Network Coding
Exercise session

I. Questions on the lecture

1. Recall what is the general principle of NC, and what is its primary goal.

Assume that a number of original packets $M^1, ..., M^n$ are generated by one or several sources. In linear network coding, each packet in the network is associated with a sequence of coefficients $g_1, ..., g_n$ in $\mathbb{F}_{2^s}$ and is equal to $X = \sum_{i=1}^{n} g_i M^i$. The summation has to occur for every symbol position, i.e., $X_k = \sum_{i=1}^{n} g_i M_k^i$, where M_k^i and X_k is the kth symbol of M^i and X respectively. In the example of Figure 1, the field is $\mathbb{F}_2 = \{0, 1\}$, a symbol is a bit, and the linear combination sent by S after receiving $M^1 = a$ and $M^2 = b$ is $M^1 + M^2$ (the + sign here is addition in $\mathbb{F}_2$, i.e., bitwise `xor`).

For simplicity, we assume that a packet contains both the coefficients $g = (g_1, ..., g_n)$, called *encoding vector*, and the encoded data $X = \sum_{i=1}^{n} g_i M^i$, called *information vector* [6]. The encoding vector is used by recipients to decode the data, as explained later. For example, the encoding vector $\mathbf{e}_i = (0, ..., 0, 1, 0, ...0)$, where the 1 is at the ith position, means that the information vector is equal to M^i (i.e., is not encoded).

Encoding can be performed recursively, namely, to already encoded packets. Consider a node that has received and stored a set $(g^1, X^1), ..., (g^m, X^m)$ of encoded packets, where g^j [resp. X^j] is the encoding [resp. information] vector of the jth packet. This node may generate a new encoded packet (g', X') by picking a set of coefficients $h_1, ..., h_m$ and computing the linear combination $X' = \sum_{j=1}^{m} h_j X^j$. The corresponding encoding vector g' is not simply equal to h, since the coefficients are with respect to the original packets $M^1, ..., M^n$; in contrast, straightforward algebra shows that it is given by $g'_i = \sum_{j=1}^{m} h_j g_i^j$. This operation may be repeated at several nodes in the network.

Using the notation and description above:

2. What is the linear system to solve in order to get the original packets in terms of the received encoded (possibly several times) packets? ***(2pts)***

3. What is the condition for this system to be solvable? What is the connection with the min-cut max-flow theorem? ***(2pts)***

4. What are the advantages of random linear network coding? ***(1pt)***

II. Inter-session network coding

The following figure represents the two-unicast butterfly network. Each arc represents a directed arc that is capable of carrying a single packet reliably per unit of time. There is one packet b_1 present at source node s_1 that we wish to communicate to sink node t_1 and one packet b_2 present at source node s_2 that we wish to communicate to sink node t_2.

1. What is the rate you can get with single routing?
2. How would you use coding and what is the rate you can get doing so?

3. Suggest a way of implementing this scheme.
4. In a lossy (wireless) network or in case of congestion at nodes, what must we pay attention to ?