Optimal and asymptotically optimal decision rules for sequential screening and resource allocation

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Abstract

We consider the problem of maximizing the expected sum of n variables X_k chosen sequentially in an i.i.d. sequence of length N. It is equivalent to the following resource allocation problem: n machines have to be allocated to N jobs of value X_k ($k = 1, \ldots, N$) arriving sequentially, the i-th machine has a (known) probability p_i to perform the job successfully and the total expected reward must be maximized. The optimal solution of this stochastic dynamic–programming problem is derived when the distribution of the X_k's is known: the optimal threshold for accepting X_k, or allocating the i-th machine to job k, is given by a backward recurrence equation. This optimal solution is compared to the simpler (but suboptimal) open–loop feedback–optimal solution for which the threshold is constant, and their asymptotic behaviors are investigated. The asymptotic behavior of the optimal threshold is used to derive a simple open–loop solution, which is proved to be asymptotically optimal ($N \to \infty$ with n fixed) for a large class of distributions. Certainty equivalence is proposed for the case where the distribution of the X_k's is unknown and estimated on–line. Simulation results indicate that open–loop rules can achieve good performance.

Keywords— certainty equivalence; dynamic programming; neutrality; resource allocation; sequential experimental design; sequential screening; stochastic assignment.

1 Introduction

Consider the situation where N items are generated sequentially, the characteristic of interest of the k-th item being measured by a scalar variable $X_k \in \mathcal{X}$. One wishes to