Model-based Bit Allocation for Normal Mesh Compression

Frédéric Payan and Marc Antonini
I3S Laboratory - CNRS - University of Nice-Sophia Antipolis
2000, route des lucioles - Sophia Antipolis, 06903, France
Phone: +33 4 92 94 27 22 Fax: +33 4 92 94 28 98
Email: fpayan@i3s.unice.fr, am@i3s.unice.fr

Abstract— **In this paper, we propose a powerful bit allocation that optimizes the quantization of the *normal mesh* geometry. This bit allocation aims to minimize the *surface-to-surface distance* [1] between the original irregular mesh and the quantized *normal* one, according to a target bitrate. Moreover, to provide a fast bit allocation, we approximate this *surface-to-surface distance* with a simple criterion depending on the wavelet coefficient distributions, and we use theoretical models. This provides a fast and low-complex model-based bit allocation yielding results better than the recent state-of-the-art methods like [2].**

I. Introduction

BIT allocation is an essential tool to provide a powerful coding of signals when a multiresolution analysis is performed. This process generally aims to optimize the trade-off between bitrate and quality, by minimizing a distortion due to the signal quantization for a specific bitrate. Among the prior works in 3D mesh compression, King and Rossignac proposed for instance a bit allocation based on relationships between the number of vertices, the bitrate per coordinate, a desired approximation error, and the bitstream size [3]. More recently, Karni and Gotsman [4] proposed to truncate their spectral coefficients according to a given RMS value. We also proposed in [5] a model-based bit allocation controlling the quantization error energy to dispatch the bits across wavelet subbands of meshes obtained with MAPS [6]. Recently, an estimation-quantization algorithm has been proposed to encode the *normal mesh* geometry [7].

In this paper, we propose a model-based bit allocation for a wavelet coder of *normal meshes* [8]. We focus on these meshes because of their compact multiresolution representation based on subdivision connectivity. The particularity of these meshes is that most of details are in the normal direction to the surface and are expressed through a single scalar (see Fig. 1). The allocation proposed for these normal meshes optimizes the rate-distortion trade-off during the encoding of the normal mesh geometry. Precisely, we aim to find the best quantization for each wavelet subband such that the global reconstruction error is minimized under a constraint on the global bitrate. A distortion measure is consequently needed to evaluate the reconstructed error of the decoded mesh.

Several distortion measures have been exploited for 3D mesh compression of irregular meshes [4], [3]. In order to measure the loss related to quantization, the authors of [4] introduce for instance a metric which captures the visual difference between the original mesh and its approximation: to this purpose, they use a criterion depending on the geometric distance and the *laplacian difference* between models. Unfortunately, this kind of *vertex-to-vertex* measures cannot be applied in our case since the proposed coder uses a remeshing technique modifying the topology of the input mesh. In that case, the most frequently quality criterion used is the so-called *surface-to-surface (S2S) distance* [1]. Based on the *Hausdorff* distance, this distance does not depend on the mesh sampling. Unfortunately, the S2S distance is a computationally intensive process which does not permit real time computation during process, particularly from wavelet coefficients.

The main contribution of this paper is to show how the S2S distance can be approximated in function of the quantization error of wavelet coefficients, and then theoretically modeled according to the wavelet coefficient distributions. This permits to design a fast and low-complex model-based bit allocation.

This paper is organized as follows. Section II introduces the normal meshes and the proposed coder. Section III deals with the approximation of the S2S distance across a wavelet coder. Section IV introduces the model-based bit allocation. Finally, we show results and conclude in section V.

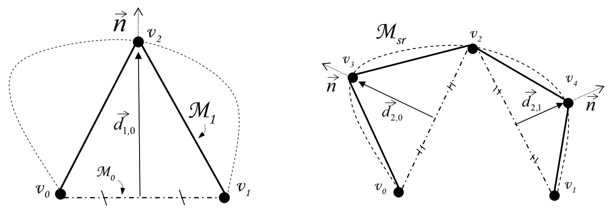
II. Backgrounds

A. Normal meshes

A *normal mesh* $\mathcal{M}_{sr} = (\mathcal{V}_{sr}, \mathcal{T}_{sr})$, where $\mathcal{V}_{sr}$ and $\mathcal{T}_{sr}$ are respectively the set of vertices and the set of triangular faces, can also be defined by a coarse mesh $\mathcal{M}_0$ and several sets of details $\{d_{i,j}\}$, i being the resolution level (see Fig. 1). Computed in function of the normal at the surface, most of the geometry information is concentrated in the coordinates z of the details (*normal components*), the coordinates x and y (*tangential components*) being infinitesimal [8].

B. Overall coding scheme

Fig. 2 presents the proposed coder. The *normal remesher* provides a *normal mesh* $\mathcal{M}_{sr}$, from the irregular input one $\mathcal{M}_{ir}$. A N-level *unlifted butterfly* wavelet transform is then applied to obtain the subbands of wavelet coefficients. This scheme corresponds to the optimal wavelet transform for the *normal meshes* [2]. The sets of tangential and normal



(a) A finer mesh $\mathcal{M}_1$ is obtained from the coarse mesh $\mathcal{M}_0$ and a detail $\vec{d}_{1,0}$.

(b) The finest mesh $\mathcal{M}_{sr}$ is obtained from $\mathcal{M}_1$ and the details $\vec{d}_{2,0}$ and $\vec{d}_{2,1}$.

Fig. 1. A *normal mesh* is obtained by successive connectivity subdivision of a coarse mesh, and some details depending on the normals at the surface.

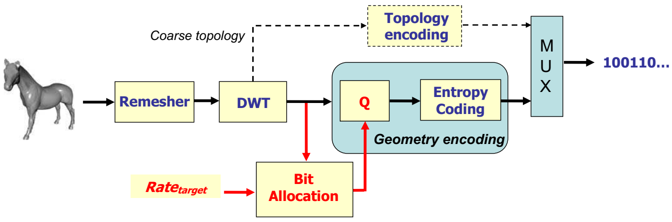


Fig. 2. Overall scheme of the proposed coder.

components are now encoded with an uniform scalar quantizer depending on the allocation process, and an entropy coder adapted to the multiresolution mesh [5]. In parallel, the connectivity of the coarse mesh is encoded with the lossless coder of Touma and Gotsman [9]. Hence, we obtain the quantized normal mesh $\hat{\mathcal{M}}_{sr} = (\hat{\mathcal{V}}_{sr}, \mathcal{T}_{sr})$, with $\hat{\mathcal{V}}_{sr}$ the set of quantized vertices.

III. Choice of the Distortion Measure

Since a remeshing technique is included in the proposed coder, we choose as distortion measure D_T the energy of the S2S distance between the irregular input mesh $\mathcal{M}_{ir}$ and the quantized *normal* mesh $\hat{\mathcal{M}}_{sr}$:

$$D_T = d_S(\mathcal{M}_{ir}, \hat{\mathcal{M}}_{sr})^2 \qquad (1)$$

where $d_S(.,.)$ represents the S2S distance.

A. Definition of the surface-to-surface distance

The S2S distance between $\mathcal{M}_{ir}$ and $\hat{\mathcal{M}}_{sr}$ is defined by

$$d_S(\mathcal{M}_{ir}, \hat{\mathcal{M}}_{sr}) = \max[\bar{d}(\mathcal{M}_{ir}, \hat{\mathcal{M}}_{sr}); \bar{d}(\hat{\mathcal{M}}_{sr}, \mathcal{M}_{ir})], \qquad (2)$$

where $\bar{d}(\mathcal{M}, \mathcal{M}')$ is the *unilateral distance* between two meshes, given by

$$\bar{d}(\mathcal{M}, \mathcal{M}') = \left(\frac{1}{|\mathcal{M}|} \iint_{p \in \mathcal{M}} d(p, \mathcal{M}')^2 d\mathcal{M} \right)^{\frac{1}{2}}. \qquad (3)$$

$|\mathcal{M}|$ is the area of $\mathcal{M}$, and $d(p, \mathcal{M}')$ is the distance between a point p belonging to a surface represented by a mesh $\mathcal{M}$

TABLE I

Mean difference between $\bar{d}(\mathcal{M}_{sr}, \hat{\mathcal{M}}_{sr})$ and $\bar{d}(\hat{\mathcal{M}}_{sr}, \mathcal{M}_{sr})$ according to the bitrate per irregular vertex (b/iv), computed for 5 typical models (Horse, Rabbit, Venus, Skull and Feline).

Bitrate (b/iv)	< 1	1-2	2-6	6-10	> 10
$\bar{d}(\mathcal{M}_{sr}, \hat{\mathcal{M}}_{sr})$	1.07e-1	1.30e-2	5.34e-3	1.72e-3	1.43e-3
$\bar{d}(\hat{\mathcal{M}}_{sr}, \mathcal{M}_{sr})$	1.02e-1	1.28e-2	5.34e-3	1.72e-3	1.43e-3
Difference (%)	3.680	1.890	0.291	0.261	0.074

and the surface represented by a mesh $\mathcal{M}'$. This distance is defined by

$$d(p, \mathcal{M}') = \min_{p' \in \mathcal{M}'} ||p - p'||_2 = ||p - \text{Proj}_{\mathcal{M}'}(p)||_2 \qquad (4)$$

with $||.||_2$ the L_2-norm, and $\text{Proj}_{\mathcal{M}'}(p)$ the orthogonal projection of p over $\mathcal{M}'$. To avoid a real computation of the S2S distance during the bit allocation, which is a computationally intensive process, we propose to approximate this distortion measure.

B. Proposed approximation of the surface-to-surface distance

First, the normal remesher provides that the irregular mesh $\mathcal{M}_{ir}$ and the *normal mesh* $\mathcal{M}_{sr}$ are visually very similar. The S2S distance between them is thus negligible, and (1) can be approximated by

$$\begin{aligned} D_T &\simeq d_S(\mathcal{M}_{sr}, \hat{\mathcal{M}}_{sr})^2 \\ &\simeq \max[\bar{d}(\mathcal{M}_{sr}, \hat{\mathcal{M}}_{sr})^2; \bar{d}(\hat{\mathcal{M}}_{sr}, \mathcal{M}_{sr})^2] \end{aligned} \qquad (5)$$

Let us study the difference of "symmetry" between the distances $\bar{d}(\mathcal{M}_{sr}, \hat{\mathcal{M}}_{sr})$ and $\bar{d}(\hat{\mathcal{M}}_{sr}, \mathcal{M}_{sr})$. Table I presents a mean of the relative errors between these two distances, computed on 5 typical models, and according to different bitrate ranges. The difference being very low ($< 4\%$) for each bitrate range, we can assume that $\bar{d}(\mathcal{M}_{sr}, \hat{\mathcal{M}}_{sr}) \simeq \bar{d}(\hat{\mathcal{M}}_{sr}, \mathcal{M}_{sr})$, and we can approximate (5) by

$$\begin{aligned} D_T &\simeq \bar{d}(\hat{\mathcal{M}}_{sr}, \mathcal{M}_{sr})^2 \\ &\simeq \frac{1}{|\hat{\mathcal{M}}_{sr}|} \iint_{p \in \hat{\mathcal{M}}_{sr}} d(p, \mathcal{M}_{sr})^2 d\hat{\mathcal{M}}_{sr}. \end{aligned} \qquad (6)$$

A *normal mesh* being densely sampled, the integral in (6) can be numerically approximated by a discrete sum [10], and (6) becomes

$$D_T \simeq \frac{1}{|\hat{\mathcal{V}}_{sr}|} \sum_{\hat{v} \in \hat{\mathcal{V}}_{sr}} d(\hat{v}, \mathcal{M}_{sr})^2. \qquad (7)$$

with $|\hat{\mathcal{V}}_{sr}|$ the number of vertices of $\mathcal{M}_{sr}$. Now, we have to approximate $d(\hat{v}, \mathcal{M}_{sr}) = ||\hat{v} - \text{Proj}_{\mathcal{M}_{sr}}(\hat{v})||_2$. Let us introduce the quantization error vector $\vec{\epsilon}(v)$ between a vertex v and its quantized version $\hat{v}$. Asymptotically, *i.e*, for optimal high bitrate coding, this error vector is mostly colinear to the normal at the surface $\mathcal{M}_{sr}$ in v, since most of tangential components of a *normal mesh* are infinitesimal [8]. Also,

$\text{Proj}_{\mathcal{M}_{sr}}(\hat{v})$ being the orthogonal projection of $\hat{v}$ over $\mathcal{M}_{sr}$, we can make the approximation: $\text{Proj}_{\mathcal{M}_{sr}}(\hat{v}) \simeq v$ [11]. Finally, we can state that $||\hat{v} - \text{Proj}_{\mathcal{M}_{sr}}(\hat{v})||_2 \simeq ||\hat{v} - v||_2 = ||\vec{\epsilon}(v)||_2$, and (7) becomes

$$D_T \simeq \frac{1}{|\mathcal{V}_{sr}|} \sum_{v \in \mathcal{V}_{sr}} ||\vec{\epsilon}(v)||_2^2. \quad (8)$$

The right part of (8) is the MSE σ^2_{Qsr} related to the *normal mesh* vertices. Finally, in case of densely sampled meshes, the energy of the S2S distance between the input mesh and the quantized normal one can be approximated by the MSE related to the quantization of the *normal mesh* geometry:

$$\boxed{D_T = d_S(\mathcal{M}_{ir}, \hat{\mathcal{M}}_{sr})^2 \simeq \sigma^2_{Qsr}.} \quad (9)$$

To design a fast and low-complexity bit allocation for a wavelet coder, this approximation has to be expressed in function of the wavelet coefficient subbands.

C. MSE across a wavelet coder

In [12], we have shown that the MSE on a 3D mesh encoded across a wavelet coder can be written as

$$\sigma^2_{Qsr} = \sum_{i=0}^{N} w_i \sigma^2_{Qi} \quad (10)$$

with σ^2_{Qi} the MSE related to the subband i, and w_i the weight due to the biorthogonality of the wavelet transform. To distinguish the tangential information from the normal one in the wavelet coefficient of a normal mesh [8], each subband of high frequency coefficients is treated as 2 independent subsets: the *tangential set*, defined by the set of coordinates x and y of the wavelet coefficients, and the *normal set* defined by the set of z-coefficients [12], [11], [7]. On the other hand, the low frequency wavelet coefficients are splitted in three scalar sets and encoded thanks to a differential coding [13]. Hence, the MSE σ^2_{Qsr} related to the normal mesh geometry across a N-level wavelet coder can be rewritten as

$$\sigma^2_{Qsr} = \sum_{i=0}^{N} w_i \sum_{j \in J_i} \sigma^2_{Qi,j} \quad (11)$$

where J_i ($i \neq 0$) is the index set of a high frequency subband i defined by $J_i = \{1, 2\}$, and J_0 the index set of the low frequency subband defined by $J_0 = \{1, 2, 3\}$. For $i \neq 0$, $\sigma^2_{Qi,1}$ and $\sigma^2_{Qi,2}$ are respectively the MSE of the tangential and normal sets of the i^{th} wavelet subband, and $\sigma^2_{Q0,j}$ is the MSE due to quantization of the j^{th} coordinate set of the low frequency coefficients.

IV. Model-based Bit allocation

A. Problem statement and solutions

The idea of the bit allocation across the wavelet coefficient subbands is to perform the best quantization of the coefficients optimizing the rate-distortion trade-off. The general purpose of the bit allocation process is precisely to *determine the best set of quantization steps* $\{q_{i,j}\}$ *that minimizes the reconstruction error* D_T*, at a given rate* R_{target}. This can be formulated by the following problem:

$$(\mathcal{P}) \begin{cases} \text{minimize} & D_T(\{q_{i,j}\}) \\ \text{under constraint} & R_T(\{q_{i,j}\}) = R_{target} \end{cases}$$

By using a lagrangian criterion and the distortion measure (11), this constrained allocation problem can be written as

$$J_\lambda(\{q_{i,j}\}) = \sum_{i=0}^{N} w_i \sum_{j \in J_i} \sigma^2_{Q_{i,j}}(q_{i,j}) + \lambda \left(\sum_{i=0}^{N} \sum_{j \in J_i} a_{i,j} R_{i,j}(q_{i,j}) - R_{target} \right), \quad (12)$$

with λ the lagrangian operator, $R_{i,j}(q_{i,j})$ the bitrate related to the i, j^{th} component set. The coefficients $a_{i,j}$ depend on the subsampling and correspond to $a_{i,j} = size(\{x_{i,j}\})/(3 \times |\mathcal{V}_{sr}|)$. The only way to allocate the bits in different subbands without pre-quantizing each subband is to perform a model-based bit allocation. This model-based bit allocation takes into account theoretical models for distortion and bitrate, depending on quantization steps and probability density functions of each data set. We have shown in [11] that distributions of tangential and normal component sets can be modeled by a *Generalized Gaussian Distribution* (GGD) [11]. Hence, according to [14], distortion and bitrate can be written as $\sigma^2_{Qi,j} = \sigma^2_{i,j}\ D(\tilde{q}_{i,j}, \alpha_{i,j})$ and $R_{i,j}(q_{i,j}) = R(\tilde{q}_{i,j}, \alpha_{i,j})$, with $\sigma^2_{i,j}$ and $\alpha_{i,j}$ respectively the variance and the GGD parameter of the i, j^{th} set, $\tilde{q}_{i,j} = \frac{q_{i,j}}{\sigma_{i,j}}$. $D(\tilde{q}_{i,j}, \alpha_{i,j})$ and $R(\tilde{q}_{i,j}, \alpha_{i,j})$ are simple functions also given by [14].

The solution of this constrained allocation problem is obtained by computing the derivatives of (12) with respect to the normalized quantization steps $\{\tilde{q}_{i,j}\}$ and λ:

$$\begin{aligned} w_i\ \sigma^2_{i,j} \frac{\partial D(\tilde{q}_{i,j}, \alpha_{i,j})}{\partial \tilde{q}_{i,j}} + \lambda a_{i,j} \frac{\partial R(\tilde{q}_{i,j}, \alpha_{i,j})}{\partial \tilde{q}_{i,j}} &= 0 \\ \sum_{i=0}^{N} \sum_{j \in J_i} a_{i,j} R(\tilde{q}_{i,j}, \alpha_{i,j}) &= R_{target}. \end{aligned}$$

Finally, we have to solve the following system of $(2N+4)$ equations with $(2N+4)$ unknowns:

$$h(\tilde{q}_{i,j}) = \frac{\frac{\partial D(\tilde{q}_{i,j}, \alpha_{i,j})}{\partial \tilde{q}_{i,j}}}{\frac{\partial R(\tilde{q}_{i,j}, \alpha_{i,j})}{\partial \tilde{q}_{i,j}}} = -\lambda \frac{a_{i,j}}{w_i \sigma^2_{i,j}} \quad (13a)$$

$$\sum_{i=0}^{N} \sum_{j \in J_i} a_{i,j} R(\tilde{q}_{i,j}, \alpha_{i,j}) = R_{target}. \quad (13b)$$

B. Model-based Algorithm

In order to solve the system (13) and to speed the bit allocation process up, Parisot *et al.* propose to use, in case of GGD, two precomputed tabulations of parametric curves $[R;\ ln(-h)]$ and $[q;\ ln(-h)]$, with h given by [14]. The first one permits to verify the constraint on the bitrate (13b), and the second one permits to compute the quantization step verifying (13a). In that case, the algorithm becomes:

1) λ is given. For each set i, j, compute the corresponding bitrate $R_{i,j}$ with precomputed tabulations of $[R;\ ln(-h)]$;
2) While (13b) is not verified, calculate a new λ by dichotomy and return to step 1;
3) For each set i, j, compute the optimal quantization step $q_{i,j}$ with precomputed tabulations of $[q;\ ln(-h)]$, and λ found in step 1;

The convergence of this algorithm is reached after few iterations, involving a fast and low-complexity process.

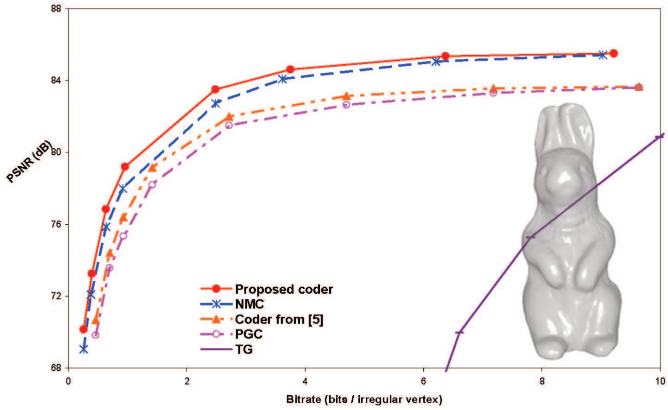


Fig. 3. Bitrate-PSNR curve for RABBIT

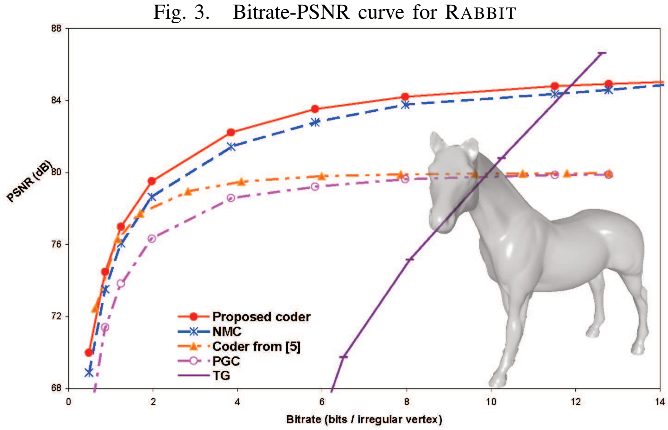


Fig. 4. Bitrate-PSNR curve for HORSE

V. RESULTS AND CONCLUSION

We compare the performances of the proposed coder with state-of-the-art coders: the multiresolution coders NMC of *Normal Mesh Compression* [2], PGC of *Progressive Geometry Compression* [15], the coder of [5] for "meshes from MAPS", and the single rate coder TG of Touma and Gotsman [9]. Fig. 3 and 4 show the resulting bitrate-PSNR curves for the models RABBIT and HORSE. The PSNR is related to the S2S distance between the irregular input mesh and the quantized normal one, normalized by the bounding box diagonal of the input mesh (computed with MESH [16]). The given bitrate is the number of bits per irregular vertex.

We observe that the proposed coder provides better results than all the state-of-the-art coders: our method provides slightly better results than NMC (up to +1.2 dB). As expected, the MSE is a good way to approximate the S2S distance between the input mesh and the reconstructed one when a normal remesher is used to obtain the semiregular mesh. Finally, we design an efficient wavelet coder for 3D meshes including a bit allocation that optimize the quantization of the wavelet coefficients according to a target bitrate.

Acknowledgments. Part of this work was supported by the IST programme of the EU in the project IST-2000-32795 SCHEMA. Models are courtesy of Cyberware. We are particularly grateful to Igor Guskov for providing us with his normal meshes, and his executable NMC.

REFERENCES

[1] P. Cignoni, C. Rocchini, and R. Scopigno, "Metro: Measuring error on simplified surfaces," *Computer graphics Forum*, vol. 2, no. 17, pp. 167–174, 1998.

[2] A. Khodakovsky and I. Guskov, "Normal mesh compression," *Geometric Modeling for Scientific Visualization, Springer-Verlag*, 2002.

[3] D. King and J. Rossignac, "Optimal bit allocation in 3D compression," *Journal of Computational Geometry, Theory and Applications*, 1999.

[4] Z. Karni and C. Gotsman, "Spectral compression of mesh geometry," *In ACM SIGGRAPH Conference Proceedings*, pp. 279–286, 2000.

[5] F. Payan and M. Antonini, "3D multiresolution context-based coding for geometry compression," in *Proceedings of IEEE International Conference in Image Processing (ICIP)*, september 2003.

[6] A. Lee, W. Sweldens, P. Schröder, P. Cowsar, and D. Dobkin, "MAPS: Multiresolution adaptive parametrization of surfaces," *SIGGRAPH*, 1998.

[7] S. Lavu, H. Choi, and R. Baraniuk, "Geometry compression of normal meshes using rate-distortion algorithms," in *Proceedings of the Eurographics/ACM SIGGRAPH symposium on Geometry processing*, 2003.

[8] I. Guskov, K. Vidimce, W. Sweldens, and P. Schröder, "Normal meshes," in *Siggraph 2000, Computer Graphics Proceedings*.

[9] C. Touma and C. Gotsman, "Triangle mesh compression," *Graphics Interface'98*, pp. 26–34, 1998.

[10] A. Gersho, "Asymptotically optimal block quantization," *IEEE Transactions on Image Theory*, no. 25, pp. 373–380, 1979.

[11] F. Payan and M. Antonini, "Model-based geometry coding of 3D multiresolution surface meshes," *special issue of CAGD journal on geometry processing*, 2004, Submitted.

[12] ——, "Mean square error for biorthogonal m-channel wavelet coder," *Trans. in Image Processing*, 2003, Submitted.

[13] ——, "Multiresolution 3D mesh compression," in *Proceedings of IEEE International Conference in Image Processing (ICIP)*, september 2002.

[14] C. Parisot, M. Antonini, and M. Barlaud, "3D scan based wavelet transform and quality control for video coding," *EURASIP journ. on Applied Signal Processing, Special issue on multimedia Signal Processing*, January 2003.

[15] A. Khodakovsky, P. Schröder, and W. Sweldens, "Progressive geometry compression," *Proceedings of SIGGRAPH*, 2000.

[16] N. Aspert, D. Santa-Cruz, and T. Ebrahimi, "Mesh: Measuring errors between surfaces using the hausdorff distance," in *Proceedings of the IEEE ICME*, vol. I, 2002, pp. 705 – 708.