Statistical Habitat Maps for Robot Localisation in Unstructured Environments

S. Rolfes and M.J. Rendas

Abstract--**In this paper we present a novel approach to mobile robot navigation in unstructured environments. Natural scenes can very often be considered as random fields where a large number of individual objects appear to be randomly scattered. This randomness can be described by statistical models. In this paper we consider that a natural scene can be interpreted as realisations of Random Closed Sets (RCS), whose global characteristics are mapped. Contrary to the feature based approach, this environment representation does not require the existence of outstanding objects in the workspace, and is robust with respect to small dynamic changes. We address the problem of estimating the position of a mobile robot, assuming that a statistical model, serving as a map of the environment, is available to it a priori. Simulation results demonstrate the feasibility of our approach.**

***Index Terms*—Mobile robot navigation, Random closed sets, Statistical description, Non-linear filtering**

I. Introduction

Last years have witnessed a significant research effort on autonomous underwater vehicles (AUV). The principal goal is to have robots, able to explore regions not accessible to human operators, or to perform unsupervised long term missions (e.g. survey of natural resources). Even if most of the vehicles actually in use are still teleoperated, for some missions, e.g. pipeline inspection, where the vehicle is artificially guided by a human-made feature, autonomous progression of the robot can be achieved.

While stable localisation methods have been proposed for indoor robots, navigation of robots in natural environments is still a challenge. We identify two reasons. First, we are confronted with large scale environments, requiring long range navigation. In the absence of external position information, pose estimation that is based only on dead-reckoning results in unbounded increase of the estimation error. Installation of artificial beacons yields good results, but the autonomy of the vehicle is limited a-priori. Perceptual information can be used in order to overcome this limitation. The robot creates a map (or uses an existing one), describing the natural landmarks observed in the workspace. This map, if dense enough, can be used to localise the robot later, when it returns to the same region. The second point concerns the structure of the environment. Natural scenes are highly unstructured (in the sense that euclidean geometry is not always appropriate in order to describe such environments). The type of maps (and of natural landmarks) differs largely from maps appropriate for indoor environments.

It is thus important to evaluate the question: 'What is the perceptual information that provides the best information for pose estimation ?'. The most common approaches for localising mobile robots are feature based, see e.g. [1][2]. Other methods use 3D elevation maps [11] (requiring non-flat sea-bottoms),.or the use of mosaics [3] based on visual information (requiring flat bottoms). Features, described by low dimensional parameter vectors, are stored in an internal map and localisation is basically done by estimating the rigid motion that matches recently observed features to those already contained in the map.

Natural environments have a random appearance. While in structured environments we are able to identify outstanding features (that we can easily distinguish from neighbouring ones), natural environments do not always present outstanding features, especially if the field of view is limited. Mismatches due to unstable feature identification, or sparseness of features preclude the use of feature maps. An additional challenge are dynamic changes of the environment (e.g. migration of flora and fauna, or simply alga, whose leaves are driven by the ocean current).

We propose a novel environment description that is suited for environments where identification of salient features is difficult. Instead of creating a detailed description of the environment as a collection of features, we propose a representation by statistical models that capture the local macroscopic characteristics of unstructured environments. Such characteristics can be (i) the number of objects per unit area, (ii) their spatial distribution, and/or (iii) the distribution of basic local morphological attributes, such as shape, colour or size. We consider this representation as an alternative to other map approaches, which, if the necessary conditions are satisfied, yield good results. The advantage of this representation is that it does not rely on individual features. Mismatch problems are thus eliminated, and the representation is robust to perturbations (small displacement of objects, or shape deformations).

The paper is organised as follows. In section II we discuss environment descriptions in general terms and propose in section III the use of RCS models as suitable descriptions for unstructured environments. Sections IV and V gives an overview of how RCS models can be used for mobile robot navigation. We assume here that such a description is available

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a-priori and do not address the problem of joint mapping and localisation for these models. In section VI we present preliminary simulation results that validate our approach and draw some conclusions in section VII.

II. ENVIRONMENT DESCRIPTIONS

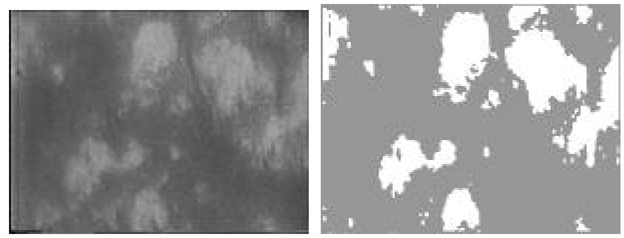
An exhaustive description of natural unstructured environments is not mandatory for the purpose of mobile robot navigation. The basic criteria for the choice of suitable environment maps are (i) simplicity of coding and (ii) robustness of recognition. The choice of the map depends thus of the nature of the environment in which the robot progresses. In general, we can induce a partition of the robot's workspace by associating to each point a mark belonging to a limited number of classes $\boldsymbol{M}$ Indoor environments can e.g. be classified into : 'corridors', 'walls', 'doors', etc.

For outdoor environments plausible classes are for example: 'stones', 'sand', 'tree', etc. This is a rather coarse classification, but still adequate for navigation if the classes are chosen in an appropriate way. This classification, based on perceptual data (using e.g. sonars or cameras), induces a series of patches on the workspace of the robot. This discretised description of the environment can be mathematically represented as the union of compact sets:

$$\Xi = \bigcup_{i=1}^{\infty} (\Xi_i + p_i, m_{ij}), \quad \Xi_i \in \boldsymbol{K}, \;\; m_{ij} \in \boldsymbol{M} \;. \qquad (1)$$

In the equation above $\boldsymbol{K}$ is a family of compact sets (set of possible shapes) and $\boldsymbol{M}$ is the mark space, designating the class to which the set belongs (other attributes than shape). Without loss of generality, we assume that the center of gravity of the sets Ξ_i is at the origin. The sum $\Xi_i + p_i$ denotes the set Ξ_i translated by the vector $p_i \in \Re^2$. The sets Ξ_i describe thus the morphological characteristics of the objects (or patches) and p_i their location in the workspace. An example is shown in Fig. 1. Fig. 1(a) shows a raw images of the sea bottom, where the white regions of the image correspond to dead 'Maerl' (coraline alga) found at the Orkney islands in the north of Scotland. This image shows well the patchy nature of this natural field. The classified version of this image ($\boldsymbol{M}$ contains just one class) is illustrated in Fig. 1(b).

Most of the approaches map the individual features (description of the shape and the location of the sets Ξ_i). If the field contains no outstanding features, the association of recently observed features to features contained in the map is subject to mismatch and leads to erroneous pose estimation.



(a) (b)

Fig. 1 Image of 'Maerl', taken in the north of Scotland. (b) the segmented image shows well the random distribution of the patches.

A different way to describe unstructured environments, by considering that the patches form a random pattern, can be formulated using the notion of random closed set (RCS) models.

III. MODELLING OF SCATTERED OBJECTS AS RANDOM CLOSED SETS

Random closed sets are mathematical models appropriate for modelling of random-like patterns. They have been frequently used in biological and physical studies in order to analyse natural patterns. Good introductions to this formalism can be found in [4][5].

A random closed set (a collection of randomly shaped compact sets, as given by equation (1)) is a doubly stochastic process, also called germ-grain model. A first random point process describes the spatial location of objects (germs), denoted by p_i in equation (1), at which realisations of a second stochastic process (grains) determine the local morphology of the field, i.e. the characteristics of the sets Ξ_i . The intersection between distinct patches can be non empty. The distribution of the germs can, for example, be clustered, structured or uniformly distributed, see Fig. 2.

We assume that the counting measure m associated to the point process (germ model) is a member of a parameterised family of distributions G_p :

$$m \in G_p = \{ m_l : l \in \Gamma \},$$

where Γ is a compact set. The vector l is the collection of parameters that determine the statistical distribution of the locations p_i . The shape process (grain model) constrains the set of possible elementary shapes (e.g. to discs of random radius, lines of random orientation or mixtures of them). Similarly to the germ process we consider that the distribution of the shapes can be parameterised by a finite number of parameters g , such that

$$k \in G_{\Xi_0} = \{ k_g : l \in \Lambda \},$$

where k is a probability measure over the space of possible shapes, Λ is a compact set and Ξ_0 is a random shape. Different model types can be obtained by considering distinct pairs of families G_p, G_{Ξ_0} (for instance, for G_p : homogeneous Poisson point process, regular pattern, etc., and for G_{Ξ_0} : discs whose radii are uniformly distributed in an interval, line segments of random length and orientation, etc.).

The random closed set model is thus given by the model type $M_{i,j}=(G_p^{(i)},G_{\Xi_0}^{(j)})$. A particular model $M\in M_{i,j}$ is specified by the parameter vector $q=(l,g)$, $M(q)=\{m_l,k_g\}$ „ where $m_l\in G_p^{(i)}$ and $k_g\in G_{\Xi_0}^{(j)}$.

The aim of the theory of random closed sets is to determine the model type $M_{i,j}$ and the model parameter $\hat{q}=(\hat{l},\hat{g})$, such that an observed scene (inside an observation window (OW) of size $n(OW)$, where $n()$ is the Lebesgue measure) is a typical realisation of the random closed set model $M(q)\in M_{i,j}$.

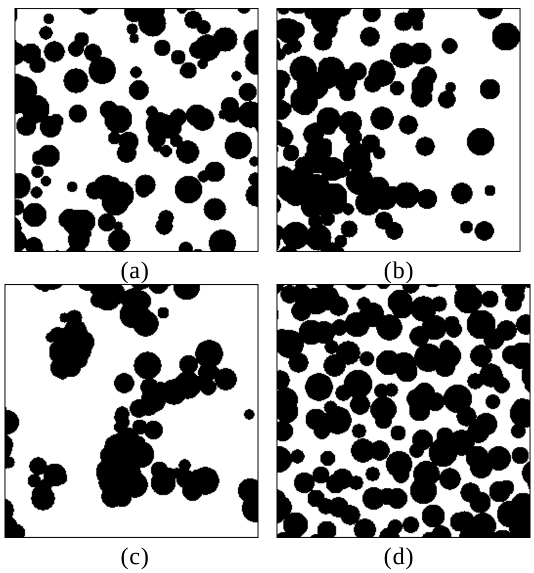


Fig. 2 Example of RCS models. with different point processes. (a) isotropic boolean model, (b) anisotropic boolean model, (c) clustered distribution and (d) regular distribution of the grains.

It is often difficult to obtain direct counting measures and estimates of the morphological characteristics of the sets Ξ_i from classified images, especially when the elementary grains Ξ_i can overlap, as illustrated in Fig. 2. Estimation of the distributions of the germ and the grain processes by direct identification of each individual shape is in these cases impossible.

We exploit here an important property of random closed sets [6], stating that the distribution of any general random closed set is uniquely determined by the **hitting capacity** which is, for each compact set K, the probability that the intersection of K with the RCS Ξ is not empty:

$$T_\Xi(K) = P(\Xi\cap K\neq\emptyset), \forall K\in\boldsymbol{K} \qquad (2)$$

The important fact is that knowledge of the hitting capacities for all $K\in\boldsymbol{K}$ is *equivalent* to the knowledge of the model parameter q (assuming the model type to be known). In the case of isotropic models (q is independent of the location and orientation of the observer) we know that $T_\Xi(K)=T_\Xi(K+p)$. Under the assumption that the RCS model is locally isotropic (inside the observation window WO) we can obtain empirical estimates of the hitting capacities from classified images.

For obvious reasons (limited computational capacities) we are able to estimate hitting capacities only for a finite collection of compact sets $K^n=\{K_1,\ldots,K_n\}$, which we call structuring elements [7]. In this case we capture just a limited number of the characteristics of Ξ_i.

For some model types we can find analytical forms of equation (2), allowing us to compute the hitting probabilities in terms of the model parameters q. This is the case for the well studied **boolean model**. The germ process is a Poisson point process, determined by the intensity parameter l, and the grains are i.i.d. realisations of compact sets. The hitting capacity for boolean models can be shown (see [4]) to be

$$T_\Xi(K)=1-\exp(-l\,E_k(n(\Xi_0\oplus\breve{K}))),$$

where $\oplus$ is the Minkowski-addition $A\oplus B=\{a+b,\forall a\in A,b\in B\}$, $E(.)$ is the statistical expectation operator with respect to the measure k of the shape process and $\breve{K}=\{-x,x\in K\}$. In this presentation of our approach to mobile robot. navigation we concentrate on Boolean models. Ongoing work concerns characterisation of other types of random closed set models as those illustrated inFig. 2 Example of RCS models. with different point processes. (a) isotropic boolean model, (b) anisotropic boolean model, (c) clustered distribution and (d) regular distribution of the grains., in particular clustered models, which seem good candidates to describe some kinds of natural scenes.

In general, the perceptual characteristics change throughout the workspace, induced by varying temperature, soil fertility, ocean current, etc. If these variations are abrupt, we can partition the workspace into disjoint areas A_k (see Fig. 3), whose macroscopic characteristics are described by different types of statistical models $M_{i_k,j_k}(q)$:

$$Workspace=\bigcup_{k=1}^{\infty}A_k,\ \ A_k\rightarrow M_k(q)=(m_l^{(i)},k_g^{(j)}),$$

where $M_k(q)$ is the model associated to area A_k. To model smooth variations of the field inside each region A_k, we let the model parameter q depend on the location x

$$\boldsymbol{Map}:\ \ x\rightarrow q(x)=(l(x),g(x));\ \ x\in A_k. \qquad (3)$$

The approach to navigation of robots in natural environments proposed here considers that the map given in the previous equation has been learned by (or given a priori to) the robot, i.e., the robot knows the partition $\{A_k\}$ and the piecewise continuous vector field defined by equation (3).

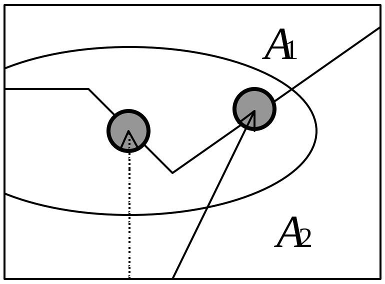


Fig. 3 The workspace is segmented in two areas. If the robot passes the boundary, localisation is possible. In the case of large pose uncertainty the set of possible locations (all along the boundary), is very large.

IV. Mobile robot navigation based on RCS models

We address now the problem of using the map defined in the previous section to estimate the robot location. According to our map representation we distinguish between localisation inside an area A_k and localisation on boundaries of adjacent areas. Localisation inside each area is possible only if the RCS model is anisotropic (the model parameter is a function of the location). An anisotropic field is illustrated in Fig. 2(b). In this case the pose error can be corrected permanently and positioning uncertainty is kept bounded. Accuracy of the localisation depend on the informativeness of the field: strong variations of the field result in more accurate pose estimates since distinction between different locations inside the area is more accurate. If the area is isotropic, localisation is only possible when the robot reaches a boundary of adjacent areas, indicated by an abrupt change of the model parameter or the model type (Fig. 2(a) illustrates an isotropic field). The problem is that during navigation inside an isotropic area the pose error growths considerably such that when reaching the boundary the uncertainty of the robot's location is large, resulting in a large set of possible true locations along the boundary. Most approaches to position estimation for mobile robots assume that the map observations are differentiable with respect to the robot position and orientation. Navigating between adjacent areas requires in this case hypothesis testing in order to determine the correct model type. Such hypothesis testing can be done using available tools for model selection, such as Generalized Maximum Likelihood or MDL [8]. We propose a method, that does not rely on differentialisation assumption and consequently do not require hypothesis testing.

We first formulate we define the general framework of the Bayesian approach to localisation. We assume that the dynamic model of the robot's state X_k and the observation model are known:

$$X_k = f(X_{k-1}) + w_k$$

$$p(H|N)=\binom{N}{H}\hat{T}(K_j)^H(1-\hat{T}(K_j))^{N-H},$$

where $\hat{T}(K_j)=\hat{H}/N$ is the estimated hitting capacity ($\hat{H}$ is the observed number of hits). The variance of $p(H|N)$ is $s_H^2=N\hat{T}(K_j)(1-\hat{T}(K_j))$. In order to guarantee that the individual hitting events are mutually independent it is important to choose an appropriate sampling number N . This number depends on the size of the observation window, the structuring element and on the RCS model. The determination of the optimal number is still an ongoing problem.. Note however that the number of hits are binomial distributed for N smaller than the optimal value. For large N the binomial distribution can be approximated by a normal distribution (see Fig. 4):

$$p(\hat{T}|q(X_k)) \hat{=} N(\hat{H}/N,\hat{T}(1-\hat{T})/N),$$

where $N(x,A)$ is the normal distribution defined by the mean x and the covariance A.

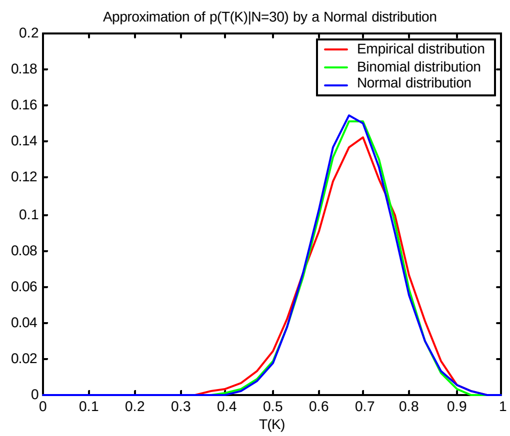


Fig. 4 Binomial distribution of the number of hits and its approximation by a normal distribution.

V. IMPLEMENTATION OF THE OPTIMAL FILTER

Direct computation of the prediction (convolution) and filtering (pointwise multiplication) steps is in practice not feasible. We saw in the previous section that the observation noise (for locally isotropic models) is well approximated by a normal distribution. Under the assumption that the noise w_k of the state space model is normal distributed we can obtain an approximation of the optimal non-linear filter by an Extended Kalman Filter (EKF). This approximation is based on the assumption that at each step the estimation error is small, allowing thus the approximation of the non-linearities of the state space model and the observation model by its linearisation around the current estimate of the robot's state. The linearisation must be a good approximation over the entire uncertainty domain which is the case when the robot is moving in informative anisotropic areas. However, navigation in isotropic areas leads to a considerable growth of the uncertainty of the robot's state and linearisation of the non-linear model may artificially shrink the estimated uncertainty of the position estimates, creating the possibility that the EKF diverges. This is particularly important when the robot reaches the boundary of adjacent areas after having crossed an isotropic area where consideration of the linearised model yields a null gain for perceptual information.

In [9] we proposed an approximation of the optimal non-linear filter by a Gaussian Mixture Model (GMM). We assume that the posterior density of the robot's state at time $k-1$ is Gaussian. We approximate the prediction density of equation (4) by a gaussian mixture:

$$p(X_k|Y^{k-1})\cong\sum_{i=1}^{N_k}P^i_{k|k-1}N(X^i_{k|k-1}|\Sigma^i_{k|k-1}),\quad \sum_{i=1}^{N_k}P^i_{k|k-1}=1.$$

Each term is has a normal distribution with covariance $\Sigma^i_{k|k-1}$ and is multiplied by a scaling parameter $P^i_{k|k-1}$. We assume that the number N_k and the locations of terms is chosen, such that linearisation of the state space model around each term is valid inside the principal support of the components. The choice of the locations and the scale parameters was discussed in [9]. The principal result is that each component can be propagated by an EKF and we proved in [9] that repeated application of the prediction and filtering step result always in a gaussian mixture. The update of the scaling parameters depends on the previous scaling parameter, the innovation and the innovation covariance (uncertainty of the perceptual observations). As a consequence, terms for which the predicted observations (hitting capacities) correspond well to the true observations (small innovations) are reinforced, otherwise they loose importance and will not contribute significantly to the posterior density. In our implementation of the mixture model, spurious terms, whose weight falls below a given treshold are eliminated and terms that are very close are fused. The number of actual terms of the mixture is an indicator of the ambiguity of the workspace inside which the robot navigates. If all but one term are eliminated ambiguity is removed completely.

If we consider the crossing of a boundary of adjacent areas the terms that are on the 'correct' side are reinforced and all other terms will loose importance, resulting in a concentration of the density on the areas inside which the robot is effectively located.

VI. RESULTS

In [10] we demonstrated navigation of a mobile robot inside anisotropic areas. The approximation of the optimal filter by an EKF was valid, since the uncertainty of the robot was maintained small during the entire trajectory. The linearisation around the estimated state was thus valid over the principal uncertainty support.

Here we present the navigation between isotropic areas, precluding thus permanent localisation. We simulated the navigation of an underwater robot equipped with proprioceptive sensors (compass and speed sensor) that are used for dead-reckoning, and with a camera pointing at the sea bottom. The robot moves at a constant altitude in an environment that was obtained by sampling from an isotropic Boolean model. The workspace is divided into two areas. The RCS model is a boolean model, where the grains are compact discs whose radius is uniformly distributed in an interval $r \in (r_1, r_2)$. The intensity of the Poisson point process is constant inside each area and changes abruptly at their frontier. The workspace (the locations of the grains) is shown in Fig. 5, along with the location of the robot and the position estimate (obtained by an EKF), indicated by a large cross (with a large error, resulting from the previous dead-reckoning period). The ellipse indicates the initial uncertainty and the square the area that is observed by the camera.

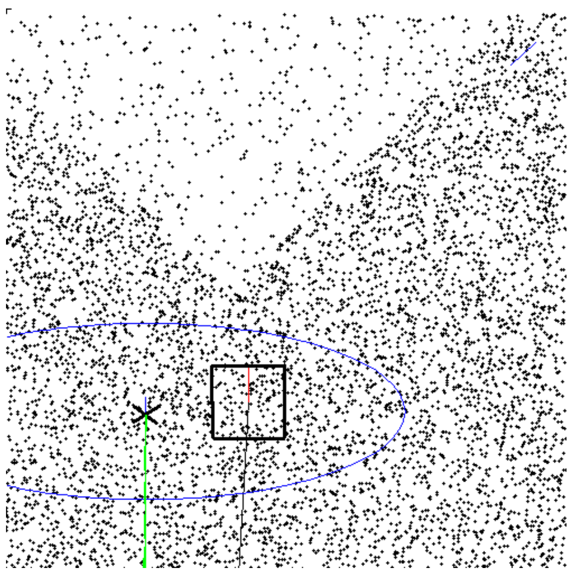


Fig. 5 Realisation of a random field, where the changement of the intensity of the point process defines the frontier between the areas.

We tested the approach for two different realisations. In the first realisation the jump of the intensity parameter at the boundary is larger; $\lambda_1 = 0.002, \lambda_2 = 0.0004$ (Fig. 5) than for the second simulation; $\lambda_1 = 0.002, \lambda_2 = 0.0008$. A simulated (non-observed) ocean current perturbates the nominal trajectory of the robot, resulting in an important drift between the true position and its dead-reckoning estimate. Throughout the trajectory, that was chosen in order to guarantee that the frontier is crossed, images are acquired at regular time intervals. The perceptual observations are empirical estimates of the hitting capacity $Z_k = \left\{\hat{T}_k(K_1), \hat{T}_k(K_2)\right\}$ for two structuring elements (squares of varying side length) that are directly obtained from the images based on a fixed number N = 15 of samples.

We assume here that we are able to determine at which time the linearisation around the estimate is not valid in order to initialise the gaussian mixture. This is the case if the boundary between the areas lies inside the principal support of the uncertainty region, indicated by the ellipse (3σ).The terms of the gaussian mixture model are indicated in Fig. 6 by the plus signs and the boundary of the principal support (coverage 99%) is traced (initially elliptic). The mean of the mixture is shown as a large circle and the trajectory (in the subsequent figures) in shown as a dashed line.

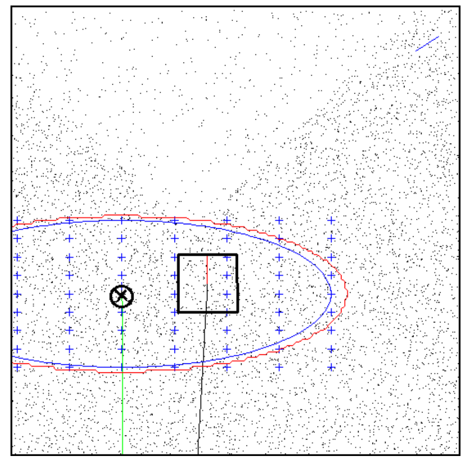


Fig. 6 Creation of the Gaussian Mixture Model. The terms are indicated by the plus signs.

When the robot progresses, some terms of the mixture cross the frontier between the areas. As long as the robot does not reach the frontier those terms loose importance and the posterior density is concentrated on the correct side of the frontier. In Fig. 7 the robot crossed the frontier, resulting in the elimination of terms on the opposite side. This is clearly indicated in the figure by the boundary of the uncertainty support of the gaussian mixture. The pose estimate obtained by the EKF is not corrected, since the predicted change of the model parameter is zero and explicit hypothesis testing is not performed. Fig. 8 shows what happens when the EKF pose estimate reaches the frontier. The map indicates a strong variation of the intensity of the point process and the density is concentrated along the frontier. The EKF estimate looses track of the true robot position, which is already inside the second area.

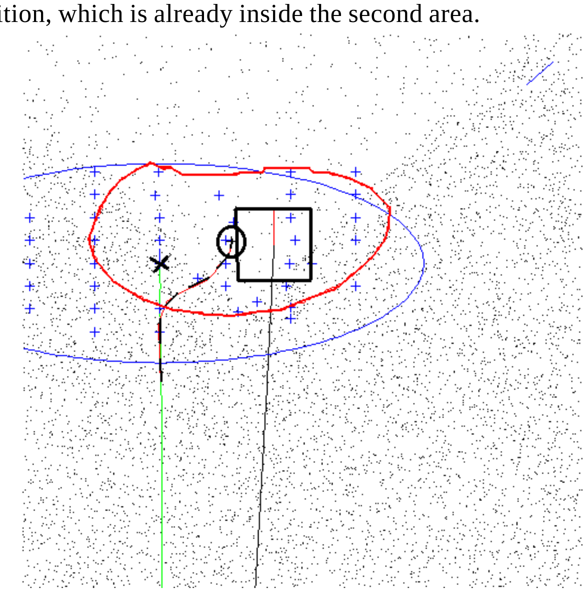


Fig. 7 The posterior density is concentrated around the terms lying on the correct side of the frontier. The significant support of this density is

indicated. The terms lying on the wrong side do not contribute significantly to the density and will be eliminated.

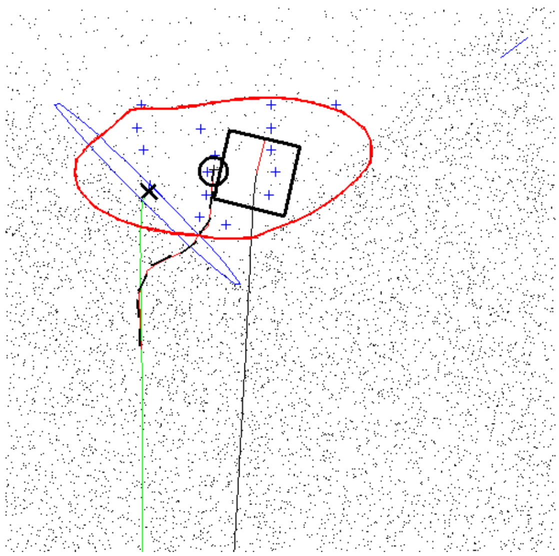


Fig. 8 The robot passed entirely to the area with lower intensity. The EKF estimate reaches the frontier, indicated by a strong gradient. of the intensity in the map. Its density is concentrated along the frontier.

Fig. 9 shows the final result. The density of the gaussian mixture is concentrated around a single term (the large circle) and the density is again gaussian. All other terms of the gaussian mixture are removed (elimination of spurious terms and fusion of closed terms) throughout the trajectory during which the robot crossed the frontier three times. A single passage is not sufficient in order to remove the initial ambiguity completely. The number of terms at each iteration is shown in Fig. 10 for both simulations. The lower curve correspond to the workspace where the intensity parameter changes strongly and the second curve corresponds to the less informative workspace. At each time when the robot passes the frontier the posterior density is concentrated on the terms that are on the correct side of the boundary, resulting in the elimination of spurious terms (Iterations 80, 120, 250). The figure shows well that the accuracy of the pose estimation depends on the information provided by the RCS map.

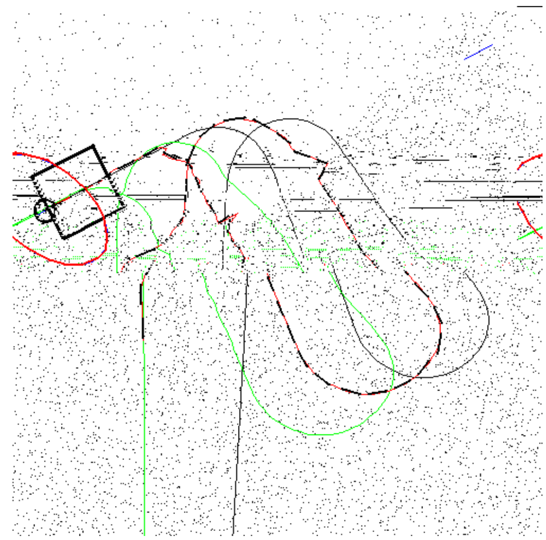


Fig. 9 After three passes of the frontier the initial ambiguity is entirely resolved. The whole density is concentrated around the single remaining term (the EKF estimate falls outside the figure (on the right)).

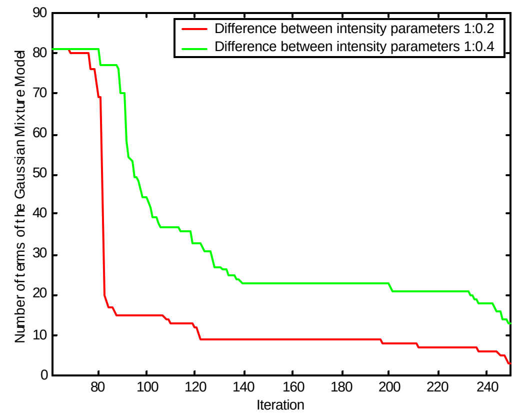


Fig. 10 The number of remaining mixture terms are indicated for both realisations (strong change by the lower curve; small change by the upper curve).

VII. CONCLUSIONS

In this paper we propose a novel environment description for robot navigation, using the formalism of random closed set models. These models capture the principal characteristics of natural environment. The approach was motivated by the fact that identification of outstanding features, on which the majority of existing approaches to robot localisation is based is not always possible. Description by statistical models does not rely on the identification of outstanding features and knowledge of exact location (or shapes) of features is not required, resulting in increased robustness with respect to small changes.

We present approximate expressions that enable definition of an approximation to the Bayesian estimator of the robot state for RCS models. We addressed the problem of ambiguity in the workspace, precluding the use of a simple EKF, when uncertainty of the estimated pose is very large. An approach that is related to multiple hypothesis, the gaussian mixture model is proposed and its feasibility is demonstrated by simulation results.

A series of open problems must still be studied more thoroughly. In particular, we need to use more complex RCS models, in order to describe clustered or regular environments. Analytical expressions for the hitting capacities need to be found for these models. Another issue concerns the gaussian mixture model. In particular we need to add additional terms to the mixture in the case where the linearisation is no longer valid. Finally the problem of joint mapping and localisation for this kind of environment representations must be addressed in order to realise fully autonomous progression of a robot in a priori unknown environments: the robot must be able to simultaneously estimate the map (the model type along with the model parameter) and its position, using the autonomously created map.

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