Possibilistic Planning Using Description Logics: A First Step

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Abstract. This paper is a first step in the direction of extending possibilistic planning to take advantage of the expressive power and reasoning capabilities of fuzzy description logics. Fuzzy description logics are used to describe knowledge about the world and about actions. Fundamental definitions are given and the possibilistic planning problem is recast in this new setting.

1 Introduction

Planning is a branch of artificial intelligence which studies how to find the most suitable sequence of actions to take a system from a given initial state into a desired state, called goal.

In a classical planning problem, it is assumed that actions are deterministic, the initial state is known, and the goal is defined by a set of final states; a solution plan is then an unconditional sequence of actions that leads from the initial state to a goal state. However, most practical problems do not satisfy these assumptions of complete and deterministic information. This is why in recent years many approaches taking into account the uncertainty in the planning problem have been proposed. In particular, a possibilistic approach to planning has been proposed in [2] which is an extension of the well-known STRIPS formalism to make possible the representation of the possibilistic uncertainty. In this formalism, the representation of the states of the world and of the effect of the actions is made using sets of literals. This fact has the advantage that reasoning for solving a planning problem is decidable but it has the disavantage that the formalism thus constructed has a severely limited expressiveness.

This paper provides a first step in the direction of extending that approach to take advantage of: (i) the expressive power of description logics [1], (ii) the decidable reasoning capabilities of description logics and (iii) recent extensions of description logics to take into account uncertainty and imprecision [5].

A relevant advantage of combining fuzzy description logics and planning is that it becomes possible to describe the planning domain using formal ontologies for expressing and organizing the (possibly vague) knowledge available about the planning domain in general and the particular problem at hand. On the one

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hand, it may be possible to stablish a level of detail for the concepts describing the domain (the world states and the effect of actions may be represented by concepts) and, on the another hand, it may be possible to make inferences about the available knowledge. As a consequence, implicit knowledge becomes available. This is possible because a formal ontology contains a structured vocabulary (its "terminology") which defines the relations between different terms.

In this work, we consider that for each planning problem we have two ontologies. One is related to the general knowledge, valid for every planning problem; the other describes knowledge about the specific problem to solve. The ontological general formalization of a planning problem must then respect two important features:

- Analitical feature: it must be easily understandable for all planning problems, i.e, the proposed formalism must be independent of the context of the particular problem.
- Engineering feature: the formalism must be flexible in order to support and to manipulate the knowledge acquired during the planning process.

In Section 2, we provide a brief introduction to description logics. In Section 3, we present a general ontology for the planning domain and a specific ontology describing a simple planning problem in the block world. In Section 4, we define the possibilistic planning problem using fuzzy description logics and Section 5 concludes.

2 Description Logics

Description logics (DL) [1] are a family of logic-based knowledge-representation formalisms, which stem from the classical AI tradition of semantic networks and frame-based systems. Description logics are characterized by a set of constructors for building complex concepts and roles starting from the primitive ones. Concepts represent classes which are interpreted as sets of objects, and roles represent relations which are interpreted as binary relations on objects.

Semantics are expressed in terms of interpretations $\mathcal{I} = (\Delta^{\mathcal{I}}, .^{\mathcal{I}})$, where:

- $\Delta^{\mathcal{I}}$ is the interpretation domain,
- $.^{\mathcal{I}}$ is the interpretation function, which maps:
  - different individuals into different elements of $\Delta^{\mathcal{I}}$,
  - primitive concepts into subsets of $\Delta^{\mathcal{I}}$,
  - primitive roles into subsets of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$.

The architecture of a description-logic-based system is composed of:

- A knowledge base with: (i) terminological knowledge (TBox), containing the definitions of concepts (for example $\mathsf{FreeBlock} \equiv \mathsf{Block} \sqcap \neg\exists \mathsf{on}^{-}.\top$ (a free block is a block such that no other object is on it)) and (ii) the knowledge about objects (ABox) containing assertions which characterize the objects and define the relations between them (for example $\mathsf{on}(\mathsf{BLOCKA}, \mathsf{TABLE})$,
- The Inference Engine,
- Applications

In order to represent uncertainty, incompleteness, and imprecision in knowledge, fuzzy description logics have been proposed that allow for no crisp concept description by using fuzzy sets and fuzzy relations. A fuzzy extension of the description logic $\mathcal{ALC}$ has been introduced in [5], with complete algorithms for solving the entailment problem, the subsumption problem, as well as the best truth-value bound problem.

For the purpose of our approach, we have added the inverse role construct to $\mathcal{ALC}$ yielding the $\mathcal{ALCI}$ language, which is suitable for most planning problems. If R is a role, R^- denotes its inverse, meaning $R^-(a,b) \equiv R(b,a)$, for all individuals a and b. For example, with the description logic $\mathcal{ALC}$, from the assertion $\mathsf{on}(\mathsf{BLOCKB}, \mathsf{BLOCKA})$ it is only possible to deduce that BLOCKB is on BLOCKA. With the $\mathcal{ALCI}$ language we may also express that BLOCKA has BLOCKB on it, because $\mathsf{on}(\mathsf{BLOCKB}, \mathsf{BLOCKA}) = \mathsf{on}^-(\mathsf{BLOCKA}, \mathsf{BLOCKB})$

A fuzzy description logic is identical in most respects to a classical description logic but it assigns a meaning to symbols by means of a fuzzy interpretation $\tilde{\mathcal{I}} = (\Delta^{\tilde{\mathcal{I}}}, .^{\tilde{\mathcal{I}}})$ such that:

- $\Delta^{\tilde{\mathcal{I}}}$ is as in the crisp case, the interpretation domain,
- $.^{\tilde{\mathcal{I}}}$ is the interpretation function mapping:
  - different individuals into different elements of $\Delta^{\tilde{\mathcal{I}}}$ as the crisp case, i.e., $a^{\tilde{\mathcal{I}}} \neq b^{\tilde{\mathcal{I}}}$ if $a \neq b$,
  - a primitive concept A into a membership function $A^{\tilde{\mathcal{I}}} : \Delta^{\tilde{\mathcal{I}}} \mapsto [0,1]$,
  - a primitive role R into a membership function $R^{\tilde{\mathcal{I}}} : \Delta^{\tilde{\mathcal{I}}} \times \Delta^{\tilde{\mathcal{I}}} \mapsto [0,1]$.

3 Domain Representation

In this section, we propose an ontological formalization for the general planning problem, with new representation for a state of the world and new representation for the actions, both using description logics. We also propose a specific ontological formalization for planning in the block world.

3.1 Representing World States

A *w*orld state is best represented as an interpretation $\mathcal{I}$ that is a model of both a TBox $\mathcal{T}$ and an ABox $\mathcal{A}$. Usually, we do not have complete information about the world, i.e., the model $\mathcal{I}$ of $\mathcal{T}$ is not known completely. All we know is some facts about the world which are represented in an ABox $\mathcal{A}$. Thus, all models of the ABox and of the TBox are considered to be possible states of the world.

Let us illustrate an example of specific ontological formalization for a planning problem in the block world. We dispose of tree blocks A, B, C and a robot arm. In the initial state (see Figure 1a), block A and block C are on the table, block B is on block A, and both block B and the robot arm are free.

To describe this problem, we define four individual names: BLOCKA, BLOCKB, BLOCKC, and TABLE, and one role on ($\mathsf{on}(a,b)$ means that a is on b).

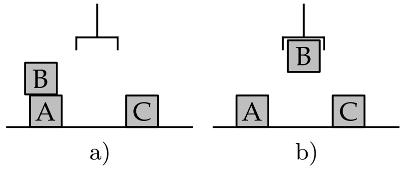

Fig. 1. Two states in the block world: a) the initial state; b) one of the effects of applying the action "grasp block B"

The TBox $\mathcal{T}$, wich represents the specific ontology, will contain definitions of a few useful concepts, namely:

$$\mathcal{T} = \{\mathsf{FreeBlock} \equiv \mathsf{Block} \sqcap \neg\exists \mathsf{on}^{-}.\top, \mathsf{InArm} \equiv \neg\exists \mathsf{on}.\top\}.$$

FreeBlock is a defined concept that characterizes a block which has no other object on it.

InArm is a defined concept that characterizes a block which is in the robot arm. Precisely, we suppose that if a block is not on any of the available objects, then it is in the robot arm.

In the initial state, the situation of Figure 1a will be described with an ABox $\mathcal{A}_0$ containing the follwing assertions:

$$\mathcal{A}_0 = \{\mathsf{on}(\mathsf{BLOCKB}, \mathsf{BLOCKA}), \mathsf{on}(\mathsf{BLOCKA}, \mathsf{TABLE}), \mathsf{on}(\mathsf{BLOCKC}, \mathsf{TABLE})\}.$$

Uncertainty about the world states is represented by means of possibility theory [3]. Possibility theory allows to represent the fact that, at a certain point of the planning process, a state is more possible than another. To represent incomplete and imprecise knowledge about individuals as well as about relations between individuals, we use fuzzy description logics. Consequently, an uncertain state may be represented by a possibility distribution on fuzzy interpretations $\tilde{\mathcal{I}}$.

3.2 Describing Actions

The syntax and the semantics of a deterministic action are those proposed by [4]. The effect of executing an action depends on the context in which it occurs. This kind of representation allows to group several actions into one action and thus contributes to keeping the complexity of the planning problem low. Let $\mathcal{T}$ be an acyclic TBox and $\mathcal{A}$ an ABox. A (deterministic) context-dependent action act is an n-tuple:

$$\mathsf{act} = \{\langle \mathrm{Context}_1, \mathrm{Effect}_1 \rangle, \ldots, \langle \mathrm{Context}_n, \mathrm{Effect}_n \rangle\},$$

in which $\mathrm{Context}_i$ is a set of assertions describing the ith context in which action act may be executed, and Effect_i is a set of primitive assertions describing the ith conditional effect that should be obtained after the execution of the action. If all assertions in $\mathrm{Context}_i$ are satisfied before executing action act, then all assertions in Effect_i should be satisfied afterwards. For all interpretation $\mathcal{I}$, there is a unique $\mathrm{Context}_i$ such that $\mathcal{I} \models \mathrm{Context}_i$.

Definition 1 (DL Possibilistic action). *An action with possibilistic effects describes an uncertain behaviour of executing an action. Its syntax and semantic are inherited from the deterministic action described above extended to take into account uncertain effects [2]. Let $\mathcal{T}$ be an acyclic TBox. A possibilistic action* pact *for $\mathcal{T}$ is an m-tuple of possibilistic effects* pe_i*:*

$$\mathsf{pact} = \left\{ \mathsf{pe}_i = \langle \mathrm{Context}_i, (\pi_{i1}, \mathrm{Effect}_{i1}), \ldots, (\pi_{in_i}, \mathrm{Effect}_{in_i}) \rangle \right\}_{i=1\ldots m},$$

in which $\pi_{ij} \in (0, 1]$ and:

- *for each state represented by an interpretation $\mathcal{I}$ there is a unique* $\mathrm{Context}_{ij}$ *such that $\mathcal{I} \models \mathrm{Context}_{ij}$.*
- *for all i, $\max_{1 \leq j \leq n_i} \pi_{ij} = 1$*

The following example illustrates a possibilistic action "grasp$_B$" as follows:

$$\mathrm{grasp_B} = \left\{ \begin{array}{l} \langle \{\mathsf{FreeBlock(BLOCKB)}, \\ \quad \neg\mathsf{InArm(BLOCKA)} \sqcap \neg\mathsf{InArm(BLOCKB)} \sqcap \neg\mathsf{InArm(BLOCKC)}\}, \\ (1, \{\neg\mathsf{on(BLOCKB, BLOCKA)}\}), (0.2, \emptyset) \rangle \\ \\ \langle \{\neg\mathsf{FreeBlock(BLOCKB)}\}, (1, \emptyset) \rangle \\ \\ \langle \{\mathsf{InArm(BLOCKA)} \sqcup \mathsf{InArm(BLOCKB)} \sqcup \mathsf{InArm(BLOCKC)}\}, (1, \emptyset) \rangle \end{array} \right\}.$$

Executing the action grasp$_B$ on a state satisfying one of the contexts of the action may results on changes on the world state. These changes are represented by updating the ABox representing the situation of the state. Precisely, applying the action grasp$_B$ on a context in which both block B and the robot arm are free results has two outcomes:

- the robot may succeed, i.e., with possibility 1, it grasps block B. This results on the "retracting" of the assertion $\mathsf{on(BLOCKB, BLOCKA)}$ on the ABox (see Figure 1b).
- The robot may fail, with possibility 0.2, thus leaving the situation unchanged.

The other two possibilistic effects cover the remaining possibilities, and both have no effect.

3.3 Reasoning About Actions

For all primitive concept name A and role name R, the result of the execution of an action act in a state $\mathcal{I}$ considering the unique context $\mathrm{Context}_i$ such that $\mathcal{I} \models \mathrm{Context}_i$, is a state $\mathcal{I}' = \mathrm{Res}(\mathrm{Effect}_{ik}, \mathcal{I})$ such that:

$$A^{\mathcal{I}'} = A^{\mathcal{I}} \cup \{b^{\mathcal{I}} : A(b) \in \mathrm{Effect}_{ik}\} \backslash \{b^{\mathcal{I}} : \neg A(b) \in \mathrm{Effect}_{ik}\}$$

$$R^{\mathcal{I}'} = R^{\mathcal{I}} \cup \{(a^{\mathcal{I}}, b^{\mathcal{I}}) : R(a, b) \in \mathrm{Effect}_{ik}\} \backslash \{(a^{\mathcal{I}}, b^{\mathcal{I}}) : \neg R(a, b) \in \mathrm{Effect}_{ik}\}$$

These definitions ensure that the resulting state $\mathcal{I}'$ is a model of both the ABox and TBox resulting after executing the action on the state $\mathcal{I}$.

Applying a possibilistic action pact on a deterministic state $\mathcal{I}$ results on a possibility distribution definded as follows:

$$\pi[\mathcal{I}'|\mathcal{I}, \mathsf{pact}] = \begin{cases} \max_k \pi_{ik} & \text{if } \mathcal{I} \models \text{Context}_i \text{ and } \mathcal{I}' = \text{Res}(\text{Effect}_{ik}, \mathcal{I}), \\ 0 \text{ otherwise.} \end{cases}$$

$\pi[\mathcal{I}'|\mathcal{I}, \mathsf{pact}]$ espresses the possibility of reaching a possible resulting state $\mathcal{I}'$ after executing the action pact in the state $\mathcal{I}$. If there is more than one path leading from $\mathcal{I}$ to a possible resulting state $\mathcal{I}'$, then the possibility associated to $\mathcal{I}'$ is the maximum of the possibilities of all such paths.

In the case in which there are incompleteness and imprecision on the knowledge about the current state, the reasoning made above must be adapted for allowing fuzzy states. Precisely, we must define the possibility associated to a resulting state when the concepts and roles describing the current state are fuzzy.

Definition 2 (Fuzzy state resulting from a possibilistic action). *Let* $\tilde{\mathcal{I}}$ *be a fuzzy interpretation for both the acyclic TBox* $\mathcal{T}$ *and the ABox* $\mathcal{A}$*. Let* pact *be a possibilistic action and* Context_i *the unique context such that* $\mathcal{S}_{\tilde{\mathcal{I}}}(\text{Context}_i) \in (0,1]$*. For each primitive concept name* A *and role name* R*, the result of executing action* pact *in* $\tilde{\mathcal{I}}$*,* $\tilde{\mathcal{I}}' = \text{Res}'(\text{Effect}_{ik}, \tilde{\mathcal{I}})$ *is such that:*

$$A^{\tilde{\mathcal{I}}'}(b) = \min(\max(A^{\tilde{\mathcal{I}}}(b), \sup_{k: A(b) \in \text{Effect}_{ik}} \pi_{ik}), 1 - \sup_{k: \neg A(b) \in \text{Effect}_{ik}} \pi_{ik})$$

$$R^{\tilde{\mathcal{I}}'}(a,b) = \min(\max(R^{\tilde{\mathcal{I}}}(a,b), \sup_{k: R(a,b) \in \text{Effect}_{ik}} \pi_{ik}), 1 - \sup_{k: \neg R(a,b) \in \text{Effect}_{ik}} \pi_{ik})$$

The possibility distribution on the new fuzzy states obtained after the execution of action pact is given by:

$$\pi[\tilde{\mathcal{I}}'|\tilde{\mathcal{I}}, \mathsf{pact}] = \begin{cases} \max_k \min(\pi_{ik}, \mathcal{S}_{\tilde{\mathcal{I}}}(\text{Context}_i)) \text{ and } \tilde{\mathcal{I}}' = \text{Res}'(\text{Effect}_{ik}, \tilde{\mathcal{I}}), \\ 0 \text{ otherwise.} \end{cases}$$

where $\mathcal{S}_{\tilde{\mathcal{I}}}(\text{Context}_i)$ expresses the degree with which the state $\mathcal{I}$ satisfies the context Context_i and $\pi[\tilde{\mathcal{I}}'|\tilde{\mathcal{I}}, \mathsf{pact}]$ espresses the possibility to arrive in fuzzy state $\tilde{\mathcal{I}}'$ after executing action pact in fuzzy state $\tilde{\mathcal{I}}$.

3.4 Plan of Actions

A sequential plan is a totally ordered set of actions $\langle \mathsf{pact}_i \rangle_{i=0}^{N-1}$ such execution transforms the interpretation representing the possible initial state to one representing possible final states satisfying the desired goal.

A partially ordered plan is a pair $\mathcal{P} = (A, O)$ where A is a set of actions and O is a set of ordering constraints between these actions. A completion of $\mathcal{P}$ is a sequential plan $\mathcal{CP} = \langle \mathsf{pact}_i \rangle_{i=0}^{N-1}$ such that $A = \{\mathsf{pact}_0, \ldots, \mathsf{pact}_{N-1}\}$ and the total ordering $\mathsf{pact}_0 < \cdots < \mathsf{pact}_{N-1}$ is consistent with O.

A consistent partially ordered plan is a plan $\mathcal{P} = (A, O)$ with a consistent set O of ordering constraints.

Executing a plan $\mathcal{P}$ means executing pact_0, pact_1, ..., pact_{N-1} in sequence, where $\langle \mathsf{pact}_i \rangle_{i=0}^{N-1}$ is a completion of $\mathcal{P}$.

The possibility of reaching a given state $\mathcal{I}_N$ by executing a sequential plan of possibilistic actions $\langle \mathsf{pact}_i \rangle_{i=0}^{N-1}$ starting in $\mathcal{I}_0$, is given by:

$$\pi[\mathcal{I}_N | \mathcal{I}_0, \langle \mathsf{pact}_i \rangle_{i=0}^{N-1}] = \max_{\langle \mathcal{I}_1 \ldots \mathcal{I}_{N-1} \rangle} \min_{i=0 \ldots N-1} \pi[\mathcal{I}_{i+1} | \mathcal{I}_i, \mathsf{pact}_i],$$

where $\langle \mathcal{I}_1 \ldots \mathcal{I}_{N-1} \rangle$ represents a sequence of states visited from $\mathcal{I}_1$ to $\mathcal{I}_{N-1}$ and $\mathcal{I}_{i+1}$ is obtained from $\mathcal{I}_i$ by applying Definition 2.

The evaluation of a solution plan is made using the necessity measure which corresponds to the certainty of reaching a goal state after applying the plan.

Let *Goals* be the set of the goal states, and π_{init} a possibility distribution over the possible initial states $\mathcal{I}_0$. The possibility and necessity measures (in possibility theory) to reach a goal state after the execution of the sequential plan $\langle \mathsf{pact}_i \rangle_{i=0}^{N-1}$ from $\mathcal{I}_0$ are given by:

$$\begin{aligned}
\Pi[Goals | \pi_{init}, \langle \mathsf{pact}_i \rangle_{i=0}^{N-1}] &= \max_{\mathcal{I}_0} \min(\Pi[Goals | \mathcal{I}_0, \langle \mathsf{pact}_i \rangle_{i=0}^{N-1}], \pi_{init}(\mathcal{I}_0)) \\
&= \max_{\mathcal{I}_0, \mathcal{I}_N \in Goals} \min(\pi[\mathcal{I}_N | \mathcal{I}_0, \langle \mathsf{pact}_i \rangle_{i=0}^{N-1}], \pi_{init}(\mathcal{I}_0)) \\
N[Goals | \pi_{init}, \langle \mathsf{pact}_i \rangle_{i=0}^{N-1}] &= 1 - \Pi[\overline{Goals} | \pi_{init}, \langle \mathsf{pact}_i \rangle_{i=0}^{N-1}] \\
&= \min_{\mathcal{I}_0, \mathcal{I}_N \in \overline{Goals}} \max(1 - \pi_{init}(\mathcal{I}_0), 1 - \pi[\mathcal{I}_N | \mathcal{I}_0, \langle \mathsf{pact}_i \rangle_{i=0}^{N-1}])
\end{aligned}$$

4 Possibilistic Planning Problem

Given a possibilistic planning described by means of a specific ontology, our objective is to construct a plan whose execution leads the world from an initial possible state to a state satisfying the goals with a given certainty. It corresponds to finding an optimal sequence of transition relation on interpretations that transforms an initial ABox representing the initial state to another ABox representing the final possible states which satisfy the goal conditions.

Definition 3 (Possibilistic Planning Problem). *A possibilistic planning problem Δ is a triple $\langle \pi_{init}, \mathrm{Goals}, A \rangle$ where π_{init} is the possibility distribution associated to the initial state,* Goals *is a set of possible goal states and A is the set of available possibilistic actions.*

Given a possibilistic problem, two criteria may be considered to define a *solution plan* $\mathcal{P}$:

- $\mathcal{P}$ is a γ-*acceptable plan* if $N[\mathrm{Goals} | \pi_{init}, \mathcal{P}] \geq \gamma$;
- $\mathcal{P}$ is an *optimally safe plan*, or simply, *optimal plan* if $N[\mathrm{Goals} | \pi_{init}, \mathcal{P}]$ is maximal among all possible sequential plans.

This definition can be extended to partially ordered sets of actions. Let $\mathcal{P}$ be a consistent partially ordered plan :

- $\mathcal{P}$ is a γ**-acceptable plan** if $N[Goals|\mathcal{I}_0, \mathcal{CP}] \geq \gamma$ for all totally ordered completion $\mathcal{CP}$ of $\mathcal{P}$;
- $\mathcal{P}$ is an **optimal plan** if $N[Goals|\mathcal{I}_0, \mathcal{CP}]$ is maximal among all possible sequential plans for all totally ordered completion $\mathcal{CP}$ of $\mathcal{P}$.

5 Conclusions

In this paper, we have proposed an initial framework for integrating description logics and possibilistic planning. The fundamental definitions for approaching possibilistic planning when knowledge about the world and actions is represented by means of fuzzy DLs have been provided, and the possibilistic planning problem has been recast in this setting. In particular, the framework we propose in this work allows the use of conceptual knowledge for describing both the states of the world and the possibilistic action. A state is represented as an interpretation in description logics; the contexts in which a possibilistic action may be executed and the effects of the action are both represented using concepts and assertions.

We have also defined the reasoning problem for the possibilistic actions thus represented. We have first considered the crisp case with a deterministic representation for a state. In a second time, we have proposed an extension of this representation for taking into account the case in which the knowledge about the state of the world is uncertain.

The main advantage of using fuzzy DLs to represent knowledge about the world and actions is that represetations may be more concise and efficient reasoning algorithms can be exploited to infer implicit knowledge.

References

1. Franz Baader, Diego Calvanese, Deborah McGuinness, Daniele Nardi, and Peter Patel-Schneider, editors. *The Description Logic Handbook: Theory, implementation and applications*. Cambridge, 2003.
2. C. da Costa Pereira, F. Garcia, J. Lang, and R. Martin-Clouaire. Planning with graded nondeterministic actions: a possibilistic approach. *Int. Journal of Intelligent Systems*, 12:935–962, 1997.
3. D. Dubois and H. Prade. *Théorie des possibilités*. Masson, 1988.
4. J. S. Penberthy and D. S. Weld. Ucpop: A sound, complete, partial order planner for adl. In B. Nebel, C. Rich, and W. Swartout, editors, *Principles of Knowledge Representation and Reasoning: Proc. of the Third International Conference (KR'92)*, pages 103–114. Kaufmann, San Mateo, CA, 1992.
5. Umberto Straccia. Reasoning within fuzzy description logics. *Journal of Artificial Intelligence Research*, 14:137–166, 2001.