SEMI-BLIND CONSTANT MODULUS EQUALIZATION WITH OPTIMAL STEP SIZE

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ABSTRACT

Channel equalization is an important problem in digital communications. This contribution studies a hybrid equalization criterion combining the constant modulus (CM) property and the minimum mean square error (MMSE) between the equalizer output and the known pilot sequence. An efficient semi-blind block gradient-descent algorithm is put forward, in which the step size globally minimizing the cost function along the search direction is algebraically computed at each iteration. The use of the optimal step size notably accelerates convergence and can further reduce the impact of local extrema on the semi-blind algorithm's performance. The proposed approach is not restricted to the CM-MMSE principle, but it can benefit alternative equalization criteria as well.

1. INTRODUCTION

The equalization of digital communication channels consists of recovering the unknown data transmitted through a distorting propagation medium. Blind equalization techniques typically rely on certain known properties of the input modulation, such as the finite alphabet or constant modulus (CM) of its data symbols [1]. Although the blind approach is versatile, bandwidth efficient and especially attractive in broadcast/multicast scenarios, the exploitation of training or pilot sequences (data symbols known by the receiver) can considerably increase equalization performance and robustness (e.g., reduce the volume of data required for successful equalization). From an alternative point of view, the semi-blind approach can also be interpreted as the regularization of the conventional training-based minimum mean square error (MMSE) receiver, whose performance degrades for insufficient pilot-sequence length [2]. The fact that current as well as future communication systems encompass training sequences in their definition standards provides another strong motivation for the development of semi-blind equalization techniques.

The CM criterion is the most widespread blind equalization principle, probably due its simplicity and flexibility [1]. Indeed, the CM criterion is easy to implement, and can also tackle non-CM modulations, at the expense of an increased misadjustment due to constellation mismatch. As its major shortcoming, the CM cost function presents local stationary points associated with spurious non-equalizing solutions. The existence of spurious solutions degrades the performance of conventional gradient- and Newton-descent procedures, which is very dependent on the initial value of the equalizer tap vector [1, 3]. Spurious convergence can be alleviated to some extent by taking into account training symbols, as shown by the semi-blind criterion of [2]. This criterion is composed of a blind part exploiting the CM property of the (unknown) data symbols and a training part based on the MMSE between the equalizer output and the pilot sequence.

Another approach to avoiding misconvergence are closed-form solutions. Both blind and semi-blind CM-based equalization can be carried out algebraically or in closed form, that is, without iterative optimization. The analytical CM algorithm (ACMA) requires a joint diagonalization stage (a costly QZ iteration) in the general case where multiple solutions exist [4], although its complexity can be relaxed if the solutions are simply delayed versions of each other [5]. The semi-blind ACMA (SB-ACMA) proposed in [6] spares the costly joint diagonalization step of its blind counterpart by constraining the spatial filter (beamformer) to lie on certain subspace associated with the pilot-sequence vector. Nevertheless, the uniqueness of this semi-blind solution remains to be studied in more detail, and so does its performance in the presence of noise. Although closed-form solutions are only exact in the noiseless case, they can always be used as judicious initial points to iterative optimization criteria.

The present contribution focuses on the semi-blind equalization principle of [2]. We propose to minimize this hybrid CM-MMSE cost function by means of an efficient gradient-descent algorithm whereby the optimal step size is computed algebraically at each iteration as the rooting of a 3rd-degree polynomial. As shown in simulations, the use of the optimal step size greatly speeds up convergence and can further reduce the impact of spurious local extrema on the equalization performance, which closely approaches the MMSE lower bound from just a few pilot symbols.

2. PROBLEM AND SIGNAL MODEL

For simplicity, we deal with the basic single-input single-output (SISO) channel model. Consider the discrete-time channel output

$$x_n = \sum_k h_k s_{n-k} + v_n \qquad (1)$$

in which s_n represents the transmitted symbols, h_k are the channel impulse-response taps, and v_n is the additive noise. The goal of channel equalization is to recover the original data symbols from the received signal corrupted by the convolutive channel effects (intersymbol interference) and noise. To achieve this objective, a baud-spaced linear equalizer with impulse response taps $\mathbf{f} = [f_1, \ldots, f_L]^{\mathrm{T}} \in \mathbb{C}^L$ is sought so that the equalizer output $y_n = \mathbf{f}^{\mathrm{H}}\mathbf{x}_n$ is a close estimate of the source symbols s_n, where $\mathbf{x}_n = [x_n, x_{n-1}, \ldots, x_{n-L+1}]^{\mathrm{T}}$. A similar signal model holds, with analogous objectives, if multiple spatially-separated sensors

[*]In receipt of a Post-Doctoral Research Fellowship awarded by the Royal Academy of Engineering, UK.

In *Proc. ICASSP*, Philadelphia, PA, March 18-23, 2005, Vol. III, pp. 577-580. Updated version

are available (spatial oversampling), or when several users simultaneously transmit, giving rise to additional co-channel interference. The results presented in this paper can readily be extended to multichannel (e.g., MIMO) configurations.

3. SEMI-BLIND CONSTANT MODULUS CRITERION

Practical communication systems typically feature pilot sequences to aid synchronization and channel equalization. Exploiting this available information can improve blind equalization performance. The minimization of the following hybrid cost function constitutes a semi-blind CM-MMSE criterion:

$$J_{\mathrm{SB}}(\mathbf{f}) = \lambda J_{\mathrm{MMSE}}(\mathbf{f}) + (1-\lambda) J_{\mathrm{CM}}(\mathbf{f}) \tag{2}$$

where

$$J_{\mathrm{MMSE}}(\mathbf{f}) = \frac{1}{N_t} \sum_{n=0}^{N_t-1} \left| y_n - s_{n-d}^{\mathrm{t}} \right|^2 \tag{3}$$

is the pilot-based MMSE cost function and

$$J_{\mathrm{CM}}(\mathbf{f}) = \frac{1}{N_b} \sum_{n=N_t}^{N-1} \left(|y_n|^2 - \gamma \right)^2 \tag{4}$$

is the CM cost function. In the above expressions, $\{s_n^{\mathrm{t}}\}$ denotes the training sequence, d represents the equalization delay, $\gamma = \mathrm{E}\{|s_n|^4\}/\mathrm{E}\{|s_n|^2\}$ is an alphabet-dependent constant, and $N_b = (N - N_t)$ is the number of equalizer output samples used in the blind part of the criterion (corresponding to unknown, or 'blind', transmitted symbols). The total number of observed symbol periods per burst is $N_d = (N + L - 1)$. Parameter $\lambda \in [0, 1]$ can be considered as the relative degree of confidence between the blind- and the training-based parts of the criterion. Without loss of generality, the training sequence is assumed to appear at the beginning of the transmitted burst.

The above semi-blind cost function (using the "CMA 1-2" cost instead of the "CMA 2-2") was first put forward in [2]. The original motivation was to overcome the deficiencies of the LS solution to (3) when the pilot sequence is not long enough, an enhancement known as regularization. On the other hand, it was also shown that the incorporation of the pilot sequence is capable of reducing the probability of convergence to spurious solutions typically arising from the non-convexity of the CM cost function.

The simple technique presented in the next section further reduces the effects of local extrema while notably accelerating convergence.

4. OPTIMAL STEP-SIZE ALGORITHM

Unconstrained optimization of cost function (2) can be performed via conventional gradient descent by updating the equalizer filter weights as:

$$\mathbf{f}_{k+1} = \mathbf{f}_k - \mu \mathbf{g}_k, \qquad k = 0, 1, \ldots \tag{5}$$

where $\mathbf{g}_k \stackrel{\mathrm{def}}{=} \nabla J_{\mathrm{SB}}(\mathbf{f}_k) = \lambda \nabla J_{\mathrm{MMSE}}(\mathbf{f}_k) + (1-\lambda)\nabla J_{\mathrm{CM}}(\mathbf{f}_k)$, and μ is the step size or adaption coefficient. We refer to this iterative method as semi-blind CMA (SB-CMA). A Newton descent is employed in [2] for the minimization of (2). However, misconvergence problems due to the non-convexity of the cost function still occur in Newton-based minimization [7].

A simple effective alternative is obtained by observing that $J_{\mathrm{SB}}(\mathbf{f} - \mu\mathbf{g})$ is a rational function in the step size parameter μ. Consequently, it is possible to perform steepest descent of function (2) by finding the optimal step size $\mu_{\mathrm{opt}} = \arg\min_{\mu} J_{\mathrm{SB}}(\mathbf{f} - \mu\mathbf{g})$ among the roots of a polynomial in μ. In effect, the derivative of $J_{\mathrm{SB}}(\mathbf{f} - \mu\mathbf{g})$ with respect to μ is the 3rd-degree polynomial

$$p(\mu) = \lambda p_{\mathrm{MMSE}}(\mu) + (1-\lambda) p_{\mathrm{CM}}(\mu) \tag{6}$$

where $p_{\mathrm{MMSE}}(\mu) = \alpha_1 \mu + \alpha_0$, with

$$\alpha_1 = \frac{1}{N_t} \sum_{n=0}^{N_t-1} |g_n|^2 \tag{7}$$

$$\alpha_0 = -\frac{1}{N_t} \sum_{n=0}^{N_t-1} \mathrm{Re}\{g_n^*(y_n - s_n^{\mathrm{t}})\} \tag{8}$$

$g_n = \mathbf{g}^{\mathrm{H}}\mathbf{x}_n$, and $p_{\mathrm{CM}}(\mu) = \beta_3\mu^3 + \beta_2\mu^2 + \beta_1\mu + \beta_0$, with

$$\beta_3 = \frac{2}{N_b} \sum_{n=N_t}^{N-1} a_n^2, \qquad \beta_2 = \frac{3}{N_b} \sum_{n=N_t}^{N-1} a_n b_n$$
$$\beta_1 = \frac{1}{N_b} \sum_{n=N_t}^{N-1} (2a_n c_n + b_n^2), \qquad \beta_0 = \frac{1}{N_b} \sum_{n=N_t}^{N-1} b_n c_n \tag{9}$$

$a_n = |g_n|^2$, $b_n = -2\mathrm{Re}(y_n g_n^*)$, $c_n = (|y_n|^2 - \gamma)$. Gradient vector $\mathbf{g}$ should be normalized beforehand in order to improve numerical conditioning. The roots of this polynomial can be found through standard non-iterative analytical procedures such as Cardano's formula, or efficient iterative methods [8]. The optimal step size corresponds to the root attaining the absolute minimum in μ of the cost function, thus accomplishing the *global* minimization of J_{SB} in the gradient direction. Once μ_{opt} has been determined, the filter taps are updated as in (5), and the process is repeated with the new filter and gradient vectors, until convergence. We refer to this algorithm as *optimal step-size semi-blind CMA (OS-SB-CMA)*. For $\lambda = 1$ the above iterative procedure reduces to the optimal step-size version of the well-known least mean squares (LMS) algorithm for supervised MMSE equalization.

The computational cost of the above sample averages is of order $\mathrm{O}(LN)$ per iteration, for data blocks composed of N sensor vectors $\mathbf{x}_n$. Alternatively, the coefficients of the step-size polynomial can be obtained as a function of the sensor-output statistics, computed once before starting the algorithm (along the lines of [9]; details are omitted here due to space limitations). The cost per iteration of this alternative procedure is of order $\mathrm{O}(L^4)$, with an additional burden of $\mathrm{O}(L^4 N)$ operations due to the computation of the sensor-output 4th-order moments.

5. EXPERIMENTAL RESULTS

A zero-mean unit-variance QPSK-modulated input excites the order-6 non-minimum phase FIR channel $H_2(z)$ of [5, Sec. V], whose output is corrupted by additive white complex circular Gaussian noise. An FIR filter with length $L = 5$ is used to equalize the channel, aiming at the optimal MMSE delay ($d_{\mathrm{opt}} = 6$ at 20-dB SNR). Bursts of length $N_d = 100$ symbols are observed at the channel output, yielding a total of $N = 96$ sensor-output vectors. We choose $\lambda = 0.5$, and $\mu = 10^{-3}$ for the constant step-size algorithms. Iterations are stopped as soon as $\|\mathbf{f}_{k+1} - \mathbf{f}_k\|/\|\mathbf{f}_k\| < 0.1\mu/\sqrt{N}$. Equalization quality is measured in terms of the symbol error rate (SER), which is estimated by averaging over 500 independent bursts. The first experiment compares several fully-blind methods ($N_t = 0$, $N_b = N$). The closed-form solution of [5, Sec. II-B] is referred to as 'DK-top'. Iterative solutions are obtained from the constant gradient-descent CMA with three different initializations: first-tap filter, center-tap filter and the DK-top solution. As a reference, a conventional receiver is simulated by computing the LS solution to the MMSE criterion assuming that 10% of the transmitted symbols are available for training. Accordingly, we refer to the LS solution with the whole burst used for training as 'MMSE bound'. Fig. 1 shows that the closed-form solution is only useful as an initial point to the blind iterative receiver, whose performance depends on the actual initialization.

In the same scenario, the performance of the SB-CMA criterion (2) with constant step size is summarized in Fig. 2. The SB-ACMA closed-form solution [6] is also considered, whereas semi-blind operation of the DK-top solution (SB-DK-top) is enabled by the SVD-based procedure described in [10, 11]. Even though the inclusion of training information enhances DK-top relative to the blind case (Fig. 1), SB-ACMA proves superior, and outperforms the conventional receiver for sufficient SNR. Nevertheless, SB-ACMA can be further improved if used as a starting point for the iterative SB-CMA, whose performance becomes nearly independent of initialization at low to moderate SNR. A flooring effect is observed at high SNR values. As observed in Fig. 4, the number of iterations for convergence increases compared to the blind scenario. This increase is probably due to the flattening of the CM cost function when training is incorporated. A similar effect in semi-blind operation (although for a different equalization criterion) is remarked in [12]. By contrast, Figs. 3–4 show that the performance of the OS-SB-CMA is virtually independent of initialization, while dramatically reducing the iteration count by about two orders of magnitude. Also, the flooring effect at high SNR observed in the constant step-size SB-CMA now disappears.

A second experiment (Figs. 5–6) evaluates the performance variation as a function of the percentage of symbols in the transmitted burst used for training, calculated as $N_t/N \times 100\%$, for 10-dB SNR. The OS-CMA using only the 'blind' symbols is also tested for two different initializations. The SB-ACMA closed-form solution only improves the conventional receiver for short pilot sequences, and always benefits from gradient-descent iterations. The OS-SB-CMA slightly improves the SB-CMA for short training and for all initializations ('$\times$': first tap; '$+$': center tap; '$\triangle$': SB-DK-top; '$\Box$': SB-ACMA), while maintaining its computational superiority across the whole training-length range. For reasonable pilot-length values, the semi-blind methods are able to attain the conventional MMSE receiver performance while increasing the spectral efficiency (decreasing the pilot length), thus improving the effective data rate. Properly initialized, fully-blind operation outperforms the semi-blind methods in short training, as if using too few pilot symbols could 'confuse' the blind receiver; a similar effect is observed for sufficient training, where the 'blind' symbols seem to divert the conventional receiver from its satisfactory solution. However, the performance of the OS-CMA in this scenario depends on initialization, although the optimal step-size approach endows the fully-blind CMA with some immunity to local extrema [9].

6. CONCLUSIONS

The semi-blind equalization criterion of [2] can be globally minimized along any given search direction. This contribution has presented the closed-form expression for the polynomial allowing the derivation of the optimal value of the step size. Experimental results demonstrate that this simple procedure remarkably accelerates convergence and can further reduce the negative impact of local extrema on the algorithm's performance. The optimal step-size strategy is not exclusive to the CM-MMSE principle but can also be incorporated to alternative equalization criteria with a rational cost function or which may be well approximated by a rational function in the adaption coefficient [7, 12].

Further work includes the comparison with alternative step-size optimality and acceleration approaches [13, 14], and the determination of the optimum value of confidence parameter λ.

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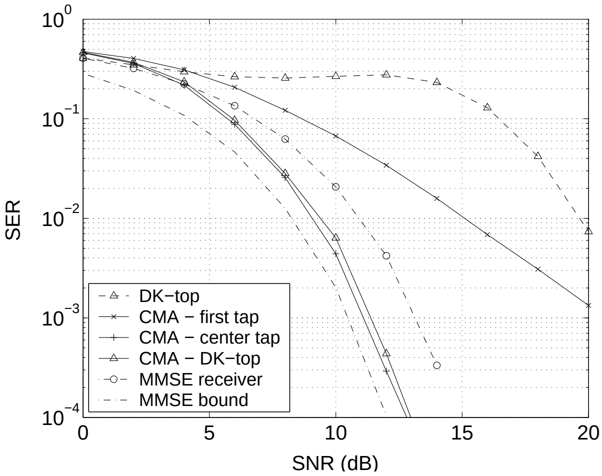

Fig. 1. Blind equalization performance. Solid lines: constant step-size gradient-descent CMA with different initializations.

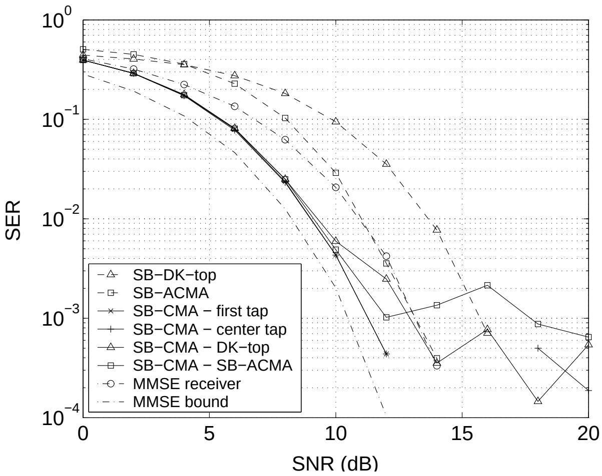

Fig. 2. Semi-blind equalization performance. Solid lines: constant step-size SB-CMA with different initializations.

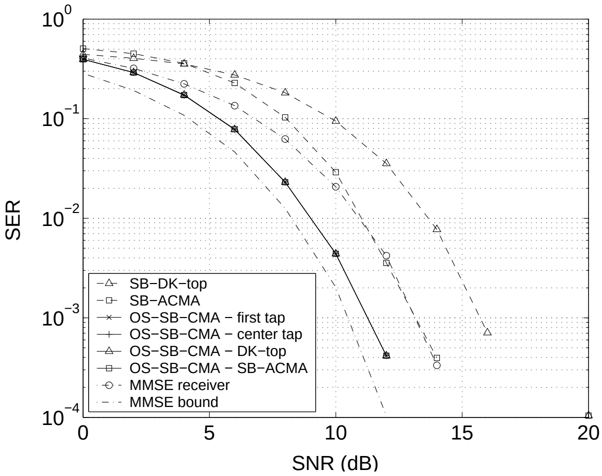

Fig. 3. Semi-blind equalization performance. Solid lines: OS-SB-CMA with different initializations.

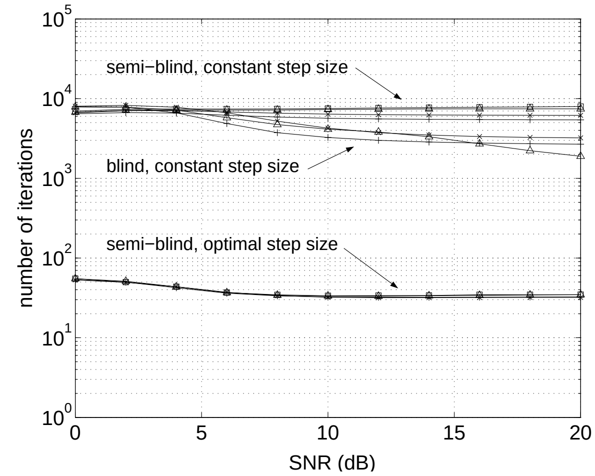

Fig. 4. Number of iterations for convergence of the iterative methods in the simulations of Figs. 1–3.

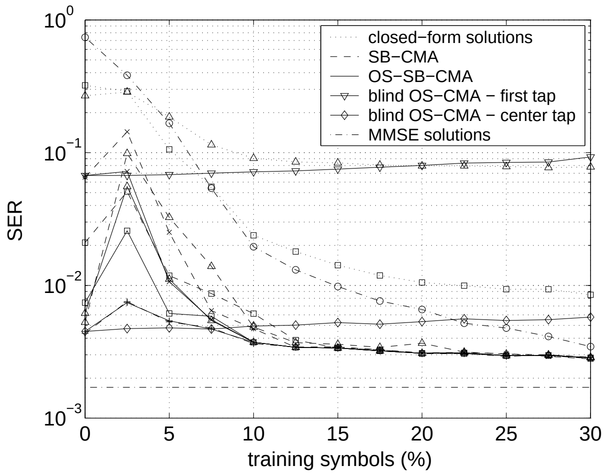

Fig. 5. Equalization performance for a varying number of pilot symbols in the transmitted burst.

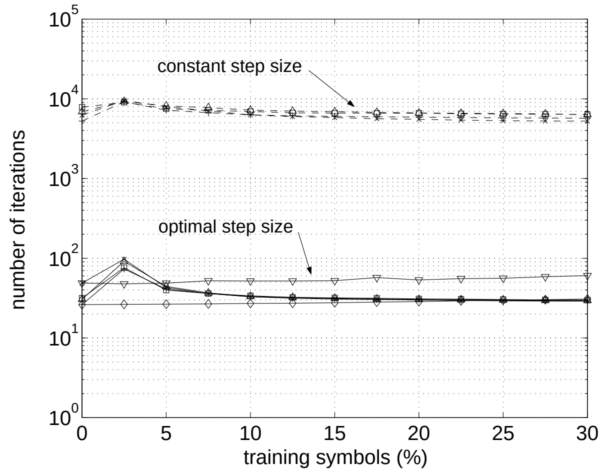

Fig. 6. Number of iterations for convergence of the iterative methods in the simulation of Fig. 5.