tasks are detailed.

II. NONLINEAR MODEL PREDICTIVE CONTROL

The use of Nonlinear Model Predictive Control (NMPC), also named receding horizon control or finite horizon optimal control, is widespread for the control of constrained nonlinear processes. The control problem (trajectory or setpoint tracking) is formulated into a nonlinear optimization problem. Based on the process model, the controller predicts the behavior of the system over a prediction horizon Np. The difference between the reference and the predicted behavior defines the cost function J to be minimized with respect to a control sequence $\tilde{u}$. Due to disturbances and model mismatches, this procedure is repeated at each sampling time. Only the first control input of the optimal control sequence is really applied to the process. At the next sampling time, when the measurements are updated, the finite horizon moves and the procedure starts again. Fig. 1 and 2 describe the principle of NMPC.

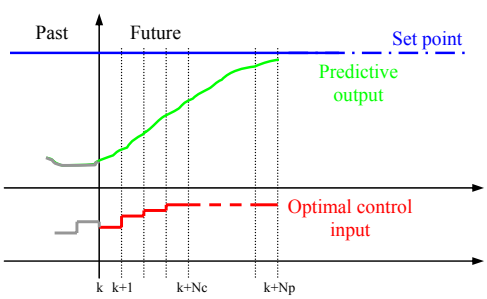

Fig. 1. At time k

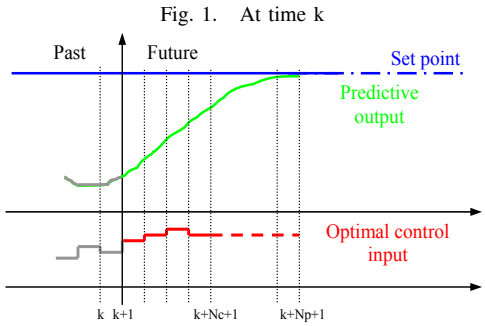

Fig. 2. At time k+1

The well-known Internal Model Control (IMC) structure (Fig. 3) is considered.

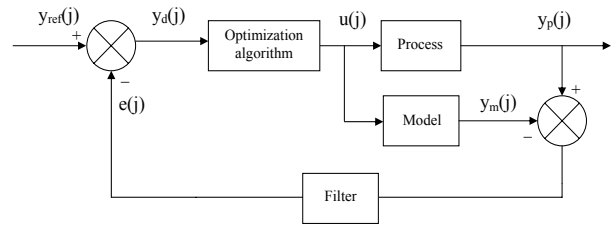

Fig. 3. Internal Model Control Structure

According to this control scheme, as the filter dynamic is very fast, we can write:

$$\begin{aligned} y_d(j) &= y_{ref}(j) - e(j) \\ y_d(j) &= y_{ref}(j) - (y_p(j) - y_m(j)) \\ y_d(j) - y_m(j) &= y_{ref}(j) - y_p(j) \end{aligned} \tag{1}$$

The tracking of the reference trajectory y_{ref} by the process output y_p is equivalent to the tracking of the desired trajectory y_d by the model output y_m.
Combined with the IMC structure, the cost function can be written in discrete-time as:

$$J(x,u) = \sum_{j=k+1}^{k+Np} [y_d(j) - y_m(j)]^T Q \, [y_d(j) - y_m(j)] \tag{2}$$

and the mathematical formulation of NMPC strategy is given by:

$$\min_{\tilde{u}} \; J(x,u) \tag{3}$$

subject to the nonlinear discrete-time model:

$$\begin{cases} x(k+1) = f(x(k),u(k)) \\ y_m(k) = h(x(k)) \end{cases} \tag{4}$$

The error $e(j)$, $j \in [k+1, k+Np]$ is assumed to be constant over the prediction horizon and equal to the measurement $e(k)$.
$\tilde{u} = \{u_k, u_{k+1}, \ldots, u_{k+Nc}, \ldots, u_{k+Np-1}\}$ is the optimal control sequence. From $u(k+Nc+1)$ to $u(k+Np-1)$, the control input is constant and equal to $u(k+Nc)$ with Nc, the control horizon ($Nc < Np$). Q is a symmetric definite positive matrix.
One of the main advantages of NMPC is the capability to explicitly take into account the constraints. Two kinds of constraints are usually taken into consideration:
- state constraints:

$$x(j) \in \mathbb{X}, \;\; j \in [k+1, k+Np] \tag{5}$$

- control constraints:

$$u(j) \in \mathbb{U}, \;\; j \in [k, k+Np-1] \tag{6}$$

$\mathbb{X}$ and $\mathbb{U}$ are respectively the sets of feasible states and inputs. These constraints are easily added to the problem (3). Numerous constrained optimization routines are available in software libraries.
Remark: The cost function (2) can be modified by adding some penalty terms :

- *a terminal constraint on the state to ensure a global asymptotic stability;*
- *a quadratic penalty term ($u^T Ru$) on the control to guarantee the smoothness of the control input.*

In the next section, the NMPC strategy is extended to image-based visual servoing problem.

III. Visual Predictive Control

Most visual servoing applications concern manipulators or mobile robots. These robotic systems are always subject to mechanical constraints such as joint bounds, actuator limitations, nonholonomic constraints, etc. Furthermore, the weak point of the classical IBVS is the visibility constraint handling. The classical control law converges if the visual features stay in the camera field of view.

The VPC strategy can easily deal with the two main problems of IBVS: the minimization of an image error and the constraint handling.

Let us define the image error at time j:

$$error(j) = image_{ref}(j) - image_p(j) \tag{7}$$

where:

- $image_{ref}$ is the reference image;
- $image_p$ is the current image.

Remark: only point like features are considered without loss of generality for the VPC approach. The point pixel coordinates in the image plane are denoted by "image_ _".

Thanks to the IMC structure (Fig. 3) combined with the VPC strategy, the image error becomes:

$$error(j) = image_d(j) - image_m(j) \tag{8}$$

where:

· $image_d(j) = image_{ref} - e(j)$ is the desired image;

· $e(j) = image_p(j) - image_m(j)$;

· $image_m(j)$ is the predicted image by the global model.

The robot system model combined with the camera model defines the global model. This nonlinear global model is a crucial element of the VPC strategy. It permits to predict the evolution of the visual features in regard to control variations over the horizon Np. It can be compared to a black box where the inputs are the controls of the robotic system and the outputs are the prediction of the visual features. The figure below (Fig. 4) illustrates the different steps of the feature configurations predicted by the global model.

Considering the usual quadratic cost function, the mathematical formulation of the 2D visual predictive controller can be summarized as follows:

$$\min_{\tilde{u}} \quad J(x,u) = \sum_{j=k+1}^{k+Np} error(j)^T \; Q \; error(j) \tag{9}$$

with:

- $error(j)$: the image error defined in (8);
- $\tilde{u} = \{u_k, u_{k+1}, ..., u_{k+Nc}, ..., u_{k+Np-1}\}$: the optimal control sequence;

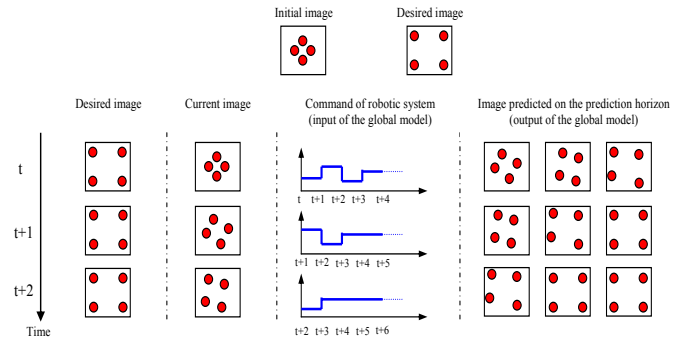

Fig. 4. Predicted evolution of the visual features by the global model

- Q : a symmetric definite positive matrix;
- Np, the prediction horizon and Nc, the control horizon;

The constraints can easily and explicitly be taken into account in the optimization problem. Two kinds of constraints are distinguished:

- Mechanical constraints such as joint limits, actuator limitations in amplitude or velocity, etc.

$$\begin{aligned} u_{\min} \leq \mathrm{u}_j \leq u_{\max} \\ \Delta u_{\min} \leq \mathrm{u}_j - \mathrm{u}_{j-1} \leq \Delta u_{\max} \end{aligned} \tag{10}$$

- Visibility constraints such as image limitations which ensure that the visual features stay in the image plane like image size for example. It can also represent forbidden areas in the image.

The constrained optimization problem (9) has to be solved at each sampling period. Only the first argument of the control sequence $\tilde{u}$ is really applied to the process. At the next sampling time, the optimization procedure is repeated from the new image measurements.

Many advantages are connected to the VPC strategy. The first is that VPC is a global method. The interaction matrix is then not necessary and consequently, inversion difficulties and singularity problems are deleted. Furthermore, the resolution of the optimisation problem provides an optimal and implicit path planning in the image plane, under visibility and mechanical constraints. Finally the VPC approach is very flexible and can be used whatever the robotic system and the camera considered, provided that a model is available. A drawback of the VPC is the computational time required for the resolution of the nonlinear constrained optimization problem. This computational burden is not a strong limitation for real time application due to the increase of PC power.

IV. SIMULATION EXPERIMENTS

The VPC strategy is applied on a 3-dof planar manipulator arm in eye-in-hand configuration. The camera embedded is a catadioptric camera. The target is composed of 3 points. The figure (Fig. 5) gives a schematic view of the application.

A. Robot model

The motions of the 3-dof planar manipulator are the rotations around the Z-axis (optical axis of the omnidirectional camera) and translations w.r.t. X-axis and Y-axis. Due to the

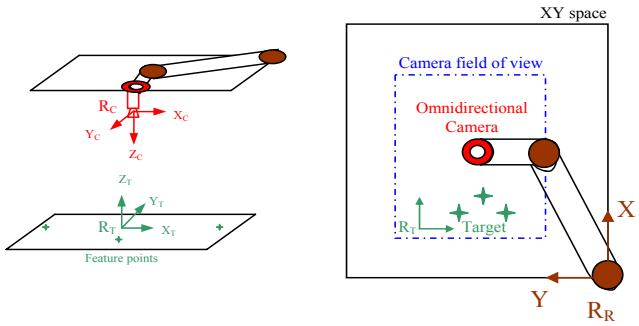

Fig. 5. 3-dof planar manipulator

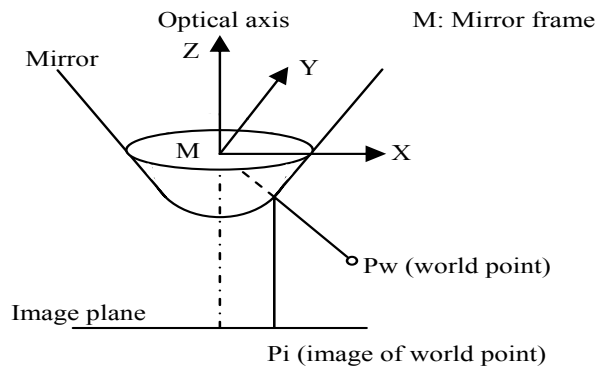

Fig. 6. Point projection: parabolic mirror and orthographic camera

low level velocity control of robotic system, the kinematic model (pure integrator) can be used:

$$q_i(k+1) = q_i(k) + u_i(k)Te \tag{11}$$

where:
- q_i is the joint position of the i articulation;
- u_i is a control input of the i articulation;
- Te is a sampling time.

The maximum speed for translations and rotations are respectively 10 cm/s and 0.1 rad/s.

B. Camera model

The modeling of catadioptric cameras has been studied in [4]. The authors show that in the case of a unique viewpoint, the projection of a world point in the image plane is given by:

$$\begin{pmatrix} u_p \\ v_p \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta X \\ \beta Y \\ 1 \end{pmatrix} \tag{12}$$

with:

$$\beta = \frac{\xi + m}{Z + \xi\sqrt{X^2+Y^2+Z^2}} \tag{13}$$

where:
- (X,Y,Z) are the coordinates of a world point (Pw) in the camera frame;
- (u_p,v_p) are the coordinates in pixels of the point (Pw) in the image frame (Pi);
- u_0, v_0, α_u, α_v are the camera intrinsic parameters;
- ξ and m are the mirror intrinsic parameters.

The initial transformation between the robot frame (R_R) and the target frame (R_T) is assumed to be known as well as the 3D model of the target. Using the direct geometric model, $\mathbf{x} = f(q)$, Cartesian coordinates of each target (X,Y,Z) expressed in the camera frame (R_C), can be calculated from joint positions. (u_p, v_p) are then deducted from (12) and (13). The robot model combined with the camera defines the global model. It ensures the prediction of the feature evolution in the image plane.

C. Simulations

Simulations have been performed on a PC pentium IV 3 GHz. The nonlinear constrained optimization problem is solved by the *fmincon* function from the optimization toolbox of Matlab software. The sampling time Te is equal to 100 ms.

Classical visual servoing controller : The interaction matrix for point like features in case of omnidirectional camera derives from the work of Barreto [5]. The knowledge of the 3D model of the target permits to estimate at each iteration the value of the depth. The control scheme is standard: an exponential decay of the signal error is ensured [6]. The λ coefficient has been set to 0.25. The control law is saturated on the constraints.

Visual predictive controller : For all simulations, the prediction horizon is chosen as 1 second (Np=10) and the control horizon is chosen as $Nc = 1$. These relevant parameters act on the accuracy and on the time response of the system. The shorter is Np, the shorter the computational time needed to solve the optimization problem. Nevertheless, the action on actuators is more important. Here Np is chosen to have the same convergence time as the classical IBVS control law. This choice is a good compromise between accuracy and computational time requirement.

Different simulation experiments are given, corresponding to different initial camera configurations. In all cases, the following information is given:
- the initial positions defined by '+' and desired positions defined by 'o' of the visual feature and the feature trajectories in the image (in the top left-hand corner);
- the initial (in green) and final configuration of the camera in the XY space (in the bottom left-hand corner);
- the control input of the planar manipulator (Tx, Ty and Wz) (in the top right-hand corner);
- the signal error between the measured and desired features in the image (in the bottom right-hand corner).

Simulation 1 (Fig. 7 and 8) : In this first experiment, the camera must move on X and Y axes and simultaneously rotate around the optical axis Z. As can be seen in Fig. 7 and 8, both control laws converge to the desired positions in the same time. However, unwanted translations of the camera in the XY plane are added in the case of classical IBVS.

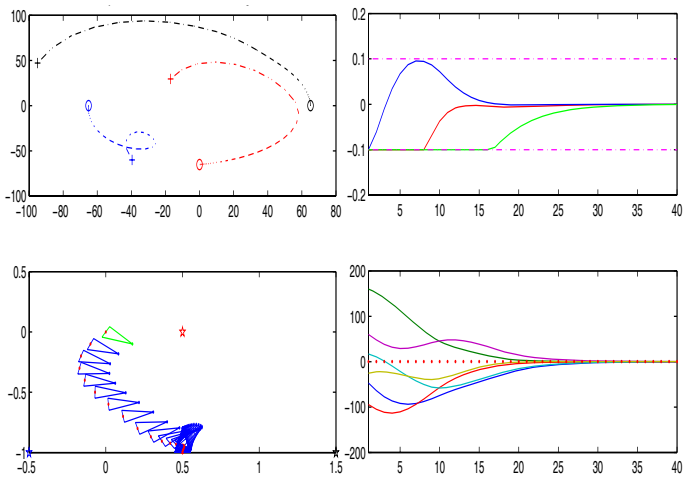

Fig. 7. Simulation 1 (AV2D)

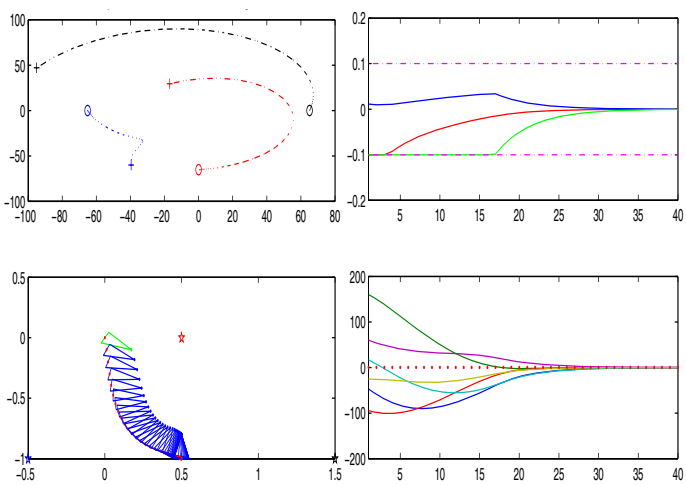

Fig. 8. Simulation 1 (VPC)

Simulation 2 (Fig. 9 and 10) : Experiment 2 requires a large displacement and a large rotation. Convergence is obtained for both controllers. The VPC results (Fig. 10) are clearly better. The camera trajectory in the XY space and the feature trajectories in the image plane are optimal.

Simulation 3 (Fig. 11) : To illustrate the capability of handling visibility constraints, a defined area in the image plane is added to the latter simulation. Now, the prediction of the visual feature motion is constrained to stay in a window defined by the following inequality :

$$\begin{bmatrix} u_{min=} - 90 \\ v_{min=} - 65 \end{bmatrix} \leq \text{image}_m(j) \leq \begin{bmatrix} u_{max} = 40 \\ v_{max} = 110 \end{bmatrix} \quad (14)$$

As can be seen in Figure 11, the VPC controller allows to satisfy both visibility constraint and control task. The camera trajectory in the work space is modified to ensure that the visual features do not get out the field of view.

Simulation 4 (Fig. 12) : In the classical IBVS approach, a 180 degree rotation around the optical axis, the Chaumette

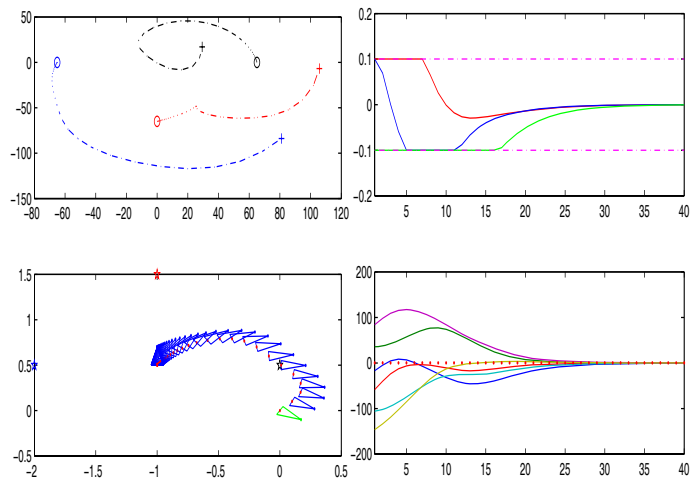

Fig. 9. Simulation 2 (AV2D)

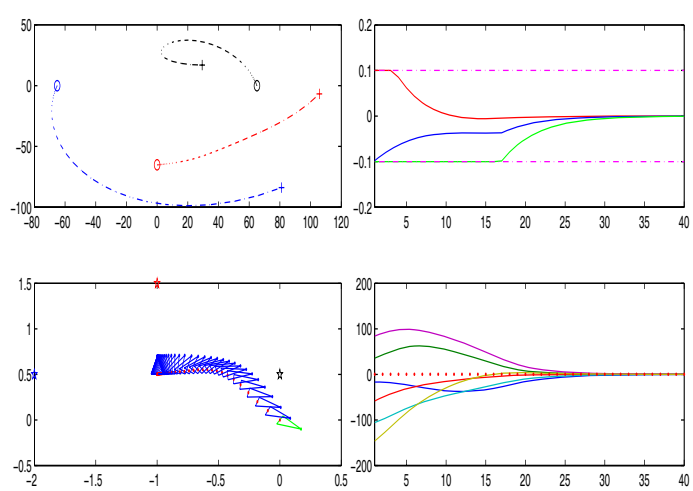

Fig. 10. Simulation 2 (CPV)

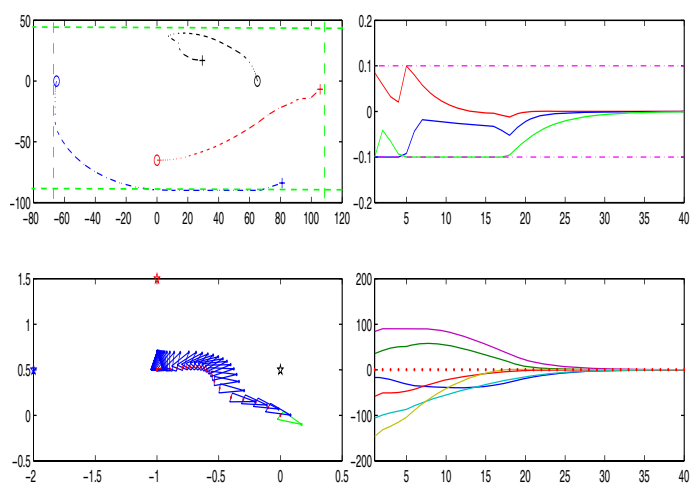

Fig. 11. Simulation 3 (CPV)

Conundrum, is known to lead to failure of the control law [8]. As IBVS strategy chooses the shortest path in image space, which is a straight line, the end-effector, that is

the camera, performs an infinite retreat, corresponding to a singularity in the interaction matrix. In contrast, the VPC strategy converges with a perfect decoupled control in figure 12. Only the rotation around the optical axis is performed.

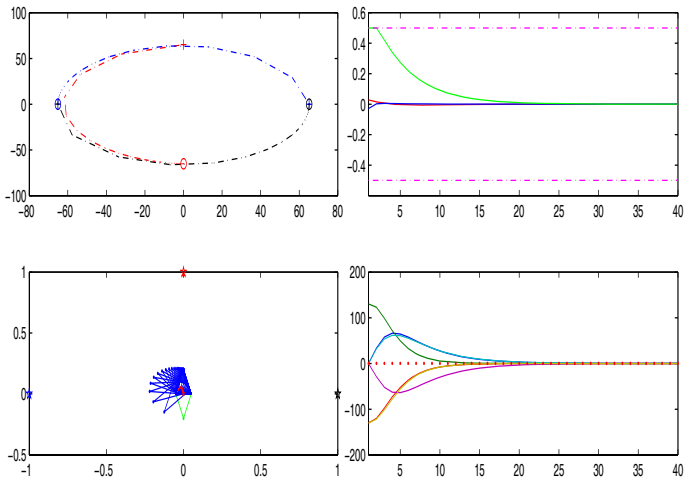

Fig. 12. Simulation 4 (VPC)

Simulation 5 (Fig. 13) : The robustness w.r.t modeling errors (20% on the intrinsic camera parameters) and disturbances (white noise added to the output) is tested. Due to the IMC structure and in spite of a large displacement associated to a rotation of 180 degrees around the optical axis, the VPC controller easily converges to the desired image.

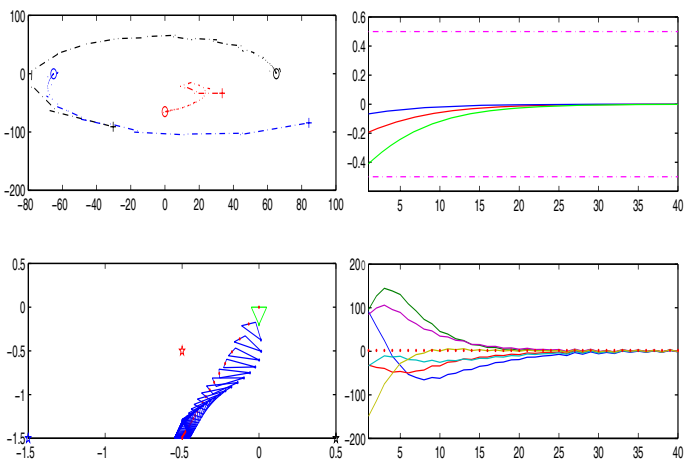

Fig. 13. Simulation 5 (VPC)

The computational time (needed time to solve the constraint optimization problem) is about 20ms per iteration for all simulations. It allows the real time application to be considered with a usual camera (25 frames per second).

V. Conclusions

The Visual Predictive Control seems to be well-adapted to dealing with 2D visual servoing tasks. The visual control objective is formulated into a constraint optimization problem in the image plane. Mechanical and visibility constraints are explicitly handled in the control law design. The optimization procedure can be compared to an implicit and optimal path planning of features in the image plane under constraints. These simulations highlight the interest and the efficiency of VPC in terms of camera and feature trajectories, robustness to modelling errors, control input feasibility, visibility constraint handling and control convergence for difficult camera configurations.

On the other hand, due to the constrained nonlinear optimization, the solution is numerical and then, the stability analysis is difficult to prove from a theoretical viewpoint. A terminal constraint can be added to the cost function. Future works will be devoted to experiment VPC strategy on a real platform and to extend this approach to 3D visual servoing.

VI. Acknowledgements

The authors would like to thank François Chaumette for many fruitful discussions and constructive comments.

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