Section–invariant numbers and generalised Goden Section optimization algorithms*

Anatoly A. Zhigljavsky†, Luc Pronzato‡, Henry P. Wynn§

Abstract.

In the present work the self–similarity property of the Golden Section is generalised to the "section–invariant numbers", collections of numbers which remain invariant under the left/right interval deletion used in the Golden Section method. This yields a family of line–search algorithms that often have faster convergence rates than the Golden Section and Fibonacci algorithms. A technique to study different performance characteristics of the convergence of such algorithms is developed and applied to a particular algorithm of the family.

Keywords. Golden Section, Fibonacci numbers, line–search, dynamic systems.

1 Section–invariant numbers

An ordered m-collection of numbers $\mathcal{U} = \{u_1, \ldots, u_m\}$ is called *section–invariant* if $0 < u_1 < \ldots < u_m < 1$ and there exists an associated m-collection of numbers

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†Dept of Mathematics, St.Petersburg University, Bibliotechnaya sq. 2, 198904, Russia

‡Laboratoire I3S, CNRS-URA 1376, Sophia Antipolis, 06560 Valbonne, France.

§Dept of Statistics, University of Warwick, Coventry, CV4 7AL, UK