Planning Robotic Exploratory Missions in the Absence of Global Positioning

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Abstract – **This paper presents a new approach to the design of exploratory (survey) robotic missions, for platforms which have no means of global positioning. Contrary to traditional approaches to this problem, which are mainly concerned with minimizing *path length*, our work explicitly addresses the expected evolution of positioning errors and their impact on mission execution.**

Imposing a non-observable departure from nominal conditions, errors in self-positioning of the robotic platform can compromise the achievement of the mission goal and even lead to platform loss. We address the problem of *a priori* characterization of the evolution of the positioning errors, for platforms with only local observability of their own position. We present general results, assuming a particular structure of the dynamical/sensors model that describes the platform. These results are then applied to the analysis of survey missions of the autonomous underwater vehicle MAUVE.

I. INTRODUCTION

The mapping/survey problem has been studied by several researchers in the last years [3]. Our work differs from the majority of these studies in that we explicitly address the effect of navigation errors in the execution of survey missions. In fact, the majority of the mapping algorithms that have been proposed attempt to minimize path length, and consider ideal perfect positioning of the platform along the exploration mission, being based on purely geometric arguments. Robotic survey missions usually have one of two main objectives: either *(i)* to make sure that if a given kind of object is present it will be detected – one obvious example is mine-hunting – and thus that *all the surface* will be observed; or *(ii)* to acquire an internal representation of the objects present in the explored region to be used in future missions – for instance, for studying the evolution of environmental conditions in a given region – in which case the reproducibility of the results is important. In either case, blind application of algorithms that assume perfect platform positioning to a platform using dead-reckoning navigation will fail to achieve the ultimate mission goal. In case *(i)* drifting will create "holes" – areas which are not observed – increasing the probability of not detecting the objects of interest. Positioning errors impose also a random deformation of the internal representation built, compromising, for case *(ii)*, the reproducibility of the results.

Under the absence of global position observability, the optimization of the exploratory mission must take into account the impact of positioning errors in the actual coverage of the target region R. *Trajectory planning* is then best cast as a **multi-criteria optimization problem**:

> Given a dynamical/observation model of the platform, find the exploratory trajectory $P = \{p(t), t \in T\}$ such that
>
> - the total energy consumption is minimized
>
> $$E(P) = \int_T g_e(\dot{p}_t)\, dt,$$
>
> where $g_e(\cdot)$ depends on the platform;
>
> - the probability of incomplete coverage is minimized
>
> $$C(P) = \max_{z \in R} \Pr\{z \text{ is not covered by } P\}. \tag{1}$$

The two constraints

$$\min E(P) \qquad \textbf{and} \qquad \min C(P),$$

are conflicting: while $C(P)$ will be small for trajectories P that are dense in R, these will necessarily correspond to large values of required energy $E(P)$. In this perspective, an optimal trajectory does not exist, and a compromise between the two criteria must be found. The non-linear dependency of both criteria on the trajectory precludes utilization of standard optimization techniques. The optimization of the definition of exploratory trajectories is made in a companion paper, and we concentrate here on the evaluation of $C(P)$ for a given trajectory.

The work presented is useful in the context of a related problem, concerning the *automatic control* of the vehicle autonomy. Since the growth of positioning errors strongly depends on the actual events that occur during the course of the mission, autonomy (in terms of maximum allowable operational range) cannot be determined a priori. To make sure that normal vehicle recovery is achievable at any time instant, we defined an on-board surveillance procedure, that checks that the probability $P_H(t)$, that the vehicle is able to return to a pre-specified homing region, is above a given threshold α, given its current level of positioning uncertainty, the accuracy of the navigation sensors and the learned environment characteristics. The evaluation of this probability uses the error prediction techniques presented in this paper.

The paper is organized as follows. In section II we present the basic results concerning the prediction of error evolution. Section III addresses the computation of $C(P)$ for a given trajectory. The results of section I are then extended and applied to the analysis of a mapping mission, in section IV. Finally, in section V we present our conclusions, and report on work currently in progress.

II. General Model

In this section we show that for positioning systems with dynamical/observation models with a particular structure, the error of some state components grows linearly in time. For positioning problems, these state components are in general related to the vehicle instantaneous position, meaning that the error in position is non-decreasing (in a statistical sense).

The structure that we consider can be abstracted using the following decomposition of the state vector into two separate subsets:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} I & F_3 \\ 0 & F_2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix} \quad (2)$$

$$z(k) = \begin{bmatrix} 0 & H \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + v(k) \ . \quad (3)$$

This model relates the observed variables $z(k)$ (output of the motion sensors, or other positioning subsystems) to the instantaneous position and velocity (and eventually higher order derivatives) of the vehicle, which are in general a no-memory transformation of the complete state vector

$$x(k)^T = \begin{bmatrix} x_1(k) & x_2(k) \end{bmatrix} .$$

The perturbation terms $w(k) = \left[w_1^T(k) \ \ w_2^T(k)\right]^T$ and $v(k)$ are stationary independent white Gaussian noise sequences with covariance matrices Q and R, respectively. The noise in the state equation (2) has a block diagonal structure, i.e.,

$$Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} \ . \quad (4)$$

In connection to this model ("complete model") we introduce the following "reduced model":

$$\begin{aligned} x_2(k+1) &= F_2 x_2(k) + w_2(k) && (5) \\ z(k) &= H x_2(k) + v(k) \ . && (6) \end{aligned}$$

The covariance matrix of the noise in the state space equation (5) of the reduced dimension model is equal to Q_2. Note that the observations are the same in both models.

Due to the particular block structure exhibited by the complete model (2)-(3), it is straightforward to show that the optimal Kalman filter for estimating $x_2(k)$ is the same for both models. Moreover, if the time-invariant reduced model (5)-(6) is completely stabilizable and detectable, the corresponding Kalman filter exhibits an asymptotic constant error behavior, i.e.,

$$\lim_{k \to +\infty} P_2(k+1|k) = P^p_{2_\infty} \ ,$$

where $P_2(k+1|k)$ stands for the covariance matrix of the predicted estimate of $x_2(k)$. The covariance matrices of predicted, $P^p_{2_\infty}$, and filtered, $P^f_{2_\infty}$, steady-state errors are given by

$$\begin{aligned} P^p_{2_\infty} &= F_2 P^f_{2_\infty} F_2^T + Q_2 \\ P^f_{2_\infty} &= \left(I - K_{2_\infty} H\right) P^p_{2_\infty} \left(I - K_{2_\infty} H\right)^T + K_{2_\infty} R K^T_{2_\infty} \end{aligned}$$

where

$$K_{2_\infty} = P^p_{2_\infty} H^T R_I^{-1} \quad (7)$$

is the steady-state Kalman gain of the reduced filter, and

$$R_I = H P^p_{2_\infty} H^T + R$$

represents the covariance matrix of the innovations process.

When the covariance matrix of the reduced filter exhibits a steady-state solution, the cross-covariance matrix, $P_{12}(k|k)$, between the filtered estimates of $x_1(k)$ and $x_2(k)$ also converges to a steady-state solution, P^f_{12}, given by

$$P^f_{12_\infty} = F_3 P^f_{2_\infty} A^T \left(I - A^T\right)^{-1}, \qquad A = \left(I - K_{2_\infty} H\right) F_2 \ .$$

Since the Kalman gain

$$K_1(k) = P_{12}(k|k-1) H^T R_I^{-1}(k),$$

associated to the estimate of the $x_1(k)$ depends only on $P_{12}(k|k-1)$, the complete filter converges, for large k, to a steady-state regime characterized by the steady-state filter gain $[K_{1_\infty} \;\; K_{2_\infty}]^T$, with

$$K_{1_\infty} = \left(P^f_{12_\infty} F_2^T + F_3 P^f_{2_\infty} F_2^T\right) H^T R_I^{-1}$$

and K_{2_∞} given by (7).

An approximation to the asymptotic regime of the covariance matrix $P(k|k)$ of the complete filter is analytically obtained by solving the Riccati equation with the Kalman gain equal to its steady-state value. For large k, one gets

$$P(k|k) \simeq \begin{bmatrix} P_1^\infty(0|0) + kC & P^f_{12_\infty} \\ P^{f^T}_{12_\infty} & P^f_{2_\infty} \end{bmatrix} \qquad (8)$$

where $P_1^\infty(0|0)$ is a constant, and

$$C \;=\; D P^f_{2_\infty} D^T + P^f_{12_\infty} D^T + D P^{f^T}_{12_\infty} + J, \qquad (9)$$

with

$$\begin{aligned} D &= F_3 - K_{1_\infty} H F_2 \\ J &= K_{1_\infty} H Q_2 H^T K^T_{1_\infty} + K_{1_\infty} R K^T_{1_\infty} + Q_1 . \end{aligned}$$

The constant $P_1^\infty(0|0)$, which depends on the transient regime of the filter, is

$$\begin{aligned} P_1^\infty(0|0) &= P_1(0|0) + D\,(I-A)^{-1}\left[P_{12}(0|0) - P^f_{12_\infty}\right]^T \\ &+ \left[P_{12}(0|0) - P^f_{12_\infty}\right](I-A)^{-T} D \\ &+ D\,(I-A)^{-1}\left[P_2(0|0) - P^f_{2_\infty}\right](I-A)^{-T} D^T . \end{aligned}$$

III. PERFORMANCE CRITERION $C(P)$

We address in this section the problem of predicting the performance index $P(C)$ defined in eq. (1).

To address this problem we must first characterize the perception sensors that the vehicle uses. Namely, to be able to define the probability that a given point be observed during a trajectory, we must specify what points of the surrounding space are observed at each point in the trajectory. In the more general case, the observed region will be dependent on both vehicle position and attitude. We define, associated to the perception sensors of the vehicle a mask O that defines the boundary of the "covered region:" $O = \{u(\theta), \theta \in \Theta\}$, such that at position p and orientation β, the vehicle covers all the points inside the closed surface

$$\begin{aligned} O[p,\beta] &= \{\text{ observed points in } (p,\beta)\} \\ &= \{z = p + \rho u(\theta+\beta) \,|\, 0 \le \rho \le 1, \theta \in \Theta\}. \end{aligned}$$

Let $u(z)$ be the first time that point z is observed:

$$u(z) = inf_t\,(t : z \in O[p(t), \beta(t)]) .$$

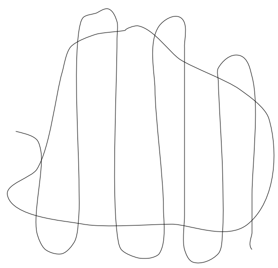

Figure 1: Comb strategy for exploratory missions.

Then, desired probability can be written in terms of the trajectory $P = \{p(t)|t \le T\}$, where T is the duration of the mission:

$$\Pr\{z \text{ is covered during } P\} = \Pr\{u(z) \le T\} .$$

We assume that the vehicle's control during its motion uses the estimated position (and attitude) of the vehicle to enforce $\hat{p}(t) = p_0(t), \forall t \le T$, i.e., that the estimated position coincides with the planned trajectory, $P_0 = \{p_0(t), t \le T\}$. Decomposing the actual trajectory $p(t)$ as $p(t) = p_0(t) + \epsilon(t)$, where $\epsilon(t)$ is the estimation error at time t,

$$\begin{aligned} u(z) &= inf_t\,(t : z \in O[p_0(t) + \epsilon(t)]) \\ &= inf_t\,(t : \epsilon(t) \in O_{t,z}) , \end{aligned}$$

where

$$O_{t,z} = \{u : u = z - p_0(t) - \rho u(\theta), 0 \le \rho \le 1, \theta \in \Theta\} .$$

The criterion $C(P)$ can be written as

$$C(P) = \max_{z \in R}(1 - \Pr\{u(z) \le T\}) .$$

We can recognize $u(z)$ as a first passage time of the time-space process $[t, \epsilon(t)]$, see [1] for a definition of these processes. The study of first passage times for multi-dimensional processes is more difficult than their one-dimensional counterparts, the majority of the existing results concerning asymptotic behavior (when t grows to infinity). Instead of computing the exact probability defined above, we derive an upper bound on $C(P)$. The bound is obtained by working with a one-dimensional projection of the error component, whose passage times are easier to compute. We present below a brief outline of the approach applied to a "comb" strategy, see Figure 1. To simplify the presentation, we consider that no unmodeled perturbations exist, i.e., that the position estimates are unbiased. Biasing effects – produced for instance by unmodeled currents in underwater applications – can be taken into account using the same approach at the cost of increased analytical complexity.

Consider the two parallel segments of linear uniform motion that are closest to point z, as shown in figure 2.

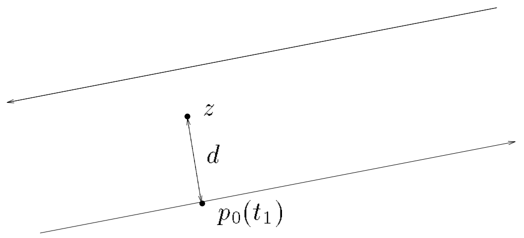

Figure 2: Parallel segments of linear uniform motion for the "comb" strategy.

To further simplify the presentation given here, we assume symmetry of the perception sensors, i.e., we admit that the observed region is a sphere of constant radius r, such that $u(\theta)$ is the normalized unit vector along direction θ, in which case

$$O[p] = \{z : \|p - z\| \leq r\}\,.$$

We assume that the nominal mission is defined such that the entire region is covered, that is that the spacing between adjacent lines, S, verifies $S \leq 2\rho$. The Markov character of the error allows us to restrict the analysis to the regions between adjacent pairs of parallel segments of the trajectory. Consider the points in a line orthogonal to the direction of motion. Let t_1 be the instant the vehicle crosses this line on the first segment, and t_2 the instant it crosses the same line on the second segment. Then, there will be "holes" in the region between the two trajectories if

$$S + \epsilon(t_1) - \epsilon(t_2) \geq 2\rho.$$

Since the error is Markov, we can make the following decomposition

$$\epsilon(t_2) = \epsilon(t_1) + \Delta\epsilon(t_2, t_1),$$

where the error variation $\Delta\epsilon(t_2, t_1)$ is Markov, independent of $\epsilon(t_1)$, and has a distribution that depends on t_2 and t_1. With the previous decomposition, the following equivalent condition is obtained:

$$S - \Delta\epsilon(t_2, t_1) \geq 2\rho \Leftrightarrow \Delta\epsilon(t_2, t_1) \leq -(2\rho - S) \leq 0\,.$$

This equation says that the maximum error accumulated over consecutive passages over the same orthogonal line must be smaller than the overlap between the regions observed in adjacent passages, and enables us to establish an upper bound:

$$\Pr\{\text{ hole in the line }\} \leq \Pr\{\Delta\epsilon(t_2, t_1) \leq -(2\rho - S)\}\,.$$

The upper bound comes from the fact that some points in this segment can be covered at other instants than t_1 and t_2.

We assume that $\Delta\epsilon(t_2, t_1)$ is a normal random variable of zero mean and covariance σ_{12}^2. $\Pr\{\Delta\epsilon(t_2, t_1) \leq -(2\rho - S)\}$ depends only on σ_{12}^2, and the maximum value occurs for the pairs (t_1, t_2) which lead to its maximum value.

We decompose the error growth between t_1 and t_2 in three separate statistically independent components: $\Delta\epsilon(t_2, t_1) = \Delta\epsilon^1 + \Delta\epsilon^{turn} + \Delta\epsilon^2$, where $\Delta\epsilon^1$ is the value of error accumulated from t_1 to the end of the first linear motion, Δ_ϵ^{turn} the error accumulated during the turning maneuver, and $\Delta\epsilon^2$ from the beginning of the second segment until time t_2. Using the statistical independence of the three components, we can write for the covariance of the total accumulated error $\sigma_{12}^2 = \sigma_1^2 + \sigma_{turn}^2 + \sigma_2^2$, where σ_i^2 is the covariance of the error component $\Delta\epsilon^i$. Let $T_i, i = 1, 2$ be the total time required for completion of the i-th segment. Then, using the Brownian characteristic of the error evolution during the linear segments, $\sigma_{12}^2 = C_1(T_1 - t_1) + \sigma_{turn}^2 + C_2 t_2$, where C_i is a constant determined from the error model predicted using the equations in the previous section for the i-the segment. The maximum of σ_{12}^2 occurs thus for the line orthogonal to the beginning of the first segment, i.e., for $t_1 = 0$ and $t_2 = T_2$, where it achieves the value $\sigma_{12}^{2*} = C_1 T_1 + \sigma_{turn}^2 + C_2 T_2$.

We can now write for the desired probability of incomplete coverage,

$$C(P) \leq \frac{1}{2}\mathrm{erfc}\left(\frac{2\rho - S}{\sqrt{2}\sigma_{12}^{2*}}\right)\,. \tag{10}$$

This equation can be used to establish the value of S that leads to a sufficiently small probability $C(P)$.

IV. APPLICATION TO AN AUV

In this section we use the general results presented in section II to find a model for the evolution of the positioning system of the autonomous underwater robot MAUVE, being developed in the framework of a MAST project. We then use these results, to compute $C(P)$ for a mapping mission.

Besides an on-board depth sensor, the MAUVE vehicle positions itself with respect to an acoustic subsystem based on the observation of the differential delay and Doppler of the signal (passively) received from two independent acoustic emitters (references) [2]. The observation vector is

$$z(k)^T = \left[R_1(k) - R_2(k)\ \dot{R}_1(k)\ \dot{R}_2(k)\right] + v(k), \tag{11}$$

where $R_i(k)$ is the distance between the vehicle and the ith reference. Assuming a locally constant radial velocity with respect to both references, a state space model of the structure presented in section II is obtained, with the following definitions:

$$F_2 = \begin{bmatrix} 1 & 0 & \Delta_t \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad F_3 = \begin{bmatrix} 0 & \Delta_t & 0 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & -1/2 \end{bmatrix}$$

with $x_1(k) = R_1(k) + R_2(k)$ and $x_2(k)^T = [R_1(k) - R_2(k)\ \dot{R}_1(k) + \dot{R}_2(k)\ \dot{R}_1(k) - \dot{R}_2(k)]$. We can conclude that the estimation error of all components of $x_2(k)$ tend to a stationary regime, while $x_1(k)$ will exhibit a linear error growth, as predicted by equation (8).

The position of the vehicle is estimated at each instant using a triangulation procedure, from the estimates of $R_1(k) + R_2(k)$ and $R_1(k) - R_2(k)$ and the output of the depth sensor. Using a local error analysis, the error of the position estimates along a desired direction can be written in terms of the errors of the filter output and of the depth sensor. Using expression (10), we can find the probability that the desired region is not covered with the required sampling rate S_r.

In the next figures, we show the results predicted for a mapping mission, where a square (3Km$\times$ 3Km) horizontal region must be sampled with a regular sampling rate of $S_r = 100$ m between samples along each direction. The trajectory of the vehicle is a "comb" trajectory, with consecutive segments spaced of $S_t = 75m$. The two acoustic references are located centrally with respect to the covered region, at a distance of $R = 500m$, with an interspacing of L. We used the analysis tools presented in the paper to analyze the effect of: *(i)* distance between references, L,*(ii)* orientation of the "comb" trajectory with respect to the line defined by the two references: orthogonal (o) or parallel (p).

In Figures 3-6, the upper plots represent, in a regular grid, the probability that the samples in the Y direction will not verify the required $S_r = 100m$. The lower plots show the determinant of the covariance matrix of the position error. In all cases, the mission starts at the point $x = -1500m$, $y = 500m$.

From the four configurations shown, Figure 6, with a trajectory orthogonal to the line connecting the two references, yields the best performance, with a probability of undersampling different from zero only in the (nealry endfire) lateral regions, where the triangulation performs poorly. Note that this does not mean that the accuracy of the sampling points is uniform over the entire mapped region, as the lower plot shows. In fact, for all configurations, the error increases monotonically during the mission, which results in high error regions at the areas more distant from the references for case (p), and at the rightmost regions for case (o). It is interesting to compare the different sensitivity of (o) and (p) with respect to the distance between references L. Comparison of Figure 3 and Figure 4 shows that even if increasing L achieves a significant position error reduction in the distant regions, the probability of coverage does not show corresponding improvement. On the contrary, see Figures 5 and 6, a similar error decrease in the last segments of the trajectory, yields a significant

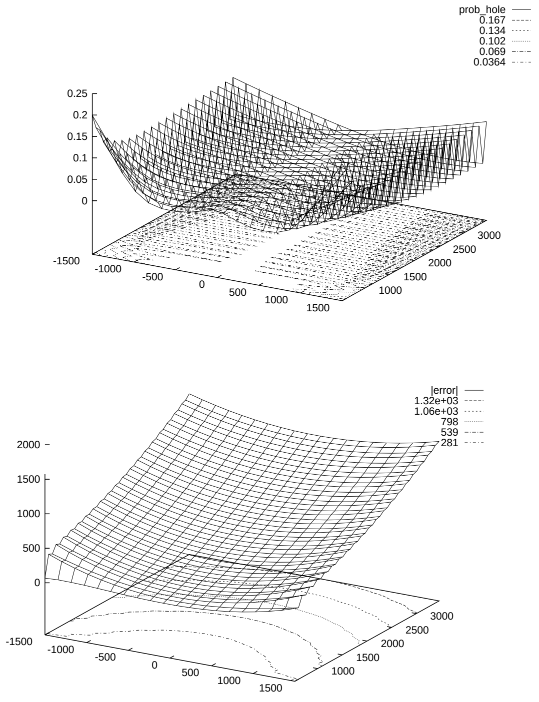

Figure 3: $\Delta_t = 10s$, $L = 100m$, (p).

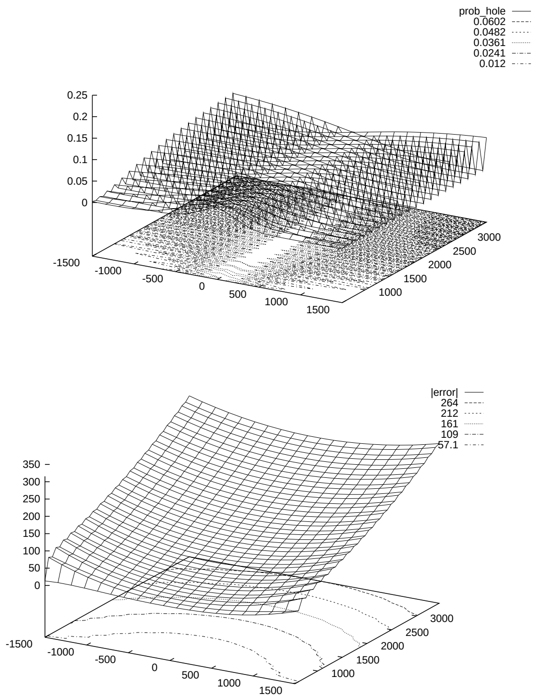

Figure 4: $\Delta_t = 10s$, $L = 500m$, (p).

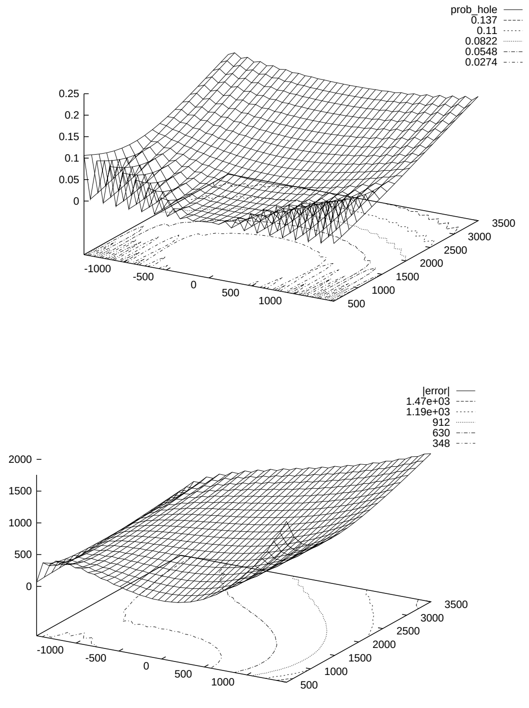

Figure 5: $\Delta_t = 10s$, $L = 100m$, (o).

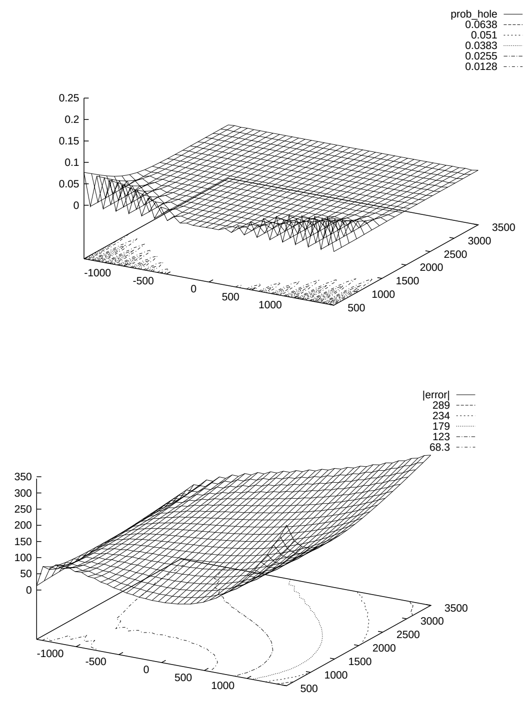

Figure 6: $\Delta_t = 10s$, $L = 500m$, (o).

reduction in the probability of undersampling.

V. CONCLUSIONS

We present in this paper a new formulation of the problem of defining exploratory trajectories, as a multi-criteria problem that explicitly evaluates the impact of positioning errors. We present a statistically based definition of a coverage performance index, that is determined from the statistical characterization of the positioning error, and analytical equations that allow the *a priori* prediction of error evolution. Finally, we apply the proposed technique to the definition of a underwater mapping mission.

The work presented in this paper is presently incorporated in the mission preparation system of the MAUVE vehicle, allowing the user to iteratively redefine a survey mission, using the multi-criteria performance analysis provided by the system. In the future, we expect to define a set of exploration strategies as the chaining of basic exploratory actions. Identification of a sufficient set of basic actions, along with the formulation of a rich set of exploration strategies sets the ground for completely *automatic mission definition.*

VI. ACKNOWLEDGMENTS

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