Combined estimation scheme for blind source separation with arbitrary source PDFs

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An alternative closed-form estimator for blind source separation based on fourth-order statistics is presented. In contrast to other estimators, the new estimator works well when the source kurtosis sum is zero. Arbitrary source PDFs are successfully treated through a combined estimation scheme based on a heuristic decision rule for choosing between the new estimator and an existing estimator.

Indexing terms: array signal processing, blind source separation, closed-form estimation, higher-order statistics, signal reconstruction.

Introduction: Recovering a set of unknown mutually independent source signals from their observed mixtures is a problem arising in a wide variety of signal processing applications, such as multi-user communications, seismic exploration and biomedical engineering [1, 2, 3, 4, 5]. In cases where the mixtures can be assumed noiseless, linear, instantaneous and real valued, the so-called blind source separation (BSS) problem consists of the estimation of the source vector $\mathbf{x} \in \mathbb{R}^q$ and the mixing matrix $M \in \mathbb{R}^{p\times q}$ $(p \geqslant q)$ from the observed sensor-output vector $\mathbf{y} \in \mathbb{R}^p$ fulfilling the linear model

$$\mathbf{y} = M\mathbf{x}. \tag{1}$$

Second-order decorrelation and normalization of the observed processes yield a set of whitened signals, related to the sources via an orthogonal transformation $Q \in \mathbb{R}^{q\times q}$ [4]:

$$\mathbf{z} = Q\mathbf{x}. \tag{2}$$

In the fundamental two-signal scenario, matrix Q becomes an elementary givens transformation,

$$Q = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}. \tag{3}$$

The source separation thus reduces to the estimation of parameter θ.

A closed-form estimator of this parameter was developed in [4], based on a complex linear combination of the whitened-output fourth-order cumulants:

$$\hat{\theta}_{\mathrm{EML}} = \frac{1}{4}\,\arg(\xi \cdot \mathrm{sign}(\gamma)), \tag{4}$$

where

$$\xi = (\kappa^z_{40} - 6\kappa^z_{22} + \kappa^z_{04}) + j4(\kappa^z_{31} - \kappa^z_{13}) = (\kappa^x_{04} + \kappa^x_{04})\,\mathrm{e}^{j4\theta} \tag{5}$$

$$\gamma = \kappa^z_{40} + 2\kappa^z_{22} + \kappa^z_{04} = \kappa^x_{40} + \kappa^x_{04}. \tag{6}$$

Notation $\kappa^z_{mn} \triangleq \mathrm{Cum}_{mn}[z_1,\, z_2]$ denotes the $(m+n)$th-order cumulant of the components of $\mathbf{z} = [z_1,\, z_2]^{\mathrm{T}}$ (using Kendall's convention for the bivariate case [6]), and similarly for κ^x_{mn}. Both ξ and γ can be compactly expressed as a function of the whitened-signal scatter-plot points:

$$\left.\begin{array}{l} \xi = \mathrm{E}[r^4 \mathrm{e}^{j4\phi}] \\ \gamma = \mathrm{E}[r^4] - 8. \end{array}\right\} \quad r\,\mathrm{e}^{j\phi} = z_1 + jz_2, \qquad j = \sqrt{-1}. \tag{7}$$

Eqn. (4) — referred to as the extended maximum-likelihood (EML) estimator — is applicable provided the source kurtosis sum (sks), γ, is not null. Among other properties [4, 5], eqn. (4) is the ML estimator of θ for symmetric sources with the same kurtosis, under the Gram-Charlier (GC) expansion of the source probability density function (PDF) truncated at fourth-order. In this sense, the EML is a generalization of the approximate ML (AML) estimator of [3].

The performance of estimator (4) degrades as the sks tends to zero [4, 5]. In this Letter this deficiency is surmounted by deriving a hybrid estimation scheme which combines the EML and another closed-form fourth-order estimator [5].

Alternative fourth-order estimator: In the spirit of the EML centroid ξ, we seek another complex linear combination of the data fourth-order statistics that provides an explicit expression for the parameter of interest. Consider

$$\xi' \triangleq (\kappa^z_{40} - \kappa^z_{04}) + 2j(\kappa^z_{31} + \kappa^z_{13}). \tag{8}$$

The relationships given in eqns. (2)–(3) together with the multilinearity property of cumulants and the source mutual independence assumption lead to $\xi' = \eta\,\mathrm{e}^{j2\theta}$, where $\eta \triangleq (\kappa^x_{40} - \kappa^x_{04})$ represents the source kurtosis difference (skd). Hence, if the skd is not null, θ may be estimated through

$$\hat{\theta}_{\mathrm{AEML}} = \frac{1}{2}\,\arg(\xi'), \tag{9}$$

that we call *alternative EML (AEML)* estimator. The source extraction is not affected by the value of η, as it can only mean an irrelevant $\pm\pi/2$ radian bias. Centroid ξ' also accepts the scatter-plot based form:

$$\xi' = \mathrm{E}\big[(z_1^2 + z_2^2)\big((z_1^2 - z_2^2) + j2z_1z_2\big)\big] = \mathrm{E}[r^4\mathrm{e}^{j2\phi}]. \tag{10}$$

The AEML is the asymptotic ML estimator of θ for symmetric sources with opposite kurtosis ($\gamma = 0$) under the source PDF fourth-order GC expansion [5], and hence its name. Other properties of this estimator are studied in detail in [5]. We only remark here that the AEML shows a performance deterioration when $\eta \to 0$, analogous to that of the EML with γ. To take advantage of the ML-optimality features displayed by these estimators while ameliorating their performance variations with their respective source statistics, a suitable scheme for combining both expressions is devised as follows.

Combined estimation: The lack of prior knowledge on the source statistics in a blind problem renders any optimal combination strategy — based, for instance, on a minimum mean squared error (MMSE) criterion — useless [5]. Instead, we look for a practical suboptimal decision rule to select between the EML and the AEML given a block of whitened-vector samples. To obtain such a decision rule, an empirical approach is adopted. The estimators' performance is evaluated over a range of sks γ and skd η by resorting to the ideas of [7], whereby a pseudorandom binary sequence (PRBS) of suitable probability can model any normalized kurtosis value. Relying on this result, we fix $\kappa_{40}^x = -2$, whereas the second source kurtosis is swept at small intervals between $\kappa_{04}^x = -2$ and $\kappa_{04}^x = 14$, providing sks values in $\gamma \in [-4, 12]$ and skd in $\eta \in [-16, 0]$. At each kurtosis value, separations are performed by the two methods from the same orthogonal mixtures of $\theta = 30^{\circ}$ composed of $T = 5 \times 10^3$ samples, over 100 independent Monte Carlo runs.[1] After each separation, the interference-to-signal ratio (ISR) performance index [5]

$$\mathrm{ISR} \triangleq \mathop{\mathrm{E}}_{i \neq j} \left\{ \frac{\mathrm{E}[\hat{x}_i^2]}{\mathrm{E}[\hat{x}_j^2]} \right\}, \tag{11}$$

where $\hat{x}_i$ denotes the ith extracted source, is computed for each method and then averaged over the MC realizations. The ISR is an approximation of the MSE $\mathrm{E}[(\hat{\theta} - \theta)^2]$ when $\hat{\theta} \approx \theta + k\pi/2$, $k \in \mathbb{Z}$, i.e., when valid separation solutions are obtained.

[1] The actual value of θ is not important, due to the estimators' orthogonal invariance property [5].

Fig. 1 shows the obtained ISR mean squared value. As expected from the theoretical analyses, the EML worsens around $\gamma = 0$ and reaches its best performance at $\eta = 0$, and conversely for the AEML. The EML outperforms the AEML when the source kurtosis have the same sign, whereas the latter improves the former for source kurtosis with opposite signs. This observation suggests the decision rule:

$$\kappa_{40}^{x}\,\kappa_{04}^{x} \underset{\text{AEML}}{\overset{\text{EML}}{\gtrless}} 0, \qquad (12)$$

which, from simple algebraic manipulations along with $|\xi| = |\gamma|$ and $|\xi'| = |\eta|$, transforms into

$$|\xi| \underset{\text{AEML}}{\overset{\text{EML}}{\gtrless}} |\xi'|. \qquad (13)$$

This empirical rule advocates the selection of the estimator with the largest centroid modulus. We refer to the associated hybrid estimation strategy as *combined EML (combEML)* estimator, whose results for the same signal realizations also appear in Fig. 1. Observe that the combEML overcomes the performance degradation of EML and AEML, consistently maintaining the best performance over all range of sks and skd. Criterion (13) is shown to be very close to the optimal MMSE principle for PRBS sources [5]. The validity of rule (13) is additionally endorsed by further experiments on continuous-distribution sources as well as by theoretical considerations on the approximate ML criterion [5].

In Fig. 2 the combEML is compared to the JADE [1] and ICA-HOEVD [2] algorithms, under the same conditions for all three methods, with $\kappa_{40}^{x} = 1$ and $\kappa_{04}^{x} \in [-2, 12]$. The three curves exhibit a similar trend, but combEML outperforms the two other procedures by $\sim$4 dB. Although it is not always the case, this improved outcome is corroborated in many other simulations on discrete as well as continuous source distributions [5].

Conclusions: Along the lines of the EML, an alternative fourth-order estimator for BSS — so-called AEML — has been proposed, which is applicable when the source kurtosis are different. A simple, heuristically-derived decision rule is used to overcome the estimators' shortcomings with respect to the source fourth-order statistics, while retaining the advantages of both. The resulting hybrid estimation scheme is shown to achieve significant improvement over well-known contrast-based BSS methods.

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References

1 CARDOSO, J.-F., and SOULOUMIAC, A.: 'Blind beamforming for non-Gaussian signals', *IEE Proceedings-F*, 1993, **140**, (6), pp. 362–370.

2 COMON, P.: 'Independent component analysis, a new concept?', *Signal Processing*, 1994, **36**, (3), pp. 287–314.

3 HARROY, F., and LACOUME, J.-L.: 'Maximum likelihood estimators and Cramer-Rao bounds in source separation', *Signal Processing*, 1996, **55**, (2), pp. 167–177.

4 ZARZOSO, V., and NANDI, A.K.: 'Blind separation of independent sources for virtually any source probability density function', *IEEE Transactions on Signal Processing*, 1999, **47**, (9), pp. 2419–2432.

5 ZARZOSO, V.: 'Closed-form higher-order estimators for blind separation of independent source signals in instantaneous linear mixtures', PhD Thesis, The University of Liverpool, 1999.

6 STUART, A., and ORD, J.K.: 'Kendall's advanced theory of statistics, Vol. 1', (Edward Arnold, London, 1994), 6th edn.

7 ZARZOSO, V., and NANDI, A.K.: 'Modelling signals of arbitrary kurtosis for testing BSS methods', *IEE Electronics Letters*, 1998, **34**, (1), pp. 29–30. (Errata: (7), p. 703)

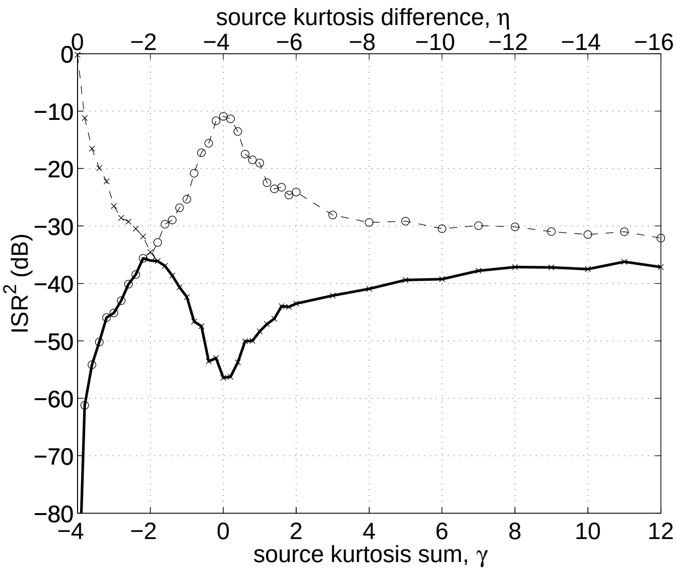

Figure 1: Performance of EML, AEML and combEML estimators against source kurtosis. PRBS sources, $\kappa_{40}^{x} = -2$, $T = 5 \times 10^3$ samples/signal, 100 Monte Carlo runs.

——— combEML -x-x-x- AEML -o-o-o- EML

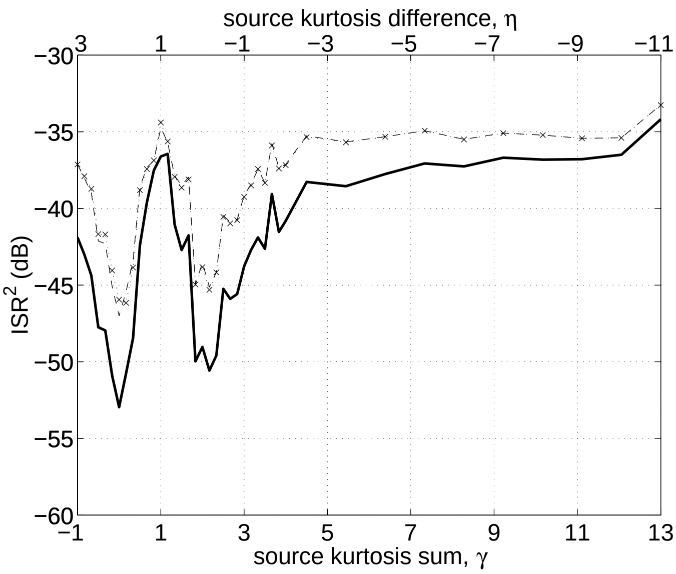

Figure 2: Performance of combEML, JADE and ICA-HOEVD against source kurtosis. PRBS sources, $\kappa_{40}^{x} = 1$, $T = 5 \times 10^3$ samples/signal, 200 Monte Carlo runs.

——— combEML -x-x-x- JADE -o-o-o- ICA-HOEVD