Semi-Regular Triangle Remeshing: A Comprehensive Study

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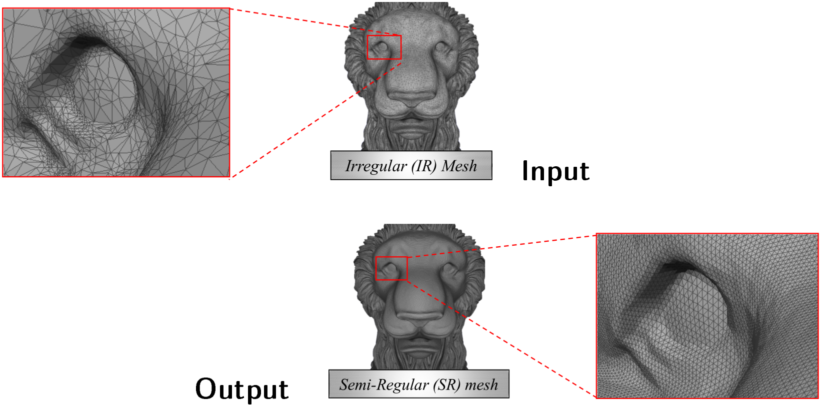
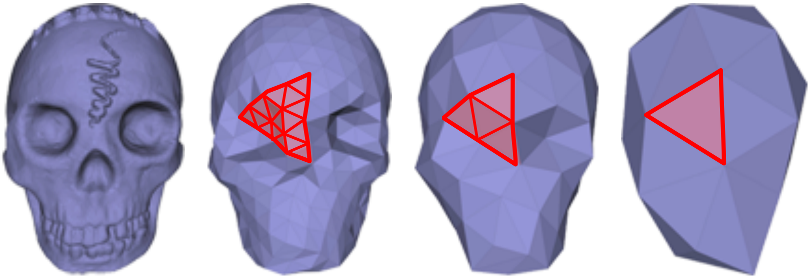


Goal: shape fidelity.

Semi-regular meshes



- A triangle mesh is semi-regular (SR) if the triangles can be merged by fours down to a low resolution mesh.
- It is a property of the mesh connectivity, sometimes called “subdivision connectivity”.
- Most vertices are regular (*i.e.* have valence 6).

Summary

1. Introduction
   - Context
   - Wavelet-based multi-resolution analysis
   - Overview of SR remeshing
2. Goals of SR remeshing
   - Shape fidelity
   - Quality of mesh elements
   - Compactness
3. Conclusions
   - Summary
   - Future works

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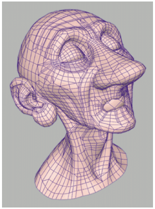
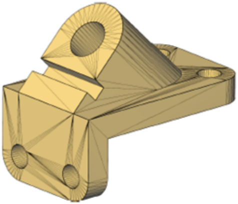
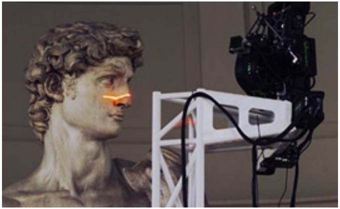
 →  → 

- 3D data → surfaces → meshes → triangle meshes.
- Triangle meshes are popular and widespread for computer graphics applications.

Context: large meshes are widespread

- Meshes are getting larger:
  - encouraged by applications;
  - supported by hardware.
- Progresses in all stages of the pipeline:
  - modeling, acquisition and reconstruction;
  - processing;
  - storage and transmission;
  - rendering.

The Digital Michelangelo Project



Goal: quality of the mesh elements.

Shape of the triangles, distribution of the vertices.

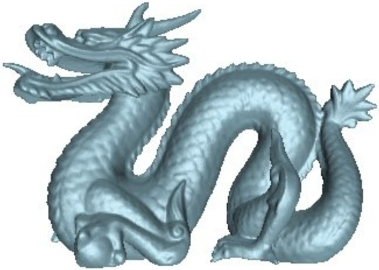
Context: large meshes are still challenging

- Limitations: storage, transmission, real-time rendering, etc.

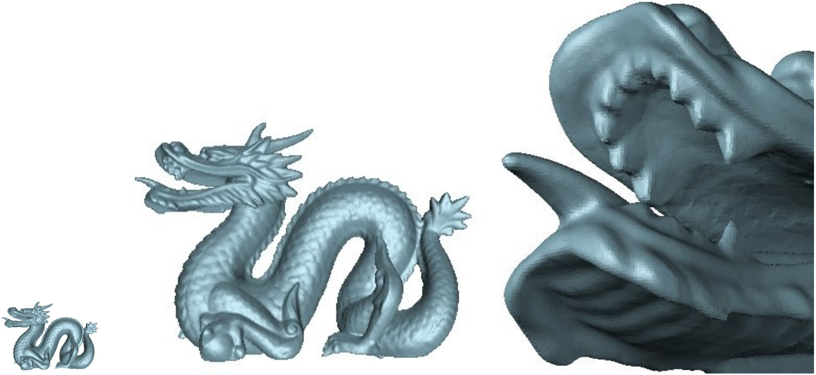
Goal: compactness of the output mesh.

It is a strength of SR meshes.

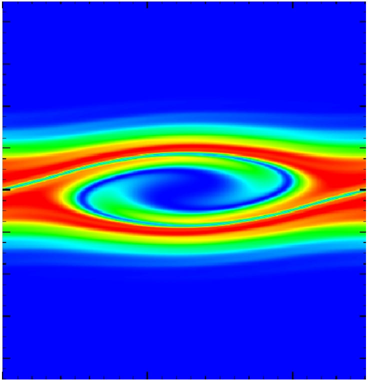
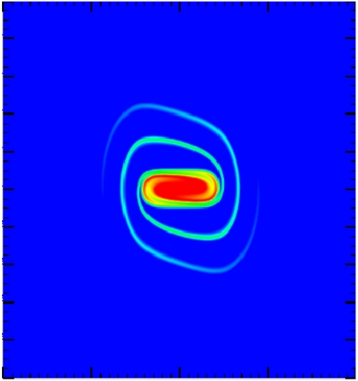
- Do you need so many details?
  It depends on the application.



When do you need details?



Where do you need details?

Plasma visualization: (position x velocity)-slice
Courtesy M. Haefele

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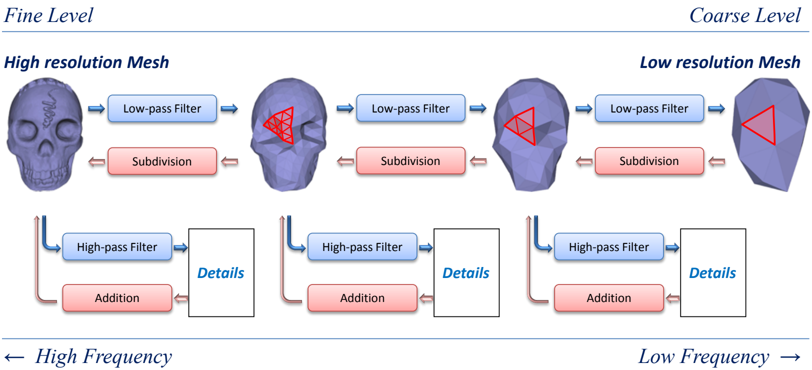
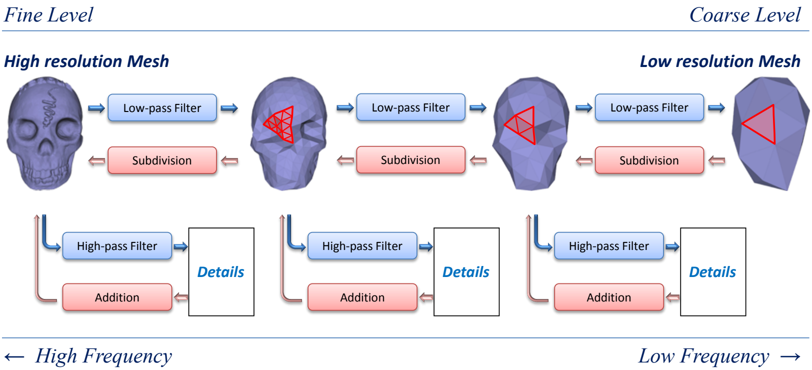
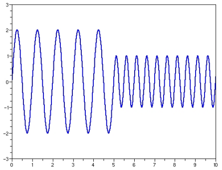
Multi-resolution analysis

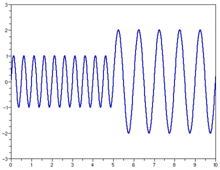


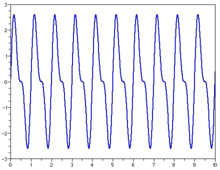
- Level-of-detail visualization and rendering [CPD*96].
- Progressive transmission [LKSS00];
- Geometry compression [KSS00, PA05];



time = space domain



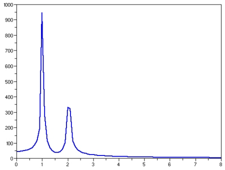




Fourier transform

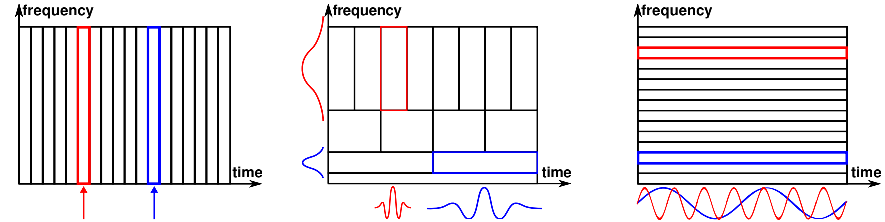
frequency domain

same norm =



From frequency to time-frequency analysis

$$s(t) = \sum_i c_i \phi_i(t)$$



Time

Time-frequency

Wavelet basis

Filterbank

Frequency

Fourier series

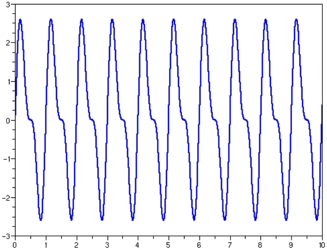
DFT

Embedding dimension

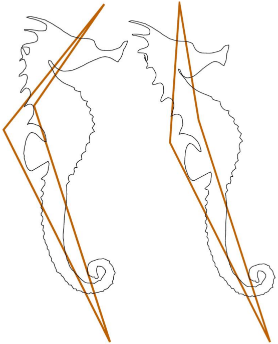
$$s(t) = \sum_i c_i \phi_i(t)$$

- Intrinsic dimension (parameter t): 1D.
- Embedding dimension (coefficients c_i):

1D sound



2D planar curve

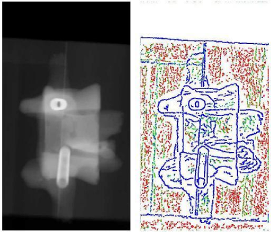


Intrinsic dimension

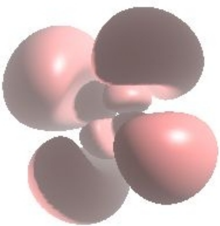
$$s(t) = \sum_i c_i \phi_i(t)$$

- Embedding dimension (coefficients c_i): 1D.
- Intrinsic dimension (parameter t):

2D image
Courtesy O. Le Cadet.



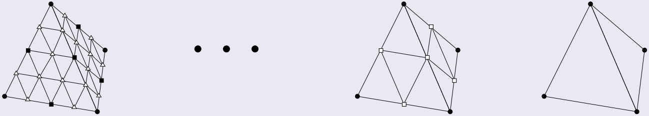
3D electronic density field
Courtesy C. Chauvin



Semi-regular meshes / subdivision connectivity

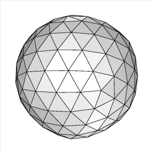
$$s(t) = \sum_i c_i \phi_i(t)$$

Topology: vertices (i) and 2D parameter domain (t)



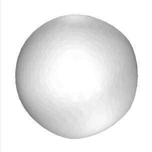
3D embedding ⇩ ⇩ Parameterization

Geometry (c_i)



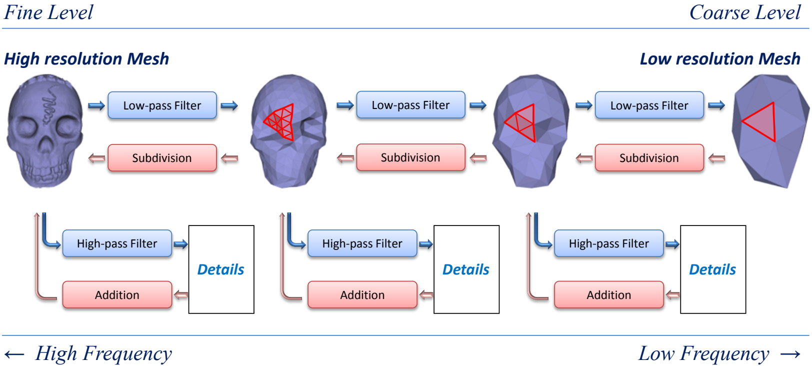
⇨ Subdivision

Limit surface (s)



Fresh look at the filterbank

$$s(t) = \sum_i c_i \phi_i(t)$$



Summary

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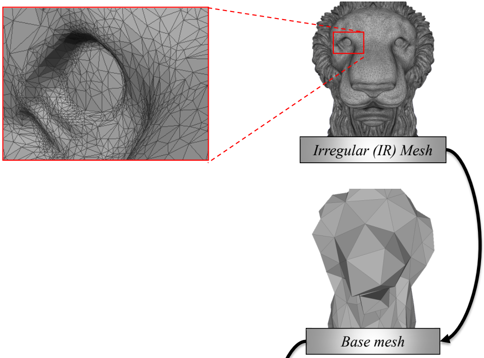
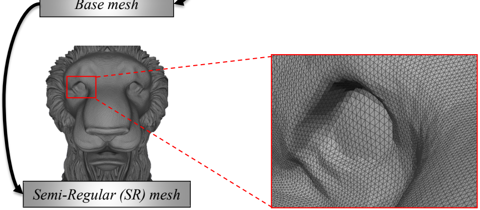
Building SR meshes

- Direct SR modeling.
- Meshing of other types of data.
- Re-meshing of irregular meshes.

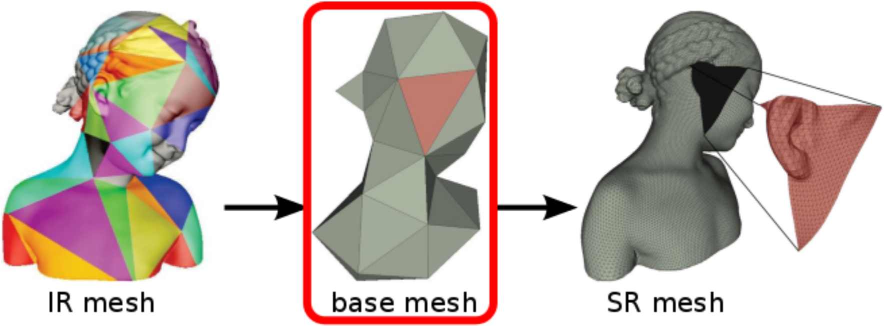


Step 1: Build the base mesh

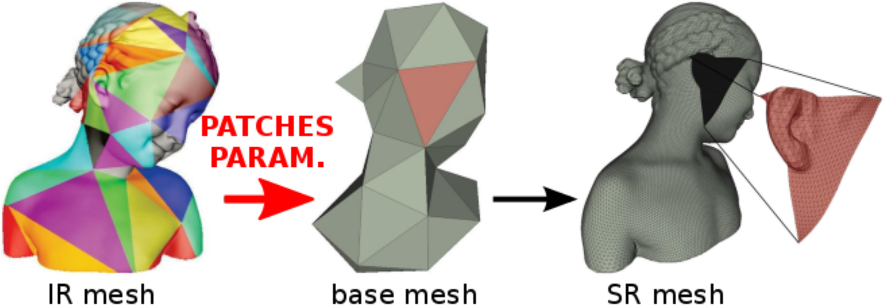
Step 2: Repeat
  Refine the SR mesh (1-to-4 split)
  Fit the SR mesh geometry to the IR mesh
Until the fine level is reached



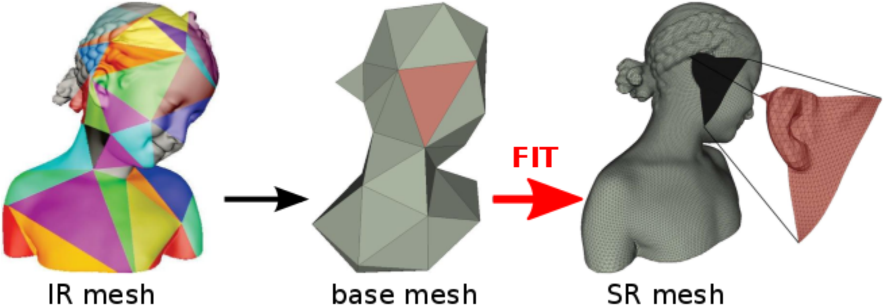
Component 1: construction of the base mesh



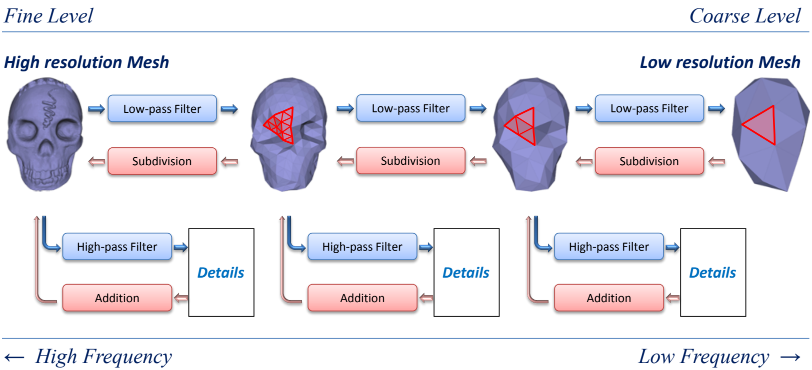
Component 2: mapping base mesh ⟶ IR mesh



Component 3: geometric fitting



Comparison with the filterbank



- The low resolution mesh (filterbank) is *not* the base mesh (remeshing).
- The low-pass filter (filterbank) is *not* the reverse of refinement + fitting (remeshing).

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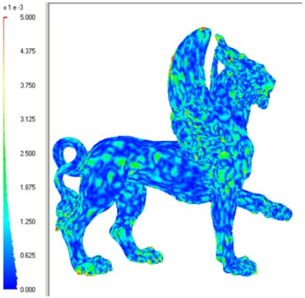
Shape fidelity : Definition

Definition

- **How a SR mesh approximates an IR mesh.**
- Minimize the **Remeshing Error (RE)**.



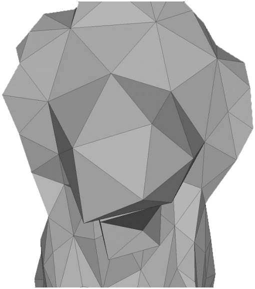
- Remeshing Error $= max(d(IR, SR), d(SR, IR))$
- with

$$d(X, Y) = \left(\frac{1}{area(X)} \int_{x \in X} d(x, Y)^2 dx\right)^{\frac{1}{2}},$$

Shape fidelity - Base mesh

Major rule

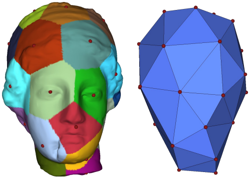
- **Base mesh has to be a coarse version of the IR mesh.**
- **Same topology** (same boundaries and genus).



Shape fidelity - Base mesh

Example

Mesh partitioning [EDD*95, KPA10, CJL11]



Example

Incremental simplification [LSS*98, GVSS00, KLS03]

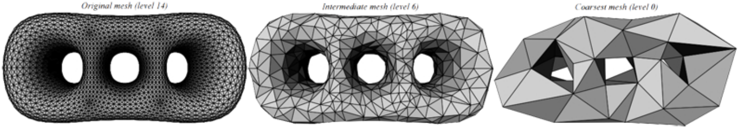


Figure: Image from [LSS*98]

Second rule

Preserving features improves the shape fidelity [Gio99, Gus07, CJL11].

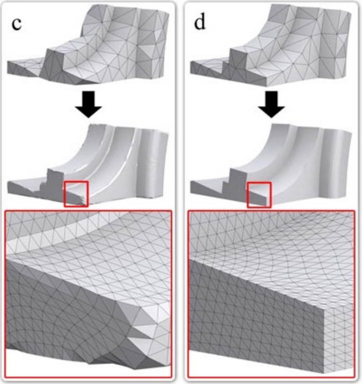


Figure: Image from [CJL11].

Example

Segmentation [CJL11]

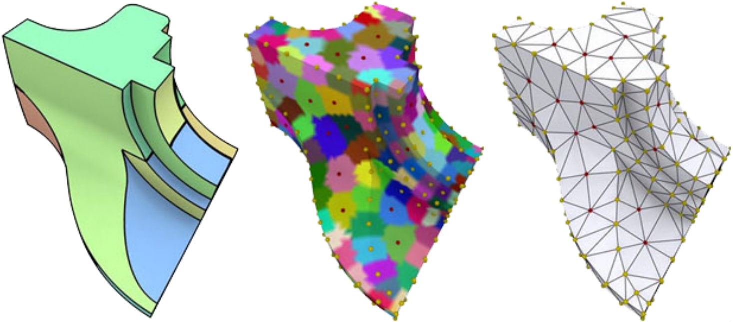


Figure: Segmentation, clustering/relaxation and triangulation.

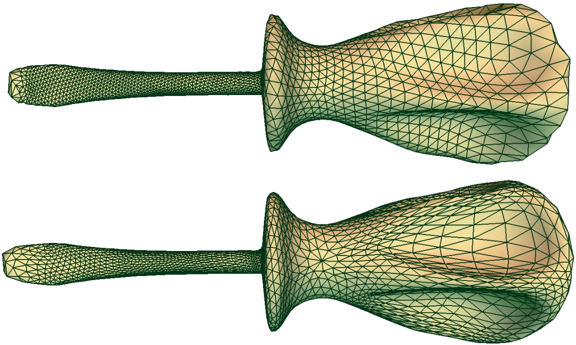
Shape fidelity - Minimize the error during fitting

Third rule

Minimize the error during refinement.

Example

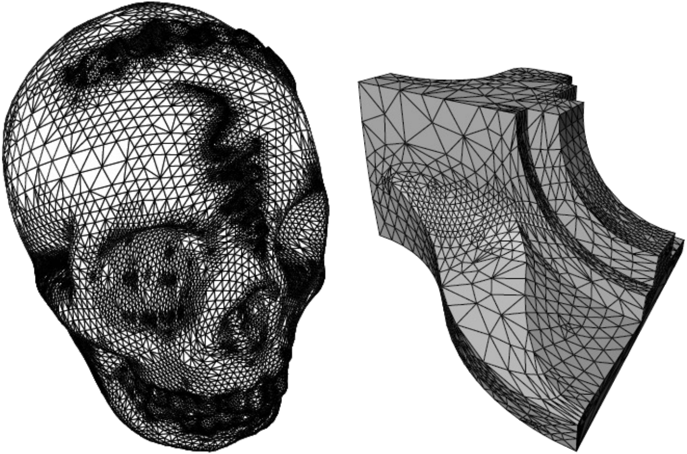
Anisotropic sampling [Gus07]



Shape fidelity - Minimize the error during fitting

Example

Adaptive sampling [LSS*98, GVSS00, HLG01, KPA10].



Shape fidelity - Minimize the error during fitting

Example

Minimizing the remeshing error at each level
[FSK04, Gus07, KPA10]

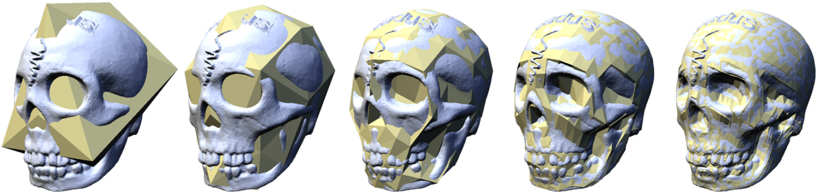


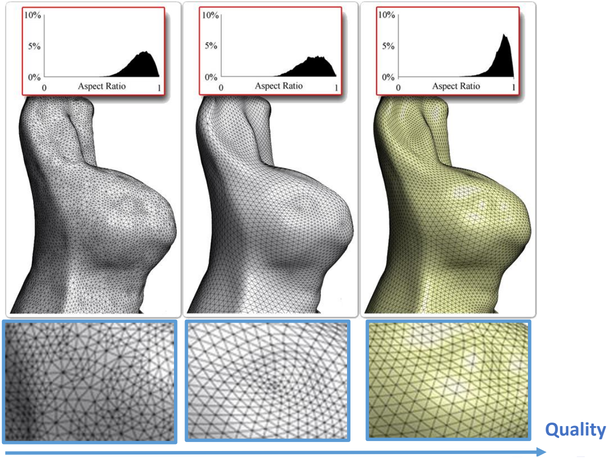
Figure: Image from [FSK04].

Summary

Quality of mesh elements

Objective

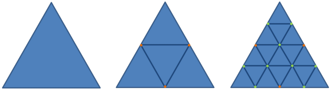
SR meshes with well-shaped triangles, and isotropic sampling with smooth gradation [AUGA08]



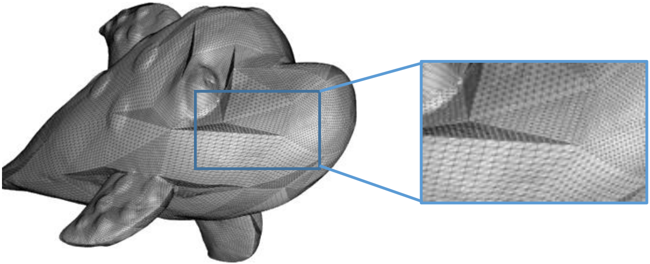
Quality of mesh elements - Semi-regularity

Starting point

Semi-regularity is well-suited to get *High Quality* (HQ) triangles.



But... it's not enough for SR remeshing of surfaces.



First rule

Build a High Quality base mesh...

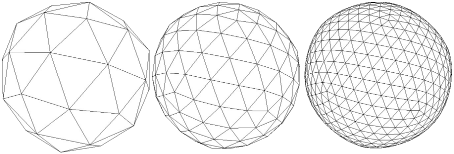


Figure: Image from [KVLS99].

But... it's **not enough with complex shapes.**

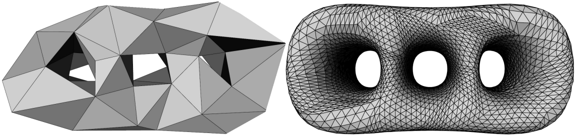


Figure: Image from [LSS*98].

Second rule

- **Patches have to be as "flat" as possible to preserve the intra-patch uniform sampling**.
- Easier if the the patch boundaries match sharp features.

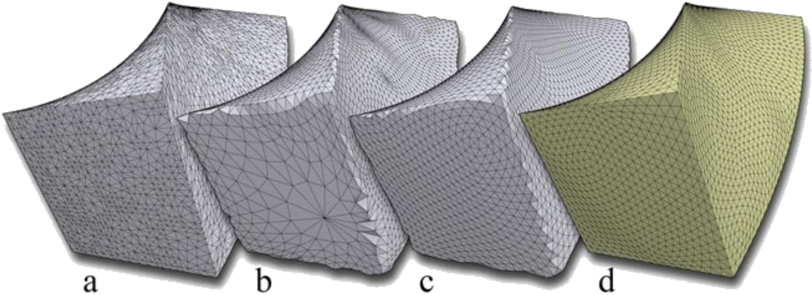


Figure: Image from [CJL11].

Quality of mesh elements - Patch boundaries

Third rule

Ensure a smooth gradation of the sampling along the patch boundaries.

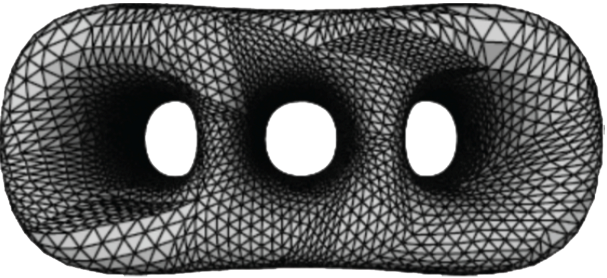


Figure: Images from [LSS*98].

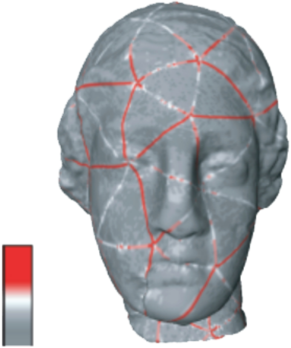


Figure: Images from [KLS03].

Quality of mesh elements - Patch boundaries

Example

Smoothing the sampling in the parametric domain
[LSS*98, KVLS99, HLG01];

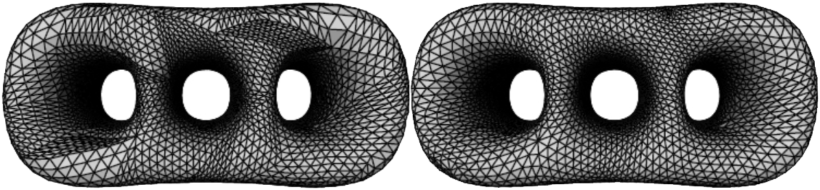


Figure: Images from [LSS*98].

Quality of mesh elements - Patch boundaries

Example

Using a **"globally smooth" parameterization** [KLS03];

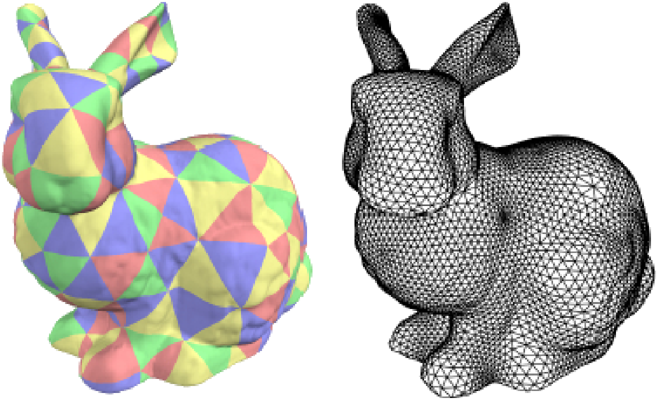
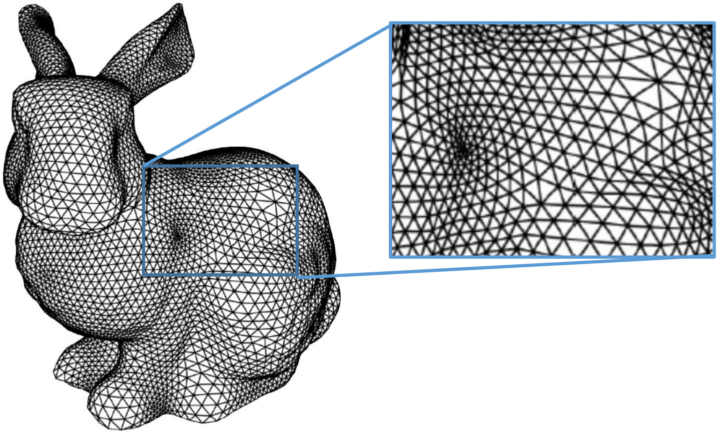


Figure: Images from [KLS03].

Quality of mesh elements - Extraordinary vertices

Last rule

Ensure a smooth gradation around the extraordinary vertices[Gus07, PTC10].



Quality of mesh elements - Extraordinary vertices

Example

Manifold-based parameterization around the extraordinary vertices [Gus07]

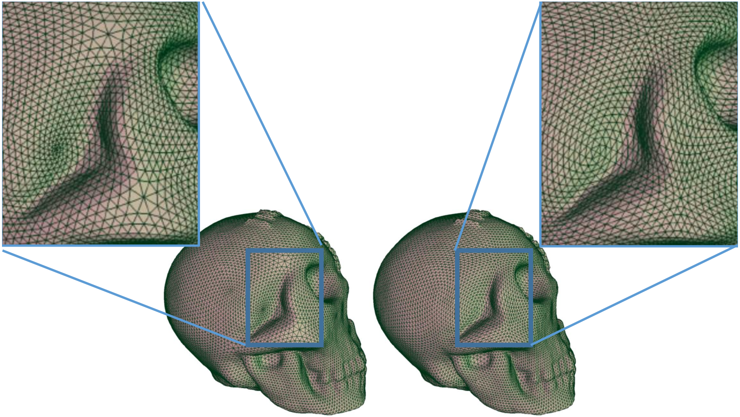


Figure: Images from [Gus07]

Quality of mesh elements - Extraordinary vertices

Example

Using an **almost isometric mesh parameterization** [PTC10].

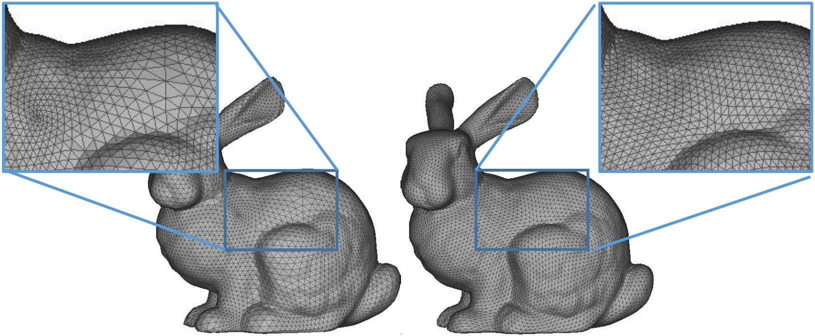
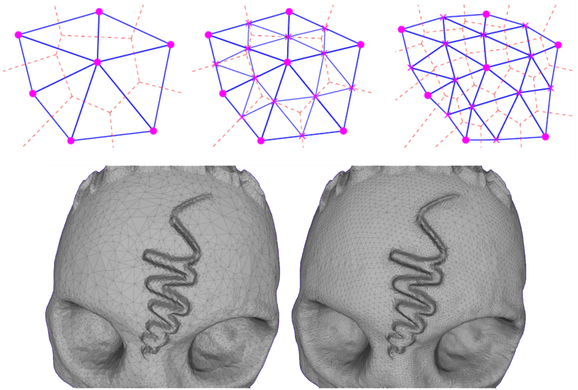


Figure: Images from [PTC10].

Example

Using a **Voronoi diagram and a relaxation** to distribute uniformly the vertices at each resolution [KPA10].



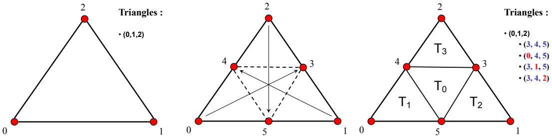
Summary

Definition

Compactness is the ability of a surface representation to encode large objects with few data.

How encoding the connectivity of SR meshes ?

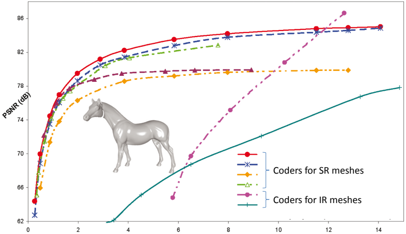
The **connectivity** of the SR meshes is **almost implicit**.



How encoding the geometry of SR meshes?

SR meshes support **efficient wavelet analysis**, that creates very sparse sets of wavelet coefficients.

"***Encoding SR meshes with wavelet-based algorithms decreases the reconstruction error by a factor 4 [...]*** , *compared to other progressive coding schemes*" [KSS00].



Bitrate-PSNR curves

- Bitrate: bits per IR vertex.
- $PSNR(dB) = 20log_{10}\frac{boundingboxdiagonal}{Recontructionerror}$

Compactness - Wavelet Coefficients

Characteristics of the Wavelet Coefficients

- **wavelet coefficients are 3D vectors computed in local frames**.
- Each coefficient is defined by a tangential component, and a normal component.
- Most of the **geometry information** is concentrated in the normal components.

Compactness - How improving the compactness?

Major Rule

Building SR meshes such as the future **sets of wavelet coefficients will be as sparse as possible**.

Example

- **Remove the tangential components of the future wavelet coefficients**, to get 1D coefficients.
- Position the SR vertices such as **the wavelet coefficients will be along the normals** [GVSS00, LMH00, LKK03, FSK04].

Example

- **Reduce the norm of the future wavelet coefficients**, to reduce the range of each set of wavelet coefficients.
- **Building a parameterization as smooth as possible** [KLS03].

Summary

- Relation between SR meshes, MR analysis, wavelets, remeshing.
- Goals:
  - shape fidelity,
  - quality of the mesh elements,
  - compactness.
- Components:
  - Building the base mesh.
  - Parameterization of IR mesh on the base mesh.
  - Geometric fitting of the SR to the IR.
- End-user? Trireme code available online [Gus07].

Further reading

- Our paper.

Ref.	Goals (contributions)			Components (methods)			Input and output (features)				Remarks
	Shape fidelity	Mesh quality	Compact-ness	Base mesh	Parametrization	Geom. fitting	Any Genus	Bnd.	Adapt.	Sharp Features	
[EDD*95]	+	+		MP	local harmonic	FI	√	√			
[LSS*98]	++	+		IS	conformal	FI	√	√	√	√	a.k.a. MAPS
[Gio99]	+		+	PP	local harmonic	FI	√	√		√	based on [EDD*95]
[KVLS99]		+		PP	implicit	FI					
[GVSS00]			++	IS	shape-preserving	FI	√		√		a.k.a. *normal meshes* or INM
[LMH00]			++	IS	no param.	FI			√		a.k.a. *displaced subd. surfaces*
[HLG01]	+	++		PP	MIPS	FI		√	√		
[KLS03]		++	+	IS	conformal	FI	√	√			based on [LSS*98], a.k.a. GSP
[LKK03]				IS	shape-preserving	FI	√	√			extension of [GVSS00] for boundaries
[FSK04]	++		+	N/A	N/A	A	√	√			param. as input, based on [GVSS00]
[AGL06]				IS	conformal	FI	√	√		√	OoC extension of [LSS*98]
[LYHL06]		+		PP	min. area disto.	FI	√	√			
[Gus07]	++	+	+	MP	mean-value	FI	√	√			a.k.a. TriReme, anisotropy
[PTC10]		++	+	IS	conformal/authalic mix	FI	√				
[KPA10]	+	+		MP	conformal	VI			√		
[DMS10]	+		+	N/A	N/A	VI					SR meshes as input
[CJL11]	+	++		MP	N/A	FI	√			√	

- Remeshing.
- Parameterization.

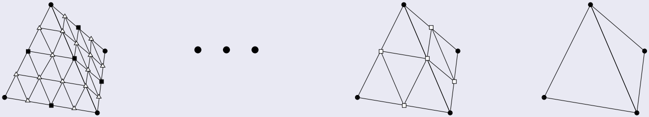
Summary

Benchmarking

- Few implementations available.
- No test dataset.
- No consensus on the quality measures.
- Timings and complexity.
- Robustness issues.

Don't confuse control mesh and limit surface

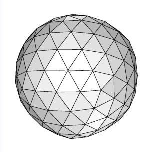
Topology: vertices (i) and 2D parameter domain (t)



3D embedding

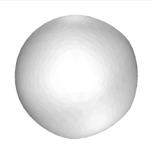
Parameterization

Geometry (c_i)



Subdivision

Limit surface (s)



Sharp features should be high frequencies

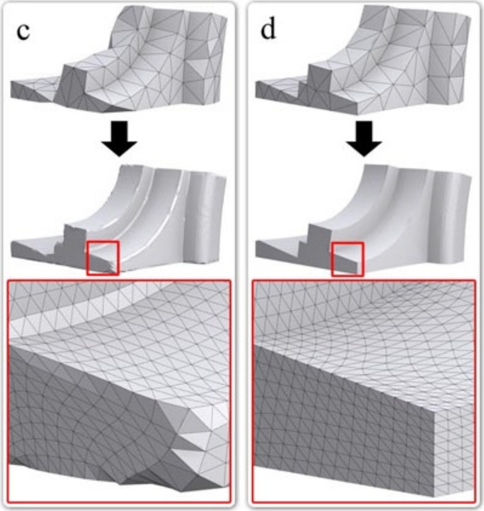
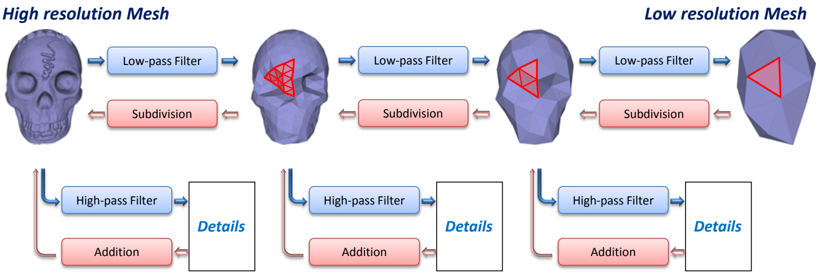


Image from [CJL11]

Should we bind the MR scheme with the mesh?

Fine Level *Coarse Level*

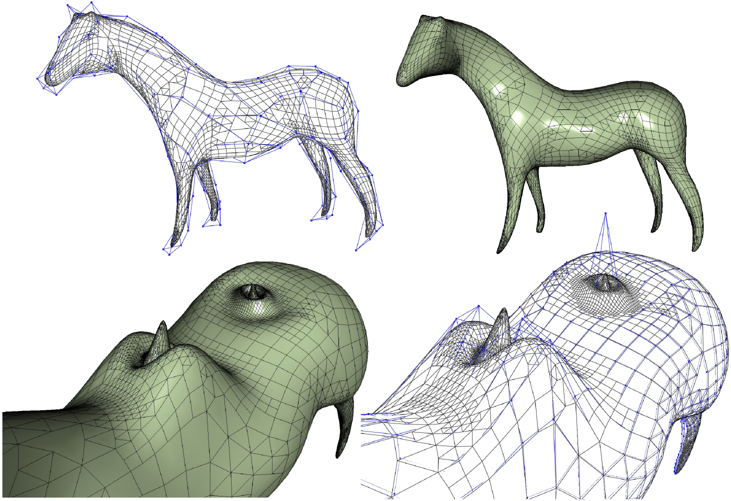


← *High Frequency* *Low Frequency* →

Guidance

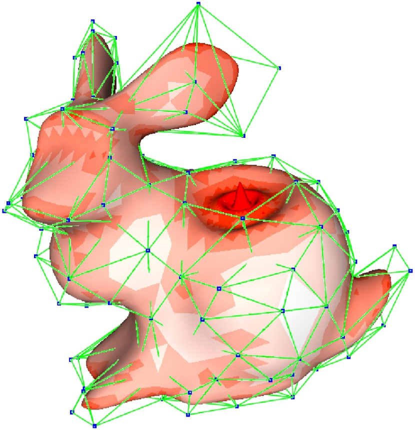
- Semi-automated: high-level user interaction.
- Automated: tangent vector fields.

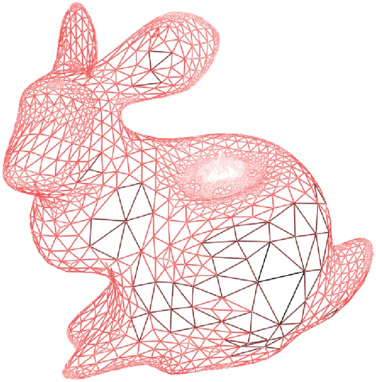
Mixed quad-triangle meshes



Courtesy P. Kraemer

$\sqrt{3}$ subdivision



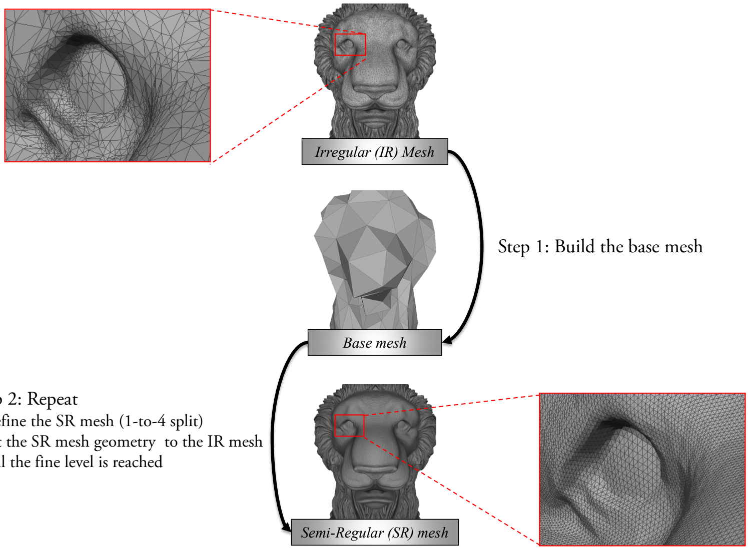


Courtesy P. Kraemer

Other possible improvements/perspectives

- Mesh attributes (color, normal, texture).
- Direct SR meshing.
- Parallel and out-of-core algorithms.

Questions ?



Step 1: Build the base mesh

Step 2: Repeat
- Refine the SR mesh (1-to-4 split)
- Fit the SR mesh geometry to the IR mesh

Until the fine level is reached

Bibliography I

[AUGA08] Alliez P., Ucelli G., Gotsman C., Attene M.:
Recent advances in remeshing of surfaces.
In *Shape Analysis and Structuring, Mathematics and Visualization* (2008), Springer.

[CJL11] Chiang C.-H., Jong B.-S., Lin T.-W.:
A robust feature-preserving semi-regular remeshing method for triangular meshes.
The Visual Computer 27, 9 (2011), 811–825.

[CPD*96] Certain A., Popovic J., DeRose T., Duchamp T., Salesin D., Stuetzle W.:
Interactive multiresolution surface viewing.
In *Proceedings of ACM SIGGRAPH'96* (1996), pp. 91–98.

Bibliography II

[EDD*95] Eck M., DeRose T., Duchamp T., Hoppe H., Lounsbery M., Stuetzle W.:
Multiresolution analysis of arbitrary meshes.
In *Proceedings of ACM SIGGRAPH'95* (1995), pp. 173–182.

[FSK04] Friedel I., Schröder P., Khodakovsky A.:
Variational normal meshes.
ACM Transactions on Graphics 23, 4 (2004), 1061–1073.

[Gio99] Gioia P.:
Reducing the number of wavelet coefficients by geometric partitioning.
Computational Geometry: Theory and Applications 14, 1-3 (1999), 25–48.

[Gus07] Guskov I.:
Manifold-based approach to semi-regular remeshing.
Graphical Models 69, 1 (2007), 1–18.

[GVSS00] Guskov I., Vidimce K., Sweldens W., Schröder P.:
Normal meshes.
In *Proceedings of ACM SIGGRAPH'00* (2000), pp. 95–102.

[HLG01] Hormann K., Labsik U., Greiner G.:
Remeshing triangulated surfaces with optimal parameterizations.
Computer-Aided Design 33, 11 (2001), 779–788.

Bibliography IV

[KLS03] Khodakovsky A., Litke N., Schröder P.:
Globally smooth parameterizations with low distortion.
In *Proceedings of ACM SIGGRAPH'03* (2003), pp. 350–357.

[KPA10] Kammoun A., Payan F., Antonini M.:
Adaptive semi-regular remeshing: A voronoi-based approach.
In *Proceedings of IEEE International Workshop on Multimedia Signal Processing* (2010), pp. 350–355.

[KSS00] Khodakovsky A., Schröder P., Sweldens W.:
Progressive geometry compression.
In *Proceedings of ACM SIGGRAPH'00* (2000), pp. 271–278.

Bibliography V

[KVLS99] Kobbelt L., Vorsatz J., Labsik U., Seidel H.-P.:
A shrink wrapping approach to remeshing polygonal surfaces.
Computer Graphics Forum 18, 3 (1999), 119–130.

[LKK03] Lee K.-Y., Kang S.-C., Kim T.-W.:
Remeshing into normal meshes with boundaries using subdivision.
Computers in Industry 50, 3 (2003), 303–317.

[LKSS00] Labsik U., Kobbelt L., Schneider R., Seidel H.-P.:
Progressive transmission of subdivision surfaces.
Computational Geometry 15, 1-3 (2000), 25–39.

[LMH00] Lee A. W. F., Moreton H., Hoppe H.:
Displaced subdivision surfaces.
In *Proceedings of ACM SIGGRAPH'00* (2000), pp. 85–94.

[LSS*98] Lee A. W. F., Sweldens W., Schröder P., Cowsar L., Dobkin D.:
MAPS: multiresolution adaptive parameterization of surfaces.
In *Proceedings of ACM SIGGRAPH'98* (1998), vol. 32, pp. 95–104.

[PA05] Payan F., Antonini M.:
An efficient bit allocation for compressing normal meshes with an error-driven quantization.
Computer Aided Geometric Design - Special issue on Geometry Mesh Processing 22, 5 (2005), 466–486.

[PTC10] Pietroni N., Tarini M., Cignoni P.:
Almost isometric mesh parameterization through abstract domains.
IEEE Transaction on Visualization and Computer Graphics 16, 4 (2010), 621–635.