Analysis of performance of symmetric second–order line search algorithms through continued fractions

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Abstract The paper makes a connection between classical dynamical systems, namely the Gauss map and the associated Farey map, and symmetric second–order line search, of which the classical Golden Section and Fibonacci search methods are particular cases. The main result is an expression for the actual finite sample rate for symmetric line search algorithms in terms of the continued fraction expansion of the first observation point. This yields formulae for asymptotic rates in the almost sure ergodic case and exact rates for the two special cases (i) the rational case when the algorithm stops and (ii) the cyclic case which corresponds to quadratic irrationality. The asymptotic rate is sub–exponential for almost all starting points. Moreover, the dynamical system suffers so–called chaos with intermittency, typical of systems which have fairly normal (exponential) expansion punctuated by periods of very slow expansion. The property follows immediately through the link with the Farey map which has slow growth close to zero.

Key words. Dynamical system, Farey map, Fibonacci, Golden–Section, Gauss map, line–search.

AMS subject classification. 90C30, 1A55, 1B57, 58F11

1 Introduction

This paper is a contribution to a series by the authors which study dynamical systems embedded within certain search algorithms [8, 9, 6]. This is a new area in that it describes kind of stochasticity different from what is commonly considered as a random algorithm, for example arising in graph theory or in Markovian algorithms such as simulated annealing or genetic algorithms.

The paper makes a surprising connection between classical dynamical systems, namely the Gauss map and the associated Farey map, and a special kind of line search. The

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