SEQUENTIAL DESIGN AND ACTIVE CONTROL

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In nonlinear situations, optimal experimental conditions generally depend upon unknown parameters to be estimated from the data collected during the experiments. A natural approach then consists in designing the experiments sequentially, that is, alternating estimation and design phases. Each design phase can be considered as a control action applied on the system. Sequential design thus corresponds to adaptive control, with, e.g., the precision of estimation as objective. Even for a purely static system, designing the experiments sequentially introduces a feedback of information, which induces dynamics into the design procedure. Several sequential schemes, corresponding to different control policies, are considered. The optimal one corresponds to closed–loop control and is the solution of a stochastic dynamic–programming problem, which is extremely difficult to solve even in very simple cases. Suboptimal strategies are thus proposed. Examples for nonlinear regression models are presented.

1. Introduction. In nonlinear situations the optimal experimental conditions generally depend upon parameters to be estimated during the experiments. Sequential design provides a method of circumventing this issue: after each observation, or each sample of n observations, the parameters are estimated, and this information is used to design the experiment for the next observation(s). One can refer to Chernoff (1972) for a monograph on sequential analysis, including the construction of stopping rules. Since stopping rules will not be discussed here, one may prefer to call the designs considered "adaptive". We shall, however, call them "sequential" since this seems to be common practice. The design phases can then be considered as control actions applied on the system, the control objective being for instance the precision of estimation. The dependence of the k-th experiment on previous observations introduces a feedback of information, and thus induces dynamics into the design procedure, even in cases where the process under study is purely static. In this sense, one can think about the k-th design step as the design of the experiment to be performed at *time* k, and sequential design can be considered as an adaptive–control problem. For instance, the usual non–Bayesian myopic approach of sequential design can be considered as *Forced Certainty Equivalence* control, see Pronzato, Walter and Kulcsár (1993), Runggaldier (1993). Relations between experimental design and control are briefly

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