Improving on Golden-Section optimisation for locally symmetric functions

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Abstract: We consider the minimisation of a uniextremal function $f(.)$ on $[0,1]$ using a "second-order" algorithm. At each iteration the current feasible region is rescaled to $[0,1]$, so that the optimizing value x^* in the initial $[0,1]$-interval varies from iteration to iteration, which defines a dynamic system. Many line-search algorithms exhibit chaotic behaviour when rescaling is applied. If $f(.)$ is symmetric around x^*, the associated dynamic system is time-homogeneous and often possesses an invariant density. In a first part, we show that the asymptotic behaviour of the classical Golden-Section algorithm is the same for locally symmetric functions as for pure symmetric functions. We believe that this property is also true for other line-search algorithms, with sometimes a better ergodic rate than the Golden-Section algorithm. In a second part, we consider the case where the number of iterations is fixed *a priori*, with a dynamic-programming approach, using a uniform prior density on $[0,1]$ for x^*.

Key words: Golden-Section, line search, ergodic behaviour, dynamic system, dynamic programming.

1 Introduction

We consider the minimisation of a uniextremal function $f(.)$ on $[0,1]$ using a "second-order" algorithm, which can be defined as follows. Let $I_n = [a_n, b_n]$ be the current interval at iteration n. Two points x' and x'' are then placed between a_n and b_n: $a_n \leq x' < x'' \leq b_n$. If $f(x') < f(x'')$, then I_n is reduced to $[x', b_n]$ because there is no minimizer in $[a_n, x']$, and conversely if $f(x') \geq f(x'')$ then I_n is reduced to $[a_n, x'']$. One can note that in both cases the next interval I_{n+1} contains either x' or x'', so that $f(.)$ only needs be evaluated once at each iteration. The way this evaluation is chosen defines the algorithm. For a more detailed introduction see for example [7].

The *Golden Section algorithm* selects x', x'' symmetrically placed between a_n and b_n, that is

$$x' + x'' = a_n + b_n \, ,$$

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