Dynamic systems in search and optimisation*

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Abstract—Even in simple cases, algorithms that are simply convergent can be considered as dynamic systems, after a suitable transformation is applied to each iteration. The main idea is to renormalise the region containing the search object x^*, for instance the point where a given function $f(.)$ is optimum, so that the target remains in a standard region $\mathcal{R}_0$. Therefore x^* is no longer fixed but moves in $\mathcal{R}_0$ at each iteration, and follows the evolution of a dynamic system. Various algorithms are considered, e.g. line search, ellipsoid, steepest descent. The associated dynamic systems show different features: in some cases the asymptotic behaviour depends on x^* and/or on the function $f(.)$, and periodic and strange attractors are typical. A main purpose is to use the theory of dynamic systems to analyse the asymptotic behaviour of the algorithms, and, when possible, to obtain optimal rates of convergence.

1 Renormalisation

There exist well known applications of dynamical systems to the study of optimisation and search algorithms. A classical example is the study of the Newton map for the cube roots of unity, where the boundaries between the basins of attraction to the roots form a fractal. The present work develops another approach, in which the main ingredient is *renormalisation*. The idea can be explained as follows.

Certain kinds of search and optimisation problems to find a target point $x^* \in \mathcal{R}$ generate a nested sequence of regions $\mathcal{R}_i$, of the same shape, containing x^*

$$\mathcal{R} \subseteq \mathcal{R}_1 \supseteq \mathcal{R}_2 \supseteq \ldots .$$

Convergence to x^* takes place if the diameter of the sets converges in some metric to zero. The nesting of the regions occurs because additional observations or data are available at each stage creating additional assumptions on the set $\mathcal{R}_n$ of allowable, or

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