An Evolutionary Approach to Multiperiod Asset Allocation

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Abstract. Portfolio construction can become a very complicated problem, as regulatory constraints, individual investor's requirements, non-trivial indices of risk and subjective quality measures are taken into account, together with multiple investment horizons and cash-flow planning. This problem is approached using a tree of possible scenarios for the future, and an evolutionary algorithm is used to optimize an investment plan against the desired criteria and the possible scenarios. An application to a real defined benefit pension fund case is discussed.

1 Introduction

The recent rise of interest in mutual funds and private pension plans in Italy has brought about a powerful drive to provide investors with more and more sophisticated, flexible and customized products.

Typically, a process of investing consists of two main steps.

- First, benchmarks for some asset categories (asset classes) are selected. These constitute what is called the *opportunity set.*
- In the second step, a mix of asset classes in the opportunity set is strategically chosen according to a specified criterion. This step is the main task of asset allocation. Following this kind of process, asset allocation policy decisions may have a relevant impact on portfolio performance, as shown by the seminal work of Brinson and colleagues [2].

Institutional investors have to manage a complex cash-flows structure. In addition, they wish to have the highest return as possible while, at the same time, guaranteeing their invested wealth to meet liabilities over time. In other words, their financial needs can be seen as a trade-off between the maximization of return and the chance of achievement of minimal targets.

The traditional asset allocation framework, based on the mean-variance approach [6], is not adequate to model the problem described above. The best way to solve the problem involves a dynamic approach, which takes into account the randomness of the time paths of the asset returns, and allocation becomes essentially a multi-period optimization problem. A sequence of portfolios is sought which maximizes the probability of achieving in time a given set of financial goals. The portfolio sequences are to be evaluated against a probabilistic representation of the foreseen evolution of a number of asset classes, the opportunity set for investment.

Evolutionary algorithms [1, 3, 4] are used to optimize the investment plan against the desired criteria and the possible scenarios. Some of the ideas presented below stem from a former study on the use of evolutionary algorithms for single-period portfolio optimization [5]. Another source of inspiration was an application of stochastic programming to dynamic asset allocation for individual investors, jointly developed by Banca Fideuram S.p.A. and Frank Russell Company (more details can be found in several papers in [8]).

2 The Problem

Objective data (expected conditions of financial markets) consist of the possible realizations of the return of the asset classes which are described by means of a scenario tree.

The multi-period optimization problem for institutional investors involves maximizing a function measuring the difference between the overall wealth and the *cost* of a failure to achieve a given overall wealth, taking into account the cash-flow structure.

Wealth (return) and the cost of falling short of achieving a predefined level (down side risk approach) are evaluated for each subperiod.

Along with objective data and criteria, which are relatively easy to quantify and account for, a viable approach to multi-period asset allocation should consider a number of subjective criteria. These comprise desired features for an investment plan, as seen from the stand point of the institutional investor and guidelines dictated by regulations or policies of the asset manager. The former are in fact criteria used to personalize the investment plan according to preferences of the customer, even at the expense of its potential return. The latter criteria may consist of specific product requirements or an *ad hoc* time-dependent risk policy.

In order to match subjective advices both from the institutional investors and the asset manager, constraints are implemented by means of the penalty functions technique (cf. Section 2.4). In this way, the relative importance of the various constraints can be fine-tuned by the asset manager to ensure a higher degree of customization for solutions. This approach makes it possible to treat advices as "soft" constraints, allowing trade-offs with other criteria. As the weight associated to a soft constraint increases, the relevant advice becomes more and more binding.

Overall, the criteria (objectives and constraints) of the multi-period portfolio optimization problem considered in this article can be grouped in five types:

1. Maximize the final wealth ($\mathrm{EW}_{\mathrm{Fin}}$);
2. Minimize the risk of not achieving the intermediate and final wealth objectives (MR);
3. Minimize turn-over from one investment period to the next one, in order to keep commissions low (TC);
4. Respect absolute constraints on the minimum and maximum weight for any asset classes or groups thereof (WAG);
5. Respect relative constraints on the weight of asset classes or groups thereof (e.g.: European bonds shall be at least twice the weight of non-European bonds), (WRG);

Optimization is carried out on a portfolio tree corresponding to the scenario tree of returns, considered as a whole. A portfolio tree can be regarded as a very detailed investment plan, providing different responses in terms of allocation, to different scenarios. These responses are hierarchically structured, forming a tree whose root represents the initial allocation of assets. Each path from the root to a leaf represents a possible evolution in time of the initial asset allocation, in response to, or in anticipation of, changing market conditions. The objective function depends on all the branches of the scenario tree (path dependence).

2.1 Notation

The following conventions will hold in the rest of the article:

N	number of asset classes;
T	number of periods (also called *levels*);
Fin	set of leaf nodes;
Int	set of internal nodes;
$\Pr(i)$	absolute probability associated with node i
$W(i)$	wealth at node i, *without* considering cash-flows
$W^*(i)$	wealth at node i, considering cash-flows
$\ell(i)$	period (level) of node i
$K(t)$	goals for downside risk calculation
$q(t)$	risk aversion coefficients
$\mathrm{ag}(t)$	set of asset groups at period t
$\|X\|$	cardinality of set X

A group of asset classes (or asset group in short) $g \in \mathrm{ag}(t)$ for some t can be thought of as a set of indices $g = \{i_0, \ldots, i_{m-1}\}$ referring to the asset classes that belong in it.

2.2 Constraints

For each asset class and each asset group, the minimum and maximum weight is given for portfolios of all periods. In addition, the maximum variation of each asset class with respect to the previous period is given. These constraints are defined by six matrices:

$W_{it}^{\min}$	minimum weight for asset class i in period t;
$W_{it}^{\max}$	maximum weight for asset class i in period t;
D_{it}^{-}	maximum negative variation for asset class i in period t;
D_{it}^{+}	maximum positive variation for asset class i in period t;
$\mathrm{WG}_{gt}^{\min}$	minimum total weight of asset group $g \in \mathrm{ag}(t)$ in period t;
$\mathrm{WG}_{gt}^{\max}$	maximum total weight of asset group $g \in \mathrm{ag}(t)$ in period t.

Beside absolute constraints like the above, relative constraints on asset groups can be defined in the form

$$\begin{array}{l} \mathrm{wrelg}(\mathbf{w}) > 0, \text{with} \\ \mathrm{wrelg}(\mathbf{w}) = k + k_0 * \mathrm{wag}(0) + \ldots + k_{m-1} * \mathrm{wag}(m-1), \end{array} \tag{1}$$

with $k, k_0, \ldots, k_{m-1}$ arbitrary constants and

$$\mathrm{wag}(g) = \sum_{i \in g} w_i$$

being the total weight of asset group g in portfolio $\mathbf{w}$; here m is the number of asset groups at the period of portfolio $\mathbf{w}$. For each period, any number of such constraints can be imposed, meaning that all portfolios at that period must satisfy them.

2.3 Wealth

The primary objective of an investor is to maximize wealth. Accordingly, the objective function, whose details are given in Section 2.4, is defined in terms of the wealth of single portfolios in the portfolio tree.

By wealth of a portfolio, we mean the quantity $W^*(\mathbf{s}, \mathbf{w})$, measuring the market value of portfolio $\mathbf{w}$ if sold in node $\mathbf{s}$ of the scenario tree. Wealth is calculated at each node while keeping into account the effect of cash-flows $F(t)$:

$$W_t^* = W_t + F(t). \tag{2}$$

2.4 Objective Function

The objective function z, to be minimized, is given by

$$z = -\mathrm{EW}_{\mathrm{Fin}} + \mathrm{MR} + \mathrm{TC} + \mathrm{WAG} + \mathrm{WRG}. \tag{3}$$

The first four terms in Equation 3 are the objectives. The first two terms ($\mathrm{EW}_{\mathrm{Fin}}$, the final wealth and MR, the mean cost of risk) are the fundamental objectives, i.e. optimization criteria, in that an investor wishes to maximize wealth while trying to minimize risk. The following term (TC, transaction cost) is an optional objective, depending on the valuations of the fund manager. All the other terms are penalties triggered by any violation of a number of constraints. Other constraints can be satisfied "by construction", as explained in Section 3.2, and therefore they need not be associated with a penalty term.

For each constraint, constant $\alpha_t^{\text{constraint}}$ is a non-negative weight allowing the fund manager to express how much the relevant penalty is to affect the search of an optimal portfolio.

The individual terms of the objective functions are discussed in the following paragraphs.

Expected Wealth The expected wealth at final nodes (the leaves of the scenario tree)

$$\mathrm{EW}_{\mathrm{Fin}} = \sum_{i \in \mathrm{Fin}} \Pr(i) \cdot W^*(i).$$

Mean Risk This term is calculated using a penalty function of a portfolio *underperformance*, which is tightly linked with the concept of *downside risk*. The motivations for this choice are illustrated and discussed in a previous work [5].

The mean risk is the mean cost of downside risk,

$$\mathrm{MR} = \sum_{i : W^*(i) < \theta_i} \alpha_{\ell(i)}^{\mathrm{MR}} \Pr(i)[k(\theta_i - W^*(i))]^{q(\ell(i))},$$

where $\Pr(i)$ is the probability of scenario i, and $\theta_i = \max\{K(\ell(i)), W_\pi(i)\}$ is the threshold for wealth under which one can speak of an underperformance of the investment plan; furthermore,

- $\ell(i)$ is the period of node i,
- $K(t)$ is a parameter provided by the investor for each period t, indicating the (intermediate or final) accumulation goal, also called *strike level* at period t,
- $W_\pi(i)$ is the value at node i of the benchmark portfolio π,
- k is a scale factor for underperformance,
- $q(t) \geq 0$ characterizes the investor's aversion to risk in period t.

Transaction Cost This takes into account the effect of transaction cost, calculated as

$$\mathrm{TC} = \sum_{i \in \mathrm{Int}} \alpha_{\ell(i)}^{\mathrm{TC}} \Pr(i) \sum_{0 \leq j < N} \Delta_{ij} \left([\Delta_{ij} > 0] c_j^{\mathrm{buy}} - [\Delta_{ij} < 0] c_j^{\mathrm{sell}} \right),$$

where $\Delta = w_{ij} - \hat{w}_{\mathrm{parent}(i),j}$ and the c_j^{buy} and c_j^{sell} are parameters specified by the fund manager, providing the percent cost respectively for the purchase and for the sale of assets in class j. Here, $\hat{w}_{\mathrm{parent}(i),j}$ stands for the weights of the parent portfolio $w_{\mathrm{parent}(i),j}$, modified by the price variations of the asset classes since the last period. Of course, for $i = 0$, $\Delta_{ij} = 0$ for all j provided that the investor does not have an initial portfolio to re-optimize, in which case this is assumed to be the parent portfolio for the calculation of Δ_{ij}.

Absolute Asset Group Constraints The degree of violation of absolute constraints on asset groups

$$\mathrm{WAG} = \sum_{0 \le t < T-1} \frac{\alpha_t^{\mathrm{WAG}}}{\|\mathrm{nal}(t)\|} \sum_{g \in \mathrm{ag}(t)} \frac{1}{\|\mathrm{ag}(t)\|} \sum_{i \in \mathrm{nal}(t)} d(g, i),$$

where $\mathrm{nal}(t)$ is the set of nodes at level t, $d(g, i)$ is the distance of the weight of asset group g from the interval $[\mathrm{WG}_{gt}^{\min}, \mathrm{WG}_{gt}^{\max}]$ for portfolio $\mathbf{w}_i$.

Relative Asset Group Constraints The degree of violation of relative constraints on asset groups

$$\mathrm{WRG} = \frac{1}{\|\mathrm{Int}\|} \sum_{i \in \mathrm{Int}} -\alpha_{\ell(i)}^{\mathrm{WRG}} \sum_{j=1}^{J_{\ell(i)}} \min(0, \mathrm{wrelg}_{\ell(i),j}(\mathbf{w}_i))$$

where all wrelg_{tj} are defined according to Equation 1, and J_t represents the number of relative constraints on asset groups at period t. All wrelg_{tj} represent constraints on asset groups at period t, the constraint being satisfied if its value is non-negative: that is why the objective function is penalized for each negative value of wrelg.

3 The Optimization Engine

The optimization engine of the Galapagos system employs evolutionary algorithms to solve a multi-period optimization problem.

3.1 The Algorithm

The optimization algorithm, which operates on an array *individual*[*popSize*] of individuals (i.e. the population) is a standard, generational replacement, elitist evolutionary algorithm.

The various elements of the algorithm are illustrated in the following subsections.

3.2 Encoding

A solution to the problem, that is a portfolio tree, is encoded as a string of $\|\mathrm{Int}\|N$ bytes. Each portfolio in the tree is encoded by a substring of N bytes, starting from the root and visiting all the nodes of the tree breadth-first. While the encoding of the root portfolio is direct, the encoding of all the children portfolios is *differential*. This means that only the changes with respect to the parent portfolio are encoded. The differential encoding makes it easy to enforce the D_{it}^- and D_{it}^+ constraints, as explained below. When these constraints are absent, the whole encoding turns out to be absolute.

The decoding of the genotype is performed in such a way that the $W_{it}^{\min}$, $W_{it}^{\max}$, D_{it}^{-} and D_{it}^{+} constraints are satisfied by construction for all asset classes i and for all periods t.

The decoding of the genotype of a portfolio tree starts by decoding the root portfolio, as follows.

Let $\mathbf{g} = (g_1, \ldots g_N)$ be the genotype substring for the root portfolio; let $\mathbf{w} = (w_1, \ldots w_N)$ and $\bar{\mathbf{w}} = (\bar{w}_1, \ldots \bar{w}_N)$ be respectively the non-normalized weight vector and the corresponding normalized vector for the same portfolio.

It is assumed

$$w_i = W_{i0}^{\min} + \left(W_{i0}^{\max} - W_{i0}^{\min}\right) \frac{g_i}{\sum_{k=1}^{N} g_k}.$$

Therefore, vector $\mathbf{w}$ satisfies the $W^{\min}$ and $W^{\max}$ constraints, but not necessarily $\sum_{i=1}^{N} w_i = 1$. The normalized vector $\bar{\mathbf{w}}$ is obtained from $\mathbf{w}$ as follows:

Let $\Delta = 1 - \sum_{i=1}^{N} w_i$, and

$$r_i = \begin{cases} W_i^{\max} - w_i \text{ if } \Delta \geq 0, \\ W_i^{\min} - w_i \text{ if } \Delta < 0. \end{cases}$$

The normalized weights are given by

$$\bar{w}_i = w_i + r_i \frac{\Delta}{\sum_{k=1}^{N} r_k}.$$

This procedure is illustrated in Figure 1 by means of a three asset class example.

The decoding then continues with the children portfolios, in an analogous way.

3.3 Initialization

The initial population is seeded with random genotypes. This means that every byte is assigned a value at random with uniform probability over $\{0, \ldots, 255\}$.

It is important to notice that, because of the complicated decoding procedure illustrated in Section 3.2, the corresponding portfolio trees will not be uniformly distributed over the space of all feasible portfolio trees.

3.4 Crossover

Uniform balanced crossover, as illustrated in [5], was adopted. Let γ and κ be substrings of length N in the two parent chromosomes corresponding to the same portfolio. For each gene i in the offspring, it is decided with equal probability whether it should be inherited from one parent or the other. Suppose that the ith gene in γ, γ_i, has been chosen to be passed on to a child genotype substring ζ. Then, the value of the ith gene in ζ is

$$\zeta_i = \min\left(255, \gamma_i \frac{\sum_{j=1}^{N} \gamma_j + \sum_{j=1}^{N} \kappa_j}{2 \sum_{j=1}^{N} \gamma_j}\right).$$

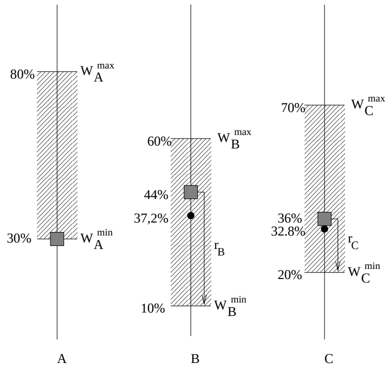

Fig. 1. Example of the derivation of a root portfolio for a problem with three asset classes, A, B and C and constraints $W_A^{\min} = 30\%$, $W_A^{\max} = 80\%$, $W_B^{\min} = 10\%$, $W_B^{\max} = 60\%$, $W_C^{\min} = 20\%$ and $W_C^{\max} = 70\%$. The feasible weight region for each asset class is indicated by the dashed texture. Assuming that the genotype be the integer vector $\mathbf{g} = (g_A, g_B, g_C) = (0, 15, 7)$, the non-normalized weights $w_A = 30\%$, $w_B = 44\%$ and $w_C = 36\%$, represented as grey squares, are calculated. Since their sum is 110%, $\Delta = -10\%$, while $r_A = 0$, $r_B = -34\%$ and $r_C = -16\%$. The normalized weights, represented as solid bullets, are therefore calculated as $\bar{w}_A = 30\%$, $\bar{w}_B = 37.2\%$ and $\bar{w}_C = 32.8\%$.

The main motivation behind this operation is to preserve the *relative* meaning of genes, which depends on their context. Indeed, genes have a meaning with respect to the other genes of the same portfolio, since the corresponding weight is obtained by normalization. Therefore, crossing the two substrings $(1, 0, 0)$ and $(0, 0, 10)$ to obtain $(1, 0, 10)$ would not correctly interpret the fact that the 1 in the first substring means "put everything on the first asset".

3.5 Mutation

Mutation perturbs genotypes by randomly changing each gene (consisting of one byte, corresponding to the weight of an asset class in one of the portfolios in the portfolio tree) with independent and identical probability p_{mut}.

There is a difference in how this random change is carried out between the first N bytes, encoding for the root portfolio, and the rest of the genotype, reflecting the fact that while the former are a direct encoding of a portfolio, the latter encode for the variation in weight from the parent portfolio.

Therefore, the first N bytes are increased or decreased by one with equal probability, while the remaining bytes are completely overwritten by a new random value from $\{0, \ldots, 255\}$. In other words, the first N bytes mutate very gradually, and the others can mutate much widely.

3.6 Fitness

The fitness of an individual (i.e. of a portfolio tree) is a positive real number f obtained from the objective function z via the transformation

$$f = \begin{cases} \frac{1}{z+1}, & \text{if } z > 0; \\ 1 - z, & \text{if } z \leq 0. \end{cases}$$

3.7 Selection

Elitist fitness proportionate selection, using the roulette-wheel algorithm, was implemented, using a simple fitness scaling whereby the scaled fitness $\hat{f}$ is

$$\hat{f} = f - f_{\text{worst}}, \tag{4}$$

where f_{worst} is the fitness of the worst individual in the current population.

Overall, the algorithm is elitist, in the sense that the best individual in the population is always passed on unchanged to the next generation, without undergoing crossover or mutation.

4 Experimental Results

4.1 A Case Study

We present a proposal-study made by Fideuram Capital to a defined benefit pension fund of the Italian subsidiary of a large multinational company at the end of December 1998.

The data set, provided by the pension fund, concerned its demographic structure and the wealth level at the moment of proposal.

Furthermore, the fund board of directors, as mandated by Italian regulations, gave some investment directives: maintain the equity exposure of the allocation plan preferably under 30%.

To begin with, we processed the demographic features of the fund members, in order to infer the pattern of cash-out flows. Furthermore, we carried out an actuarial analysis of the liabilities, to calculate their modified duration. The synopsis of cash flows and duration is given in Table 1.

Year	1	2	3	4	5	...
Cash-flow (in millions)	3593.979	3559.601	3522.412	3482.310	3439.196	...
Duration	10.02					

Table 1. Cash flows for the first five years and duration of liabilities for all years.

4.2 Problem Formulation

In this subsection we show how to approach the problem illustrated above using our model. The generation of the scenario tree was based on the data provided by the econometric model for forecasting, jointly developed by the Research Department of Fideuram Capital and Frank Russell Company. The holding period of the investment plan is five years.

It is common sense that matching the duration of asset portfolio with the liability duration minimizes future funding uncertainty [7]. This widespread methodology leads to portfolios having a return approximately equal to the yield of a bond with the same maturity as the duration. We took this return as the minimal return for an optimal allocation plan. Likewise, given the attitude toward risk of the institutional investor, we try to perform better than an alternative suitable risk-free investment. Because of the defined benefit nature of the pension fund, an optimal plan is supposed to outperform the "duration maturity" bond *each* year, once again to reduce uncertainty. In order to translate this methodology in the language of the Galapagos system, we calculated the strike levels (target returns) in terms of wealth, assuming that the initial wealth is one.

The preferences communicated by the fund board of directors were implemented in terms of "soft" constraints, by associating a penalty to plans with equity exposure above the suggested threshold. However, we still admit investment plans with a higher equity exposure, provided that violation is compensated for by a much higher performance.

For this particular application, transaction costs (TC) were not taken into account. Parameter q, modeling the investor's aversion to risk was set to 2, reflecting the fact that a typical pension fund has a conservative investment profile.

The values for the other parameters were dictated by sensitivity analysis studies previously carried out by the asset allocation bureau at Fideuram Capital. The parameter setting is summarized in Table 2.

4.3 Results

The evolutionary algorithm, with a population size of 100 and a scenario tree of 14808 nodes, was run several times for the problem presented in Section 4.1 on a Pentium 233 MHz PC with the Microsoft Windows NT operating system and 128 Mbytes of RAM. All runs converged to the same optimal investment plan, whose details are given in Tables 3, 4 and 5 for the first five years of the overall holding periods. Average running time before convergence was little more than 8 hours.

The solution abtained was judged completely satisfactory by experts, because it fulfilled all requirements with a high chance of achievement and, in addition, provided a fair level of expected annual returns, as an inspection of Table 3 can confirm.

Year t	0	1	2	3	4	5
cash flow $F(t)$	-1	0.07037	0.06969	0.06896	0.06817	0.06733
recursivity	1	1	1	1	1	1
strike level $K(t)$	n/a	0.9556	0.9108	0.8649	0.8192	0.7732
aversion $q(t)$	n/a	2	2	2	2	2
α_t^{MR}	n/a	4	4	4	4	4
$\mathrm{WG}_{\mathrm{EQUITY},t}^{\max}$	n/a	30%	30%	30%	30%	30%
α_t^{WAG}	n/a	10	10	10	10	10
tree branching	50	6	4	3	2	0
D_{it}^{+} $(\forall i)$	n/a	40%	40%	40%	40%	40%
D_{it}^{-} $(\forall i)$	n/a	40%	40%	40%	40%	40%

Asset groups: {CASH, BOND, EQUITY}

Table 2. Summary of parameter values.

Date	expected wealth	stdev of wealth	cash flows	expected return	stdev of return	chance of achievement
01-Jan-99	51,075,957,450	n/a	n/a	n/a	n/a	n/a
01-Jan-00	51,539,522,840	3,593,979,000	6.23%	7.94%	6.23%	85.33%
01-Jan-01	52,162,291,989	3,559,601,000	10.85%	8.07%	9.76%	86.28%
01-Jan-02	52,232,981,114	3,522,412,000	16.71%	6.96%	12.31%	87.36%
01-Jan-03	52,712,022,519	3,482,310,000	23.02%	7.70%	14.38%	88.41%
01-Jan-04	54,052,204,567	3,439,196,000	30.46%	9.16%	16.94%	90.83%

Table 3. Expected Wealth and Wealth Objectives for the optimal investment plan. Wealth is expressed in December 1998 real ITL.

Date	01-Jan-99	01-Jan-00	-Jan-01	-Jan-02	-Jan-03
JPM_EMU_6M	20.97%	16.54%	15.08%	8.55%	9.91%
SSB_EMU_S	3.71%	10.64%	0.00%	14.57%	8.67%
SSB_EMU_M	3.73%	19.59%	18.36%	8.55%	0.39%
SSB_EMU_L	0.00%	0.00%	8.31%	3.18%	5.84%
SSB_US	15.08%	8.33%	0.87%	14.71%	8.35%
SSB_JP	22.28%	7.40%	7.15%	5.50%	13.08%
SSB_UK	0.00%	0.48%	19.81%	5.24%	10.74%
COMIT	3.41%	0.85%	2.71%	3.18%	6.51%
MSUK	3.94%	11.37%	2.71%	9.41%	6.34%
MSEUXUK	16.39%	15.99%	13.33%	12.59%	11.96%
MSUS	3.61%	7.50%	1.06%	3.98%	8.29%
MSJP	6.88%	1.31%	10.63%	10.54%	9.91%

Table 4. Asset Class Weights for the sequence of portfolios corresponding to the mean-return scenarios, i.e. those scenarios in which the returns for each asset class correspond to the mean of the probability distribution.

5 Conclusions

This paper proposes a new approach to solve a multiperiod asset allocation problem. If we take into account factors like regulatory constraints, individual investors requirements, non-trivial indices of risk and subjective quality measures, portfolio construction becomes a very complicated highly non-linear problem. As the complexity of the problem increases, traditional optimization techniques are no longer useful tools for finding satisfactory solutions. This is so because their performance does not scale nicely and they typically lack flexibility, whereby accommodating new criteria for optimization along with old ones almost always requires recoding large parts of the algorithms.

Evolutionary algorithms are particularly attractive for their flexibility because they do not make any assumptions on the objective function. The user who decides about criteria and constraints can consider them in any form, i.e. he or she is not obliged to choose them linear or derivable because all kind of functions are allowed.

Date	CASH	BOND	EQUITY
01-Jan-99	24.68%	41.10%	34.22%
01-Jan-00	27.18%	35.80%	37.02%
01-Jan-01	15.08%	54.49%	30.43%
01-Jan-02	23.12%	37.18%	39.70%
01-Jan-03	18.58%	38.40%	43.02%

Table 5. Weights of the asset groups in the portfolio sequence of Table 4.

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