Erratum to: Stable periodicity and negative circuits in differential systems

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December 2, 2014

In a private communication, Frederic Beck (University of Mainz) pointed out that, in [1], when proving that Ω contains a stable periodic solution we use a wrong argument, the following: $(\partial f_i/\partial x_i)(x) < 0$ for all $x \in \Omega$, $i = 1, 2$. Indeed, for instance, if $x_1 \in]2-\varepsilon, 2+\varepsilon[$ and $x_2 \in \times[1+\varepsilon, 3-\varepsilon]\backslash]2-\varepsilon, 2+\varepsilon[$ then $(\partial f_i/\partial x_i)(x) = 4\varphi_2'(x_1) - 1$, and this term can not be less than zero for all $x_1 \in]2-\varepsilon, 2+\varepsilon[$ because φ_2' has to be greater than $1/2\varepsilon > 1$ somewhere in this region. So there are $x \in \Omega$ with $(\partial f_i/\partial x_i)(x) > 0$.

However, using slightly more involved arguments, we proved here that the system is still a counter-example of Conjecture 2'. More precisely, we prove that if $\varepsilon \leq 1/8$ then there is a stable periodic solution in the domain $\Omega' = [0,4]^2 \setminus \Gamma$, where Γ is the interior of the convex hull of the set containing the points $A = (1-\varepsilon, 3-\varepsilon)$, $B = (2-\varepsilon, 3+\varepsilon)$, $C = (3-\varepsilon, 3+\varepsilon)$, $D = (3+\varepsilon, 2+\varepsilon)$, $E = (3+\varepsilon, 1+\varepsilon)$, $F = (2+\varepsilon, 1-\varepsilon)$, $G = (1+\varepsilon, 1-\varepsilon)$, and $H = (1-\varepsilon, 2-\varepsilon)$; see Fig. 1 for an illustration.

First, since $\Omega' \subseteq \Omega$, there is no equilibrium point in Ω'. Suppose now that $\varepsilon \leq 1/8$, and let us prove that all the solutions starting in Ω' remain in Ω'. As showed in [1], no solution starting in $[0,4]^2$ leaves $[0,4]^2$, thus it is sufficient to prove that no solution starting in Ω' reaches the interior of the convex hull Γ. Consider first the line segment L with endpoints A and B. For all $x \in L$ we have

$$\begin{aligned} f_1(x) &= 4\varphi_3(x_2) - x_1 \leq 4 - x_1 \leq 3 + \varepsilon \\ f_2(x) &= 4 - x_2 \geq 1 - \varepsilon. \end{aligned}$$

Thus for all $x \in L$ the scalar product between $f(x)$ and the vector $v = (-2\varepsilon, 1)$ is at least $-2\varepsilon(3+\varepsilon) + (1-\varepsilon) = 1 - 7\varepsilon - 2\varepsilon^2$, and this term is positive since $\varepsilon \leq 1/8$. Since v is orthogonal to L and is pointing outside Γ, this means that if a solution starts in Ω', then it cannot reach Γ by crossing the line segment $L = AB$. Also, for all x that lies in the line segment BC we have $f_2(x) = 1 - \varepsilon > 0$. Thus if a solution starts in Ω', then it cannot reach Γ by crossing BC. Reasoning similarly with the segments CD, DE, EF, FG, GH, and HA, we deduce that all the solutions starting in Ω' remains in Ω'. Thus, following the Poincaré-Bendixon theorem, there exists a periodic solution ψ of period $T > 0$ starting in Ω'.

Finally, let us prove that ψ is stable. For all $x \in \mathbb{R}^2$ we have $(\partial f_1(x)/\partial x_1)(x) = 4\varphi_2'(x_1)(\varphi_1(x_2) - \varphi_3(x_2)) - 1$. Thus $(\partial f_1(x)/\partial x_1)(x) \geq 0$ implies $4\varphi_2'(x_1)(\varphi_1(x_2) - \varphi_3(x_2)) > 0$ which implies that x belongs to the domain $]2-\varepsilon, 2+\varepsilon[\times]1-\varepsilon, 3+\varepsilon[$ which is disjoint from Ω'. Thus $(\partial f_1/\partial x_1)(x) < 0$

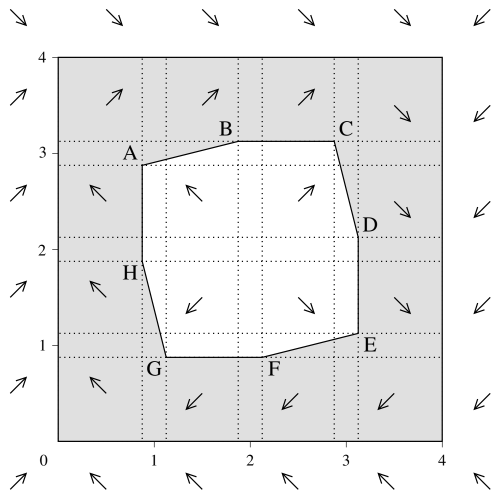

Figure 1: The gray region is an illustration of Ω'.

for all $x \in \Omega'$, and we prove with similar arguments that $(\partial f_2/\partial x_2)(x) < 0$ for all $x \in \Omega'$. Thus

$$\int_0^T \frac{\partial f_1}{\partial x_1}(\psi(t)) + \frac{\partial f_2}{\partial x_2}(\psi(t))dt \;<\; 0$$

and we deduce that ψ is stable.

References

[1] A. Richard and J.-P. Comet. Stable pariodicities and negative circuits in differential systems. *Journal of Mathematical Biology*, 63:593–600, 2011.