Possibilistic Test of OWL Axioms under the Open-World Assumption

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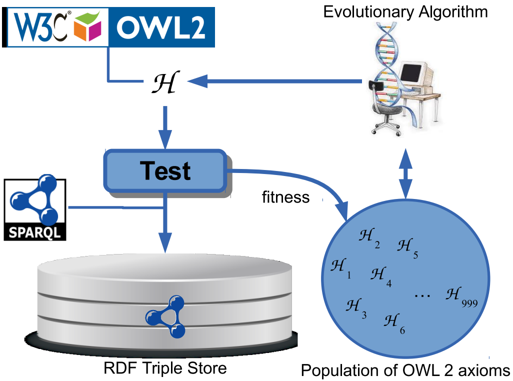
WASPIQ, Arras, June 27, 2017

Motivation: RDF Mining



Problem:

How to test OWL axioms under the open-world assumption?

Agenda

⇒ A possibilistic scoring heuristic
(joint work with Catherine Faron-Zucker and Fabien Gandon)

Basic Intuition

Evaluate the *credibility* of OWL axioms based on RDF *evidence*.

- Theory of a possibilistic framework for OWL axiom testing
  1. Development and logical content of an axiom
  2. Support, confirmation and counterexample of an axiom
  3. Possibility and necessity of an axiom
  4. Acceptance/rejection index (ARI) combining them
- Practical application: test `SubClassOf` axioms against DBpedia.

Problem Statement

Axiom Testing

given a *hypothesis* about the relations holding among some entities of a domain, evaluate its credibility based on the available *evidence*

$$h \overset{?}{\Longleftarrow} e$$

h		e
OWL		RDF

Fundamental problem in epistemology, with ramifications in statistical inference, data mining, inductive reasoning, medical diagnosis, judicial decision making, and even the philosophy of science.

Confirmation is central to this problem

Extended hypothetico-deductivism:

- e confirms h if $h \models e$
- e disconfirms h if $e \models \neg h$

Hempel's Development

Given a body of evidence, a hypothesis h can be *developed* into a finite ground formula.

Definition (Development of a hypothesis)

Let C be a finite set of individual constants of $\mathcal{L}$. The *development* $D_C(h)$ of hypothesis $h \in \mathcal{L}$ according to C, such that $h \models D_C(h)$, is defined recursively as follows: for $\phi, \psi \in \mathcal{L}$,

1. if $C = \emptyset$ or ϕ is atomic, then $D_C(\phi) = \phi$;
2. otherwise,
    1. $D_C(\neg\phi) = \neg D_C(\phi)$;
    2. $D_C(\phi \vee \psi) = D_C(\phi) \vee D_C(\psi)$;
    3. $D_C(\phi \wedge \psi) = D_C(\phi) \wedge D_C(\psi)$;
    4. $D_C(\forall x \phi) = \bigwedge_{c \in C} D_C(\phi\{c/x\})$;
    5. $D_C(\exists x \phi) = \bigvee_{c \in C} D_C(\phi\{c/x\})$.

$\phi\{c/x\}$: ϕ with all free occurrences of x replaced by constant c.

Development of an OWL 2 Axiom

We define a transformation which translates an OWL 2 axiom into a FOL formula based on the OWL direct semantics.

Definition (OWL 2 to FOL Transformation)

Let $t(\cdot; x, y)$ be recursively defined as follows:

- Entities:
  - if d is a data value (a literal), $t(d; x, y) = (x = d)$;
  - if a is an individual name (an IRI), $t(a; x, y) = (x = a)$;
  - if C is an atomic concept, $t(C; x, y) = C(x)$;
  - if D is an atomic datatype, $t(D; x, y) = D(x)$;
  - if R is an atomic relation, $t(R; x, y) = R(x, y)$;

. . . continued on the following slides

Development of an OWL 2 Axiom (continued)

Definition (OWL 2 to FOL Transformation (continued))

- Expressions:
  - $t(R^{-}; x, y) = t(R; y, x)$;
  - $t(C_1 \sqcap \ldots \sqcap C_n; x, y) = t(C_1; x, y) \wedge \ldots \wedge t(C_n; x, y)$;
  - $t(C_1 \sqcup \ldots \sqcup C_n; x, y) = t(C_1; x, y) \vee \ldots \vee t(C_n; x, y)$;
  - $t(\neg C; x, y) = \neg t(C; x, y)$;
  - $t(\{a_1, \ldots, a_n\}; x, y) = t(a_1; x, y) \vee \ldots \vee t(a_n; x, y)$;
  - $t(\exists R.C; x, y) = \exists y(t(R; x, y) \wedge t(C; y, z))$;
  - $t(\forall R.C; x, y) = \forall y(\neg t(R; x, y) \vee t(C; y, z))$;
  - $t(\exists R.\{a\}; x, y) = t(R; x, a)$;
  - $t(\exists R.\mathrm{Self}; x, y) = t(R; x, x)$;
  - $t(\geq nR.\top; x, y) = (\|\{y \mid t(R; x, y)\}\| \geq n)$;
  - $t(\leq nR.\top; x, y) = (\|\{y \mid t(R; x, y)\}\| \leq n)$;
  - $t(= nR.\top; x, y) = (\|\{y \mid t(R; x, y)\}\| = n)$;
  - $t(\geq nR.C; x, y) = (\|\{y \mid t(R; x, y) \wedge t(C; y, z)\}\| \geq n)$;
  - $t(\leq nR.C; x, y) = (\|\{y \mid t(R; x, y) \wedge t(C; y, z)\}\| \leq n)$;
  - $t(= nR.C; x, y) = (\|\{y \mid t(R; x, y) \wedge t(C; y, z)\}\| = n)$;

Development of an OWL 2 Axiom (continued)

Definition (OWL 2 to FOL Transformation (continued))

- Axioms:
  - $t(C_1 \sqsubseteq C_2; x, y) = \forall x(\neg t(C_1; x, y) \vee t(C_2; x, y))$;
  - $t(C_1 \equiv C_2; x, y) =$ $\forall x((t(C_1; x, y) \wedge t(C_2; x, y)) \vee (\neg t(C_1; x, y) \wedge \neg t(C_2; x, y)))$;
  - $t(\mathrm{Dis}(C_1, \ldots, C_n); x, y) =$ $\bigwedge_{i=1}^{n} \bigwedge_{j=i+1}^{n} (\neg t(C_i; x, y) \vee \neg t(C_j; x, y))$;
  - $t(S \sqsubseteq R; x, y) = \forall x \forall y(\neg t(S; x, y) \vee t(R; x, y))$;
  - $t(S_1 \ldots S_n \sqsubseteq R; x, y) = \forall x \forall z_1 \ldots \forall z_{n-1} \forall y(\neg t(S_1; x, z_1) \vee \neg t(S_2; z_1, z_2) \vee \ldots \vee \neg t(S_n; z_{n-1}, y) \vee t(R; x, y))$;
  - $t(R_1 \equiv R_2; x, y) =$ $\forall x \forall y((t(R_1; x, y) \wedge t(R_2; x, y)) \vee (\neg t(R_1; x, y) \wedge \neg t(R_2; x, y)))$;
  - $t(\mathrm{Dis}(R_1, \ldots, R_n); x, y) =$ $\bigwedge_{i=1}^{n} \bigwedge_{j=i+1}^{n} (\neg t(R_i; x, y) \vee \neg t(R_j; x, y))$;
  - etc.

Development of an OWL 2 Axiom (continued)

Let us consider the following OWL 2 axiom:

$$\phi = \texttt{SubClassOf}(\texttt{dbo:LaunchPad dbo:Infrastructure}),$$

Its transformation into FOL is:

$$\begin{aligned}
&t(\phi, x, y) = \\
&t(\texttt{dbo:LaunchPad} \sqsubseteq \texttt{dbo:Infrastructure}, x, y) = \\
&\forall x(\neg t(\texttt{dbo:LaunchPad}, x, y) \vee t(\texttt{dbo:Infrastructure}), x, y)) = \\
&\forall x(\neg \texttt{dbo:LaunchPad}(x) \vee \texttt{dbo:Infrastructure})(x))
\end{aligned}$$

Development of an OWL 2 Axiom (continued)

Definition (Development of an Axiom)

Let ϕ be an OWL 2 axiom and let $\mathcal{K}$ be an RDF dataset. The *development* $D_{\mathcal{K}}(\phi)$ of ϕ wrt $\mathcal{K}$ is defined as follows:

1. Let $\hat{\phi} = t(\phi; x, y)$;
2. Let $I(\mathcal{K})$ be the finite set of individuals in $\mathcal{K}$;
3. $D_{\mathcal{K}}(\phi) = NF(\hat{D}(\hat{\phi}))$, where
   - $\hat{D}(\cdot)$ is recursively defined as follows:
     1. if $\hat{\phi}$ is atomic, then $\hat{D}(\hat{\phi}) = \hat{\phi}$,
     2. $\hat{D}(\neg\hat{\phi}) = \neg\hat{D}(\hat{\phi})$,
     3. $\hat{D}(\hat{\phi} \vee \hat{\psi}) = \hat{D}(\hat{\phi}) \vee \hat{D}(\hat{\psi})$,
     4. $\hat{D}(\hat{\phi} \wedge \hat{\psi}) = \hat{D}(\hat{\phi}) \wedge \hat{D}(\hat{\psi})$,
     5. $\hat{D}(\forall x\hat{\phi}) = \bigwedge_{c \in I(\mathcal{K})} \hat{D}(\hat{\phi}\{c/x\})$,
     6. $\hat{D}(\exists x\hat{\phi}) = \bigvee_{c \in I(\mathcal{K})} \hat{D}(\hat{\phi}\{c/x\})$;
   - $NF(\cdot)$ transforms its input into either CNF or DNF (whichever has the greatest number of basic statements).

Content of an Axiom

Definition (Content of Axiom ϕ)

Given an RDF datset $\mathcal{K}$, $content_{\mathcal{K}}(\phi)$, is defined as the set of all the basic statements of $D_{\mathcal{K}}(\phi)$.

E.g., $\phi = \texttt{dbo:LaunchPad} \sqsubseteq \texttt{dbo:Infrastructure}$

Let us assume $\mathcal{K}$ = DBpedia; then

$$
\begin{aligned}
t(\phi; x, y) &= \forall x(\neg \texttt{dbo:LaunchPad}(x) \vee \texttt{dbo:Infrastructure}(x)) \\
D_{\mathcal{K}}(\phi) &= \bigwedge_{c \in I(\mathcal{K})} (\neg \texttt{dbo:LaunchPad}(c) \vee \texttt{dbo:Infrastructure}(c)) \\
content(\phi) &= \{\neg \texttt{dbo:LaunchPad}(c) \vee \texttt{dbo:Infrastructure}(c) : \\
&\qquad c \text{ is a resource occurring in DBPedia}\}
\end{aligned}
$$

By construction, for all $\psi \in content(\phi)$, $\phi \models \psi$.

Confirmation and Counterexample of an Axiom

Given $\psi \in \mathit{content}(\phi)$ and an RDF dataset $\mathcal{K}$, three cases:

1. $\mathcal{K} \models \psi$: $\longrightarrow$ ψ is a *confirmation* of ϕ;
2. $\mathcal{K} \models \neg\psi$: $\longrightarrow$ ψ is a *counterexample* of ϕ;
3. $\mathcal{K} \not\models \psi$ and $\mathcal{K} \not\models \neg\psi$: $\longrightarrow$ ψ is neither of the above

Selective confirmation: a ψ favoring ϕ rather than $\neg\phi$.
$\phi = \mathtt{Raven} \sqsubseteq \mathtt{Black} \longrightarrow \psi =$ a black raven (vs. a green apple)

Idea

Restrict $\mathit{content}(\phi)$ just to those ψ which can be counterexamples of ϕ. Leave out all ψ which would be trivial confirmations of ϕ.

Support, Confirmation, and Counterexample of an Axiom

Definition

Given axiom ϕ, let us define

$$u_\phi = \|content(\phi)\| \text{ (a.k.a. the } \textit{support} \text{ of } \phi)$$
$$u_\phi^+ = \text{the number of confirmations of } \phi$$
$$u_\phi^- = \text{the number of counterexamples of } \phi$$

Some properties:

- $u_\phi^+ + u_\phi^- \leq u_\phi$ (there may be ψ s.t. $\mathcal{K} \not\models \psi$ and $\mathcal{K} \not\models \neg\psi$)
- $u_\phi^+ = u_{\neg\phi}^-$ (confirmations of ϕ are counterexamples of $\neg\phi$)
- $u_\phi^- = u_{\neg\phi}^+$ (counterexamples of ϕ are confirmations of $\neg\phi$)
- $u_\phi = u_{\neg\phi}$ (ϕ and $\neg\phi$ have the same support)

Probability-Based Axiom Scoring

Score from statistical inference: $\Pr(\phi \text{ is true} \mid \text{evidence})$

- Simple statistics: $\hat{p}_\phi = u_\phi^+ / u_\phi$
- Refinements are possible, e.g., confidence intervals

⚠ Implicit assumption that we know how to estimate the conditional probabilities in the RHS of

$$\Pr(\phi \mid e) = \frac{\Pr(e \mid \phi)\Pr(\phi)}{\Pr(e \mid \phi)\Pr(\phi) + \Pr(e \mid \neg\phi)\Pr(\neg\phi)}$$

... But do we?

$\Rightarrow$ Alternative scoring heuristics based on possibility theory, weaker than probability theory

Possibility Theory

Definition (Possibility Distribution)

$$\pi : \Omega \to [0, 1]$$

Definition (Possibility and Necessity Measures)

$$\begin{aligned} \Pi(A) &= \max_{\omega \in A} \pi(\omega); \\ N(A) &= 1 - \Pi(\bar{A}) = \min_{\omega \in \bar{A}} \{1 - \pi(\omega)\}. \end{aligned}$$

For all subsets $A \subseteq \Omega$,

1. $\Pi(\emptyset) = N(\emptyset) = 0, \quad \Pi(\Omega) = N(\Omega) = 1;$
2. $\Pi(A) = 1 - N(\bar{A})$ (duality);
3. $N(A) > 0$ implies $\Pi(A) = 1, \quad \Pi(A) < 1$ implies $N(A) = 0.$

In case of complete ignorance on A, $\Pi(A) = \Pi(\bar{A}) = 1$.

Postulates for the Possibility and Necessity of an Axiom

1. $\Pi(\phi) = 1$ if $u^-_\phi = 0$ or, if $D(\phi)$ is disjunctive, $u^+_\phi > 0$,
2. $N(\phi) = 0$ if $u^+_\phi = 0$ or, if $D(\phi)$ is conjunctive, $u^-_\phi > 0$,
3. let $u_\phi = u_\psi$; then $\Pi(\phi) > \Pi(\psi)$ iff $u^-_\phi < u^-_\psi$ and, if $D(\phi)$ is disjunctive, $u^+_\psi = 0$,
4. let $u_\phi = u_\psi$; then $N(\phi) > N(\psi)$ iff $u^+_\phi > u^+_\psi$ and, if $D(\phi)$ is conjunctive, $u^-_\phi = 0$,
5. let $u_\phi = u_\psi = u_\chi$ and $u^+_\psi = u^+_\phi = u^+_\chi = 0$,
$$u^-_\psi < u^-_\phi < u^-_\chi \Rightarrow \frac{\Pi(\psi) - \Pi(\phi)}{u^-_\phi - u^-_\psi} > \frac{\Pi(\phi) - \Pi(\chi)}{u^-_\chi - u^-_\phi},$$
6. let $u_\phi = u_\psi = u_\chi$ and $u^-_\psi = u^-_\phi = u^-_\chi = 0$,
$$u^+_\psi < u^+_\phi < u^+_\chi \Rightarrow \frac{N(\phi) - N(\psi)}{u^+_\phi - u^+_\psi} > \frac{N(\chi) - N(\phi)}{u^+_\chi - u^+_\phi},$$

Possibility and Necessity of an Axiom with conjunctive development

If $u_\phi > 0$ and $D(\phi)$ is conjunctive,

$$\Pi(\phi) = 1 - \sqrt{1 - \left(\frac{u_\phi - u_\phi^-}{u_\phi}\right)^2}; \tag{1}$$

$$N(\phi) = \begin{cases} \sqrt{1 - \left(\frac{u_\phi - u_\phi^+}{u_\phi}\right)^2}, & \text{if } u_\phi^- = 0, \\ 0, & \text{if } u_\phi^- > 0; \end{cases} \tag{2}$$

Possibility and Necessity of an Axiom with disjunctive development

If $u_\phi > 0$ and $D(\phi)$ is disjunctive,

$$\Pi(\phi) = \begin{cases} 1 - \sqrt{1 - \left(\frac{u_\phi - u_\phi^-}{u_\phi}\right)^2}, & \text{if } u_\phi^+ = 0, \\ 1, & \text{if } u_\phi^+ > 0; \end{cases} \tag{3}$$

$$N(\phi) = \sqrt{1 - \left(\frac{u_\phi - u_\phi^+}{u_\phi}\right)^2}; \tag{4}$$

(5)

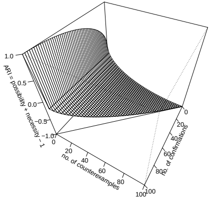
Acceptance/Rejection Index

Definition

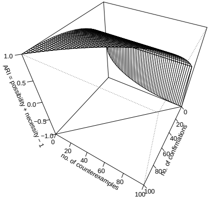
$$\mathrm{ARI}(\phi) = N(\phi) - N(\neg\phi) = N(\phi) + \Pi(\phi) - 1$$

- $-1 \leq \mathrm{ARI}(\phi) \leq 1$ for all axiom ϕ
- $\mathrm{ARI}(\phi) < 0$ suggests rejection of ϕ ($\Pi(\phi) < 1$)
- $\mathrm{ARI}(\phi) > 0$ suggests acceptance of ϕ ($N(\phi) > 0$)
- $\mathrm{ARI}(\phi) \approx 0$ reflects ignorance about the status of ϕ

CNF:



DNF:



OWL 2 → SPARQL

To test axioms, we define a mapping $Q(E, x, y)$ from OWL 2 expressions to SPARQL graph patterns, such that

$$\texttt{SELECT DISTINCT ?x ?y WHERE } \{\ Q(E, \texttt{?x}, \texttt{?y})\ \}$$

returns $[Q(E, x, y)]$, all known instances of class expression E and

$$\texttt{ASK } \{\ Q(E, a, b)\ \}$$

checks whether $E(a, b)$ is in the RDF base.

For an atomic concept A (a valid IRI),

$$Q(A, \texttt{?x}, \texttt{?y}) = \texttt{?x a } A \texttt{ .}$$

Concept Negation: $Q(\neg C, ?\mathrm{x}, ?\mathrm{y})$

Problem

Open-world hypothesis, but no ¬ in RDF!

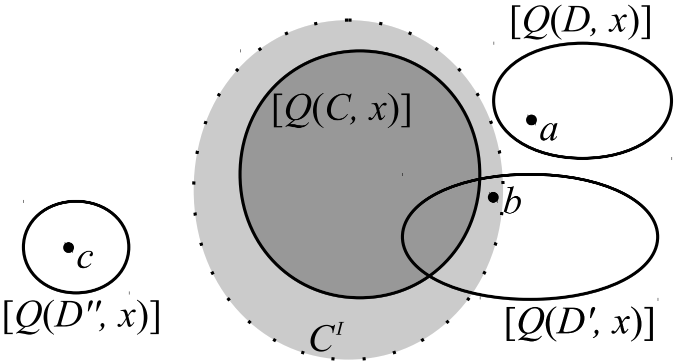
We approximate an open-world semantics as follows:

$$Q(\neg C, ?\mathrm{x}, ?\mathrm{y}) = \{\ \texttt{?x a ?dc .}\ \texttt{FILTER NOT EXISTS}\ \{\ \texttt{?z a ?dc .}\quad Q(C, ?\mathrm{z}, ?\mathrm{y1})\ \}\ \} \tag{6}$$

For an atomic class expression A, this becomes

$$Q(\neg A, ?\mathrm{x}, ?\mathrm{y}) = \{\ \texttt{?x a ?dc .}\ \texttt{FILTER NOT EXISTS}\ \{\ \texttt{?z a ?dc .}\quad \texttt{?z a}\ A\ \}\ \}. \tag{7}$$

Concept Negation: Discussion



SubClassOf(C D) Axioms

To test SubClassOf axioms, we compute their logical content based on their development

$$D_{\mathcal{K}}(C \sqsubseteq D) = \bigwedge_{a \in I(\mathcal{K})} (\neg C(a) \vee D(a))$$

whence, following the principle of selective confirmation,

$$u_{C \sqsubseteq D} = \|\{D(a) : \mathcal{K} \models C(a)\}\|,$$

because, if $C(a)$ holds,

$$C(a) \Rightarrow D(a) \equiv \neg C(a) \vee D(a) \equiv \bot \vee D(a) \equiv D(a)$$

Support, Confirmations and Counterexamples of $C \sqsubseteq D$

$u_{C \sqsubseteq D}$ can be computed by

SELECT (count(DISTINCT ?x) AS ?u) WHERE $\{Q(C, ?x)\}$.

As for the computational definition of $u^{+}_{C \sqsubseteq D}$ and $u^{-}_{C \sqsubseteq D}$:

- confirmations: a s.t. $a \in [Q(C, x)]$ and $a \in [Q(D, x)]$;
- counterexamples: a s.t. $a \in [Q(C, x)]$ and $a \in [Q(\neg D, x)]$.

Therefore,

- $u^{+}_{C \sqsubseteq D}$ can be computed by

  SELECT (count(DISTINCT ?x) AS ?numConfirmations)
  WHERE $\{ \ Q(C, ?x) \ Q(D, ?x) \ \}$

- $u^{-}_{C \sqsubseteq D}$ can be computed by

  SELECT (count(DISTINCT ?x) AS ?numCounterexamples)
  WHERE $\{ \ Q(C, ?x) \ Q(\neg D, ?x) \ \}$

Test a `SubClassOf` axiom (plain version, w/o time cap)

Input: ϕ, an axiom of the form `SubClassOf`$(C\ D)$;
Output: $\Pi(\phi)$, $N(\phi)$, confirmations, counterexamples.

Compute u_ϕ using the corresponding SPARQL query;
compute u_ϕ^+ using the corresponding SPARQL query;
if $0 < u_\phi^+ \leq 100$ **then**
    query a list of confirmations;
if $u_\phi^+ < u_\phi$ **then**
    compute u_ϕ^- using the corresponding SPARQL query;
    if $0 < u_\phi^- \leq 100$ **then**
        query a list of counterexamples;
else
    $u_\phi^- \leftarrow 0$;
compute $\Pi(\phi)$ and $N(\phi)$ based on u_ϕ, u_ϕ^+, and u_ϕ^-.

Experiments

Experimental Setup:

- DBpedia 3.9 in English as RDF fact repository
- Local dump (812,546,748 RDF triples) loaded into Jena TDB
- Method coded in Java, using Jena ARQ and TDB
- 12 6-core Intel Xeon CPUs @2.60GHz (15,360 KB cache), 128 GB RAM, 4 TB HD (128 GB SSD cache), Ubuntu 64-bit OS.

Two experiments:

1. Explorative test of systematically generated subsumption axioms
2. Exhaustive test of all subsumption axioms in the DBpedia ontology.

Results at `http://www.i3s.unice.fr/~tettaman/RDFMining/`.

Explorative Experiment

Systematically generate and test SubClassOf axioms involving atomic classes only

- For each of the 442 classes C referred to in the RDF store
- Construct all $C \sqsubseteq D$: C and D share at least one instance
- Classes D are obtained with query

$$\texttt{SELECT DISTINCT ?D WHERE } \{Q(C, \texttt{?x}).\ \texttt{?x a ?D}\}$$

722 axioms have been tested this way (but this took 290 CPU days).

Explorative Experiment: Results

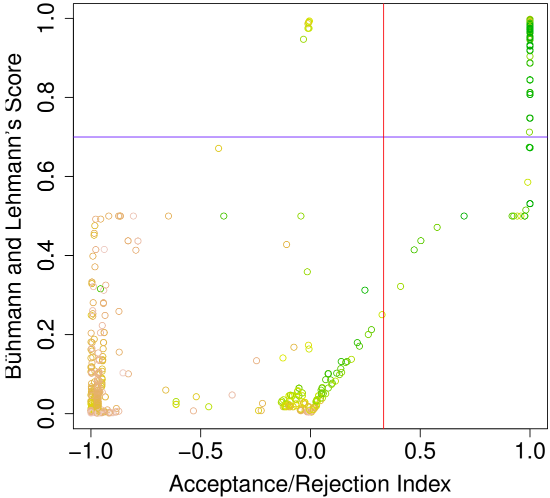
Assessment:

1. sort the first 380 tested axioms by their ARI
2. manually tag each of them as either *true* or *false*

Findings:

- $\mathrm{ARI}(\phi) > 1/3$ as the optimal acceptance criterion for ϕ
- This would yield 4 FP and 6 FN (97.37% accuracy)
- Misclassifications to blame on mistakes in DBpedia
- Pr score w/ 0.7 threshold yields 13 FN (+7) and 4 FP (=)

Comparison with a Probability-Based Score



Exhaustive Experiment

Test all SubClassOf axioms in DBpedia ontology

- Functional syntax, with query

```
SELECT DISTINCT
  concat("SubClassOf(<", str(?x),
          "> <",str(?y),">)")
WHERE { ?x a owl:Class .  ?x rdfs:subClassOf ?y }
```

- 541 axioms
- Testing “only” took 1 h 23 min 31 s

Exhaustive Experiment: Results

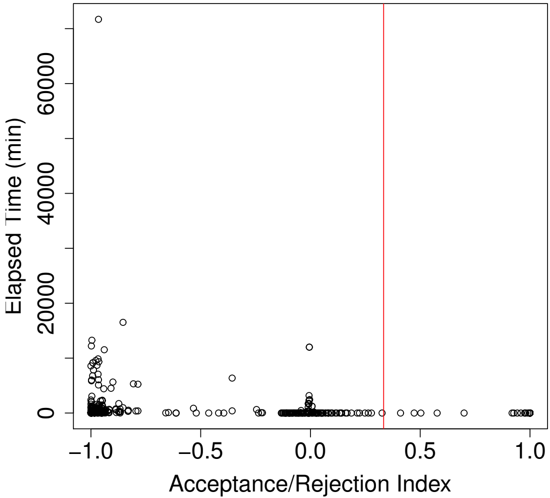
- For 143 axioms, $u_\phi = 0$ (empty content!): $\mathrm{ARI}(\phi) = 0$
- For 28 axioms, $\mathrm{ARI}(\phi) < 0 \Rightarrow \exists$ erroneous facts

Examples of axioms $C \sqsubseteq D$ with their conterexamples:

Axiom	**Counterexamples**
dbo:LaunchPad $\sqsubseteq$ dbo:Infrastructure	:USA
dbo:Brain $\sqsubseteq$ dbo:AnatomicalStructure	:Brain [*sic*]
dbo:Train $\sqsubseteq$ dbo:MeanOfTransportation	:New_Jersey_Transit_rail_operations, :ALWEG
dbo:ProgrammingLanguage $\sqsubseteq$ dbo:Software	:Ajax
dbo:PoliticalParty $\sqsubseteq$ dbo:Organisation	:Guelphs_and_Ghibellines, :-, :New_People's_Army, :Syrian

N.B.: counterexamples are instances a such that $C(a)$ and $E(a)$ with $E^{\mathcal{I}} \cap D^{\mathcal{I}} = \emptyset$: in this case, either $C(a)$ is wrong or $E(a)$ is.

$T(\phi) = O\left((1+\mathrm{ARI}(\phi))^{-1}\right)$ or $O\left(\exp(-\mathrm{ARI}(\phi))\right)$



Time Predictor

How much time should we allow in order to be reasonably sure we are not throwing the baby out with the bathwater, while avoiding to waste time on hopeless tests?

Studying the elapsed times for accepted axioms, we observed that the time it takes to test $C \sqsubseteq D$ tends to be proportional to

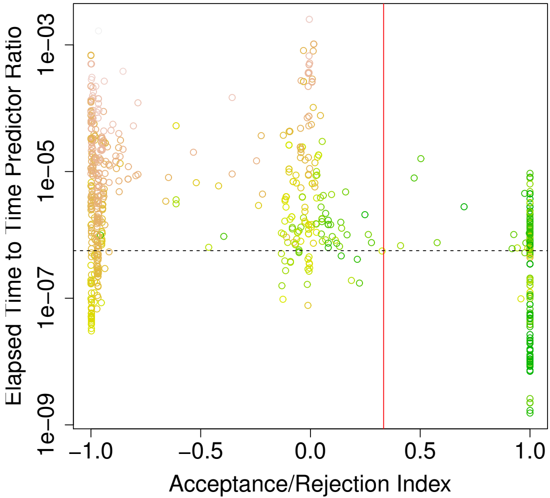
$$\mathrm{TP}(C \sqsubseteq D) = u_{C \sqsubseteq D} \cdot nic_C,$$

where nic_C denotes the number of intersecting classes of C.
A computational definition of nic_C is the following SPARQL query:

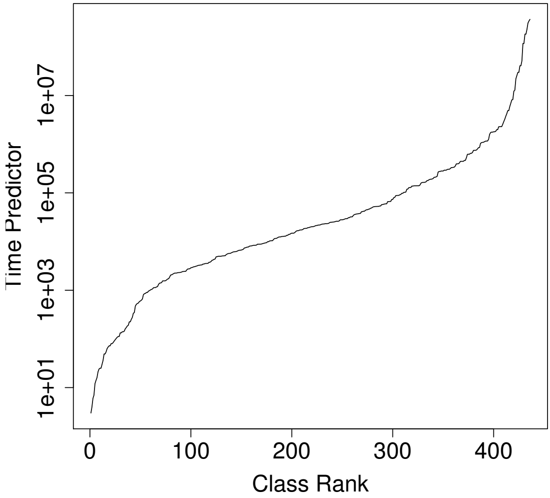
```
SELECT (count(DISTINCT ?A) AS ?nic)
WHERE { Q(C,?x)  ?x a ?A . }
```

where A represents an atomic class expression.

$T(C \sqsubseteq D)/\mathrm{TP}(C \sqsubseteq D)$



$\mathrm{TP}(C \sqsubseteq D)$ as a function of the cardinality rank of C



Test a SubClassOf axiom (time-capped version)

Input: ϕ, an axiom of the form `SubClassOf(`C D`)`;
a, b, the coefficients of the linear time cap equation.
Output: $\Pi(\phi)$, $N(\phi)$, confirmations, counterexamples.

Compute u_ϕ and *nic* using the corresponding SPARQL queries;
$\mathrm{TP}(\phi) \leftarrow u_\phi \cdot nic$;
compute u_ϕ^+ using the corresponding SPARQL query;
if $0 < u_\phi^+ \leq 100$ **then**
    query a list of confirmations;
if $u_\phi^+ < u_\phi$ **then**
    $t_{\max}(\phi) \leftarrow a + b \cdot \mathrm{TP}(\phi)$
    waiting up to $t_{\max}(\phi)$ min **do**
        compute u_ϕ^- using the corresponding SPARQL query;
    if time-out then
        $u_\phi^- \leftarrow u_\phi - u_\phi^+$;
    else if $0 < u_\phi^- \leq 100$ **then**
        query a list of counterexamples;
else
    $u_\phi^- \leftarrow 0$;
compute $\Pi(\phi)$ and $N(\phi)$ based on u_ϕ, u_ϕ^+, and u_ϕ^-.

Experiments

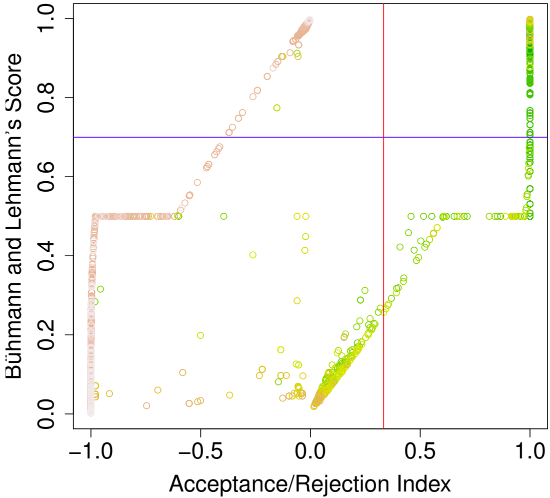
Experimental Setup:

- DBpedia 3.9 in English as RDF fact repository
- Local dump (812,546,748 RDF triples) loaded into Jena TDB
- Method coded in Java, using Jena ARQ and TDB
- 12 6-core Intel Xeon CPUs @2.60GHz (15,360 KB cache), 128 GB RAM, 4 TB HD (128 GB SSD cache), Ubuntu 64-bit OS.

Systematically generate and test SubClassOf axioms involving atomic classes only

- For each of the 442 classes C referred to in the RDF store
- Construct all $C \sqsubseteq D$: C and D share at least one instance
- Test these axioms in increasing time-predictor order
- Compare with scores obtained w/o time cap

Time-Capped Score vs. Probability-Based Score



Results

Speed-up:

- Testing 722 axioms w/o time cap took 290 days of cpu time
- We managed to test 5,050 axioms in $<$ 342 h 30' (244 s/axiom) w/ time cap
- 142-fold reduction in computing time

Precision loss due to time capping:

- 632 axioms were tested both w/ and w/o time cap
- Outcome different on 25 of them: a 3.96% error rate

Absolute accuracy:

- Comparison to a gold standard of DBpedia Ontology `SubClassOf` axioms + `SubClassOf` axioms that can be inferred from them
- Of the 5,050 tested axioms, 1,915 occur in the gold standard
- 327 (17%) get an $\mathrm{ARI} < 1/3$
- 34 (1.78%) get an $\mathrm{ARI} < -1/3$

Conclusions & Future Work

Contributions

- Axiom scoring heuristics based on possibility theory
- A framework based on the proposed heuristics

Experimental Results

- ARI gives a highly accurate assessment axiom validity
- Human evaluation suggests most axioms accepted by mistake are inverted subsumptions or involve ill-defined concepts

Where Can We Go From Here?

- Use scoring heuristics as fitness of an EA for RDF Mining
- Generalize to Possibilistic Test of Hypothesis?

The End

Thank you for your attention!