Sequential selection of observations in randomly generated experiments

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Abstract

We consider a situation where experimental conditions are observed sequentially but are uncontrolled. The length of the observed sequence is N, and only $n < N$ experiments can be performed. The problem is to select on–line the most informative ones. More precisely, we assume that parameters $\boldsymbol{\theta} \in I\!\!R^p$ have to be estimated in a regression model with observations $y_k = \eta(\boldsymbol{\theta}, \xi_k) + \epsilon_k$, with $\{\epsilon_k\}$ a sequence of i.i.d. errors. The experimental conditions ξ_k are i.i.d. random variables, independent of $\{\epsilon_k\}$ and are observed sequentially. The decision of observing y_k or not must be made on–line. The criterion to be maximized is $\det[\boldsymbol{\Omega}^{-1} + \mathbf{M}]$, with $\mathbf{M}$ the Fisher information matrix, calculated at a prior nominal value $\hat{\boldsymbol{\theta}}^0$ of the model parameters, and $\boldsymbol{\Omega}$ a given positive–definite matrix. When θ is a scalar parameter the problem corresponds to the maximization of the sum of n i.i.d. random variables $x_k = [z(\hat{\theta}^0, \xi_k)]^2$, with $z(\theta, \xi) = \partial\eta(\theta, \xi)/\partial\theta$, which are observed sequentially in a sequence of length N. The optimal decision rule, which is solution of a stochastic dynamic–programming problem, is is compared to the simpler (but suboptimal) open–loop feedback–optimal rule. The situation is far more complex when $\boldsymbol{\theta}$ is p-dimensional, because the information is not additive. However, the computation of the expected value of the determinant of the information matrix allows one to construct the open–loop feedback–optimal rule. Forced certainty-equivalence is proposed for the case where the distribution of the ξ_k's is unknown and estimated on–line. Certainty equivalence is also forced in the case where θ can be estimated on–line: at step k, the prior nominal value $\hat{\theta}^0$ is replaced by an estimate $\hat{\boldsymbol{\theta}}^{k-1}$. Illustrative examples are presented.

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