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Using Renyi entropies to measure uncertainty in search problems

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Abstract. Search methods are a becoming increasingly important from an industrial point of view. Search engines form the core of many computing applications from high level Artificial Intelligence suites to low-level search engines for data bases and networks. In so far as optimisation is a type of search it must of major importance in any area where performance needs to be optimised or has to meet target specifications, for example in product and system design. The connections beween search and basic ideas in information theory are perhaps not so well known despite the central importance of information theory in communication, coding and related fields.

The present paper, in fact, makes a three-way link between dynamic search, optimisation and entropy (negative information) and uses, to do this, the theory of dynamical systems. Although the contribution has a theoretical priority, genuinely useful line-search algorithms are presented, and it is hoped that the principals are applicable more widely. A key suggestion is that Renyi entropies are suitable measures of uncertainty in search problems. The asymptotic behavior of these entropies for sequences of partitions generated by different search algorithms are considered.

1. Renyi entropies and their properties

1.1. Renyi entropies of discrete distributions. Let P be a discrete distribution on a finite or countable set:

$$P=\{p_1,p_2,\dots\}:\ \ p_i\geq 0,\ i=1,\dots,N,\ \ \sum_{i=1}^{N}p_i=1,\ N\leq\infty\,.$$

The Renyi entropy of order $\alpha\neq 1$ of the distribution P is defined as

$$H_\alpha(P)=\frac{1}{1-\alpha}\log_2\sum_i p_i^\alpha,\quad -\infty<\alpha<\infty,\quad \alpha\neq 1\,. \tag{1.1}$$

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