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RESEARCH ARTICLE

Blind channel estimation based on maximizing the eigenvalue spread of cumulant matrices in (2×1) Alamouti's coding schemes

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ABSTRACT

The popular Alamouti orthogonal space time code attains full transmit diversity in multiple antenna systems. This paper addresses the problem of blind channel identification in (2×1) Alamouti coded systems. Under the assumption of independent symbol substreams, the channel can be estimated from the eigendecomposition of matrices composed of second- or higher-order statistics (cumulants) of the received signal. The so-called joint approximate diagonalization of eigenmatrices (JADE) method for blind source separation *via* independent component analysis is optimal in that it tries to simultaneously diagonalize a full set of fourth-order cumulant matrices. To reduce computational complexity, we perform the eigenvalue decomposition of a single cumulant matrix, which is judiciously chosen by maximizing its expected eigenvalue spread. Simulation results show that the resulting technique outperforms existing blind Alamouti channel estimation methods and achieves a performance close to JADE's at a fraction of the computational cost. Copyright © 2010 John Wiley & Sons, Ltd.

KEYWORDS

OSTBC; Alamouti coding; blind source separation; channel estimation

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1. INTRODUCTION

In the last decade, a large number of Space–Time Coding (STC) techniques have been proposed in the literature to exploit spatial diversity in systems with multiple elements at both transmission and reception (see, for instance, References [1,2] and references therein). Orthogonal Space Time Block Coding (OSTBC) is remarkable in that it is able to provide full diversity gain with linear decoding complexity [3–5]. The basic premise of OSTBC is the encoding of the transmitting symbols into a unitary matrix so as to spatially decouple their Maximum Likelihood (ML) detection, which can be seen as a matched filter followed by a symbol-by-symbol detector.

In addressing the issue of decoding complexity, Alamouti has proposed a popular OSTBC scheme for transmission in systems with two antennas at the transmitter and only one at the receiver [3]. This scheme is the only OSTBC capable of achieving full spatial rate for complex constellations. Other OSTBCs have been proposed for more than two transmitting antennas, but they suffer from severe spatial rate loss [4,5]. The Alamouti code can also be used in systems with multiple antennas at reception. At first glance, it seems that using several receiving antennas is beneficial because this increases the diversity gain and provides array gain. However, the signal structure imposed by the Alamouti code reduces the constrained channel capacity limit when there is more than one receiving antenna [6]. Thus, (2×1) Alamouti coded systems are most attractive in wireless communications due to their simplicity and their ability to provide maximum diversity gain while preserving channel capacity. Because of these advantages, the Alamouti code has been incorporated in the IEEE 802.11 and IEEE 802.16 standards [7].

Coherent detection in (2×1) Alamouti coded systems requires the identification of a (2×2) unitary channel matrix. The transmission of pilot symbols, also referred to as training symbols, is often used to perform channel estimation required for a coherent detection of OSTBCs [8,9]. However, training symbols reduce the throughput and such schemes are inadequate when the bandwidth is scarce. Several strategies have been proposed recently to avoid these limitations. Among the most popular is the so-called Differential STBC (DSTBC) [10,11], which incurs a 3-dB penalty compared to the coherent ML receiver.

Another class of decoding strategies has recently arisen by interpreting the decoding of OSTBCs as a Blind Source Separation (BSS) problem: the transmitted symbol substreams can be considered as unknown sources to be recovered from their mixtures observed at the receiving antenna output, whereas the channel matrix can be seen as the mixing transformation between the sources and the observations [12–15]. The term *blind* (or unsupervised) refers to the fact that little or nothing is known or assumed about the sources and the mixing matrix structure in a general BSS scenario. Under the assumption of statistical independence between the transmitted symbol substreams, Independent Component Analysis (ICA) techniques can be used to tackle this problem. Hence, many existing ICA algorithms (e.g., References [12–14]) would be able to identify the channel matrix and recover the transmitting symbols. However, in order to reduce the computational load, specific algorithms taking advantage of the special structure of these codes can be designed instead [16–19].

The present work focuses on blind algorithms for channel identification in (2×1) Alamouti's OSTBC based on the eigenvalue decomposition (EVD) of matrices containing fourth-order statistics of the observations [16,19]. These algorithms can be considered as particular cases of the popular ICA method known as Joint Approximate Diagonalization of Eigenmatrices (JADE) [20], which can be regarded as optimal in that it exploits all fourth-order information. To reduce computational cost, we propose to perform the eigendecomposition of a single cumulant matrix. A simple fully blind criterion is proposed to determine the cumulant matrix with optimum eigenvalue spread. As demonstrated by numerical experiments, the novel estimation method derived from this criterion achieves similar error probabilities to JADE's but presents reduced computational cost.

The material is structured as follows. Section 2 briefly describes Alamouti's coding scheme. An overview of the BSS/ICA approach to blind channel identification is then presented in Section 3. In Section 4, we determine the closed form of the matrix that maximizes the eigenvalue spread of fourth-order cross-cumulant matrices. A novel blind channel estimation method in (2×1) Alamouti systems is derived from this result. Section 5 summarizes some numerical experiments to evaluate and compare the performance of the new algorithm. Finally, Section 6 brings the paper to an end with some concluding remarks.

2. THE (2 × 1) ALAMOUTI'S CODING SCHEME

Figure 1 shows the baseband representation of Alamouti OSTBC with two antennas at the transmitter and one antenna at the receiver. A digital source in the form of a binary data stream, b_i, is mapped to complex modulation symbols which are separated in two substreams, s_1 and s_2. Each pair of symbols $\{s_1, s_2\}$ is then transmitted in two adjacent periods using a simple strategy: in the first period s_1 and s_2 are transmitted from the first and the second antenna, respectively, and in the second period $-s_2^*$ is transmitted from the first antenna and s_1^* from the second one, the symbol $(\cdot)^*$ denoting complex conjugation. In the sequel, we assume that the symbol substreams are complex-valued, zero-mean, stationary, non-Gaussian distributed and statistically independent; their exact probability density functions are otherwise unknown.

The transmitted symbols (sources) arrive at the receiving antenna through fading paths h_1 and h_2 from the first and second transmitting antenna, respectively. Hence, the signal received during the first symbol period has the form

$$z_1 = s_1 h_1 + s_2 h_2 + n_1 \tag{1}$$

If the channel remains constant during two periods, the observation in the second period is given by

$$z_2 = s_1^* h_2 - s_2^* h_1 + n_2 \tag{2}$$

In the above expressions, n_i denotes the additive white Gaussian noise (AWGN). By defining the observation vector as $\mathbf{x} = [x_1, x_2]^{\mathrm{T}} = [z_1, z_2^*]^{\mathrm{T}}$, symbol $(\cdot)^{\mathrm{T}}$ standing for the transpose operator, the relationship between the observation vector $\mathbf{x}$ and the source vector $\mathbf{s} = [s_1, s_2]^{\mathrm{T}}$ is given by

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{3}$$

where $\mathbf{n} = [n_1, n_2^*]^{\mathrm{T}}$ is the noise vector and $\mathbf{H}$ represents the (2×2) channel matrix:

$$\mathbf{H} = \left[\,\mathbf{h}_1 \middle| \mathbf{h}_2\,\right] = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}. \tag{4}$$

It is interesting to note that matrix $\mathbf{H}$ is unitary up to a scalar factor, i.e.,

$$\mathbf{H}\mathbf{H}^{\mathrm{H}} = \mathbf{H}^{\mathrm{H}}\mathbf{H} = \|\mathbf{h}\|^2 \mathbf{I}_2 \tag{5}$$

where $\|\mathbf{h}\|^2 = |h_1|^2 + |h_2|^2$ is the squared Euclidean norm of the channel vector, $\mathbf{I}_2$ is the (2×2) identity matrix and $(\cdot)^{\mathrm{H}}$ is the Hermitian operator. It follows that the transmitted symbols can be recovered, up to scale, as $\hat{\mathbf{s}} = \hat{\mathbf{H}}^{\mathrm{H}}\mathbf{x}$, where $\hat{\mathbf{H}}$ is a suitable estimate of the channel matrix. As a result, this scheme supports ML detection based only on linear processing at the receiver. As developed throughout the rest of the paper, the unitary character of matrix $\mathbf{H}$ can be exploited to identify the channel using EVD-based techniques.

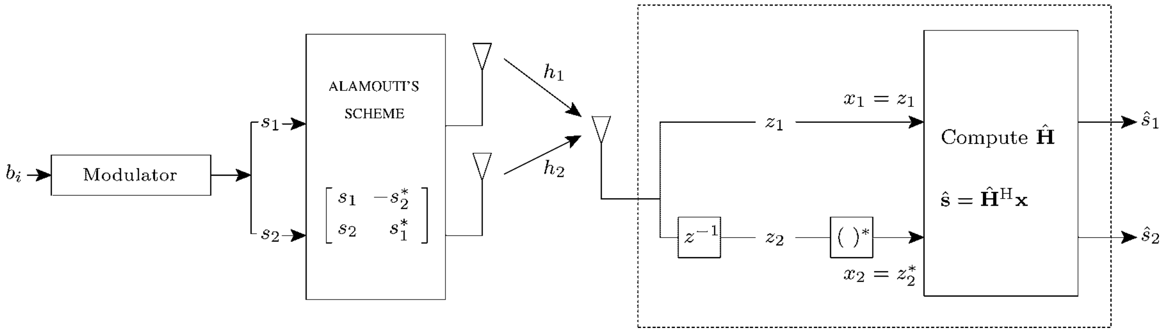

Figure 1. Alamouti's coding scheme.

3. CHANNEL ESTIMATION STRATEGIES

The performance of communication systems which make use of the Alamouti's coding scheme, like most other coding strategies, depends on the accurate estimation of the channel matrix $\mathbf{H}$. The standard way to estimate this matrix is through the transmission of pilot symbols [8,9]. However, the inclusion of pilot symbols reduces the system throughput (equivalently, it reduces the system spectral efficiency) and wastes transmission energy because training sequences do not convey information. Strategies that avoid this limitation include the so-called Differential STBC (DSTBC) [11] which is a signaling technique that generalizes differential modulations to the transmission over MIMO channels. DSTBCs can be incoherently decoded without the aid of channel estimates but they incur a 3-dB performance penalty when compared to coherent detection.

Pilot symbols can also be avoided by using blind approaches. BSS algorithms can estimate the mixing matrix $\mathbf{H}$ and the realizations of the source vector $\mathbf{s}$ from the corresponding realizations of the observed vector $\mathbf{x}$. This lack of prior knowledge may limit the achievable performance, but makes blind approaches more robust to calibration errors (i.e., deviations of model assumptions from reality) than conventional array processing techniques [20]. A property commonly exploited in BSS is the statistical independence of the sources. Depending on the degree of independence considered, two main group of techniques can be distinguished: Principal Component Analysis (PCA), which are based on Second-Order Statistics (SOS), and Independent Component Analysis (ICA), which exploits Higher-Order Statistics (HOS). A number of PCA and ICA approaches rely on the eigendecomposition of certain matrix or tensor structures.

3.1. SOS-based blind channel estimation

The PCA methods are based on the observations covariance matrix

$$\mathbf{Q}_{\mathbf{x}}^{(2)} = \mathrm{E}[\mathbf{x}\mathbf{x}^{\mathrm{H}}] = \mathbf{H}\mathbf{R}_{\mathbf{s}}\mathbf{H}^{\mathrm{H}} \tag{6}$$

where $\mathbf{R}_{\mathbf{s}} = \mathrm{E}[\mathbf{s}\mathbf{s}^{\mathrm{H}}]$ represents the source covariance matrix. Due to the independence assumption, the components of source vector $\mathbf{s}$ are in particular second-order uncorrelated, giving rise to a diagonal covariance matrix $\mathbf{R}_{\mathbf{s}} = \mathrm{diag}(\sigma_1^2, \sigma_2^2)$, in which σ_i^2 denotes the ith-source power. The EVD [21] of $\mathbf{Q}_{\mathbf{x}}^{(2)}$ reads

$$\mathbf{Q}_{\mathbf{x}}^{(2)} = \mathbf{U}\boldsymbol{\Delta}\mathbf{U}^{\mathrm{H}} \tag{7}$$

where the columns of $\mathbf{U}$ are the eigenvectors and the diagonal matrix $\boldsymbol{\Delta}$ contains the eigenvalues of $\mathbf{Q}_{\mathbf{x}}^{(2)}$. Comparing expressions (6) and (7), the mixing matrix may be readily identified, up to scale, as $\hat{\mathbf{H}} = \mathbf{U}$. However, it is well-known that the matrix $\mathbf{U}$ can be found only when the associated eigenvalues are different [21]. In order to guarantee this condition, several authors have proposed to use a linear precoder before to Alamouti's encoder to unbalance the source power [22] or to color the sources [17]. Experiment results reported in Reference [19] show that the global performance is degraded when the power source is unbalanced because, although the mean probability is adequate, the error probability of the sources with lower power is excessively high for some real applications. Another way to guarantee the identifiably condition consists in transmitting an odd number of real symbols at each block [18]. This approach, however, produces a loss in the transmission rate because some symbols must be ruled out.

3.2. HOS-based blind channel estimation

The higher-order independence of the source signals is exploited by the ICA approach. Independence is typically measured by means of HOS such as the higher-order cumulants: the absolute value of the marginal cumulants is to be maximized or, equivalently, that of the cross-cumulants minimized, subject to the appropriate constraints. In Comon's pioneering ICA contribution [12], the initial source estimates provided by PCA are further processed *via* Givens rotations aiming at maximizing the fourth-order independence of the transformed signals. The optimal rotation angles are obtained by rooting a low-degree polynomial whose coefficients are computed from the fourth-order cumulants of the signal pair. Several sweeps over all signal pairs are necessary for convergence. This pairwise scheme can be seen as the generalization to fourth-order cumulant tensors (higher-order arrays) of the well-known Jacobi technique for matrix diagonalization.

Research into higher-order eigen-based approaches began with Cardoso's early work on the so-called quadricovariance, a folded version of the fourth-order moment array, and culminated in the popular JADE method [20], which can be summarized as follows. For the sake of simplicity, we restrict the exposition to zero-mean distributions and circular statistics. These are commonly encountered in the application at hand.

Given a random vector $\mathbf{x} = [x_1, x_2, \ldots, x_n] \in \mathbb{C}^n$, its second-order cumulants are simply defined as $\mathrm{cum}(x_i, x_j^*) = \mathrm{E}[x_i x_j^*]$, and the fourth-order cumulants as

$$\mathrm{cum}(x_i, x_j^*, x_k, x_\ell^*) = \mathrm{E}[x_i x_j^* x_k x_\ell^*] - \mathrm{E}[x_i x_j^*]\mathrm{E}[x_k x_\ell^*] - \mathrm{E}[x_i x_\ell^*]\mathrm{E}[x_j x_k^*] - \mathrm{E}[x_i x_k]\mathrm{E}[x_j^* x_\ell^*]. \tag{8}$$

Given a matrix $\mathbf{M} \in \mathbb{C}^{n\times n}$, the fourth-order cumulant matrix $\mathbf{Q}_{\mathbf{x}}^{(4)}(\mathbf{M})$ is defined as the $(n \times n)$ matrix with components [20]

$$\left[\mathbf{Q}_{\mathbf{x}}^{(4)}(\mathbf{M})\right]_{ij} = \sum_{k,\ell=1}^{n} \mathrm{cum}(x_i, x_j^*, x_k, x_\ell^*) m_{\ell k} \tag{9}$$

where $m_{\ell k} = [\mathbf{M}]_{\ell k}$. Cumulants verify a multi-linearity property [23] whereby, if the sources in $\mathbf{s}$ are statistically independent, we have

$$\mathrm{cum}(x_i, x_j^*, x_k, x_\ell^*) = \sum_p h_{ip}\, h_{jp}^*\, h_{kp}\, h_{lp}^*\, \rho_p \tag{10}$$

where $\rho_p = \mathrm{cum}(s_p, s_p^*, s_p, s_p^*)$ is the marginal fourth-order cumulant (kurtosis) of the source s_p and h_{ip} is the element in the ith row, pth column of $\mathbf{H}$. Note also that the kurtosis of a Gaussian distributed signal is zero, $\mathrm{cum}(n_i, n_i^*, n_i, n_i^*) = 0$. As a consequence, under an AWGN linear model like Equation (3) with statistically independent sources and unitary mixing matrix,[†] the cumulant matrix takes the form [20]

$$\mathbf{Q}_{\mathbf{x}}^{(4)}(\mathbf{M}) = \mathbf{H}\boldsymbol{\Delta}(\mathbf{M})\mathbf{H}^{\mathrm{H}} \tag{11}$$

matrix $\boldsymbol{\Delta}(\mathbf{M})$ being diagonal with

$$[\boldsymbol{\Delta}(\mathbf{M})]_{ii} = \rho_i \mathbf{h}_i^{\mathrm{H}} \mathbf{M} \mathbf{h}_i. \tag{12}$$

Since the sources provide from the same modulated bit stream, they have equal power and kurtosis, i.e., $\rho = \rho_1 = \rho_2$.

Hence, the eigendecomposition of the matrix defined in Equation (9) allows the identification of the remaining unitary part of $\mathbf{H}$ if the eigenvalues of $\mathbf{Q}_{\mathbf{x}}^{(4)}(\mathbf{M})$ are different, i.e., if matrix $\boldsymbol{\Delta}(\mathbf{M})$ contains different entries: $\rho\mathbf{h}_i^{\mathrm{H}}\mathbf{M}\mathbf{h}_i \neq \rho\mathbf{h}_j^{\mathrm{H}}\mathbf{M}\mathbf{h}_j,\ \forall i \neq j$. To increase robustness to eigenspectrum degeneracy, a set $\{\mathbf{Q}_{\mathbf{x}}^{(4)}(\mathbf{M}_k)\}_{k=1}^{m}$, may be (approximately) jointly diagonalized. The full set comprises $m = n^2$ linearly independent (e.g., orthonormal) matrices $\{\mathbf{M}_k\}_{k=1}^{m}$. A simplified version of the algorithm is obtained by considering the set of matrices verify $\mathbf{Q}_{\mathbf{x}}^{(4)}(\mathbf{M}_k) = \lambda_k \mathbf{M}_k$. As there are only n such eigenmatrices, this version is, in theory, computationally more efficient. JADE can be efficiently implemented in terms of the Jacobi technique for matrix diagonalization.

In Alamouti's coding scheme, the channel matrix is essentially unitary with $n = 2$ and can therefore be identified by this procedure since, up to the scale indeterminacy, $\mathbf{H}$ can be determined from the EVD of $\mathbf{Q}_{\mathbf{x}}^{(4)}(\mathbf{M})$ if this matrix has different eigenvalues. The orthogonality property of the channel matrix used in OSTBC makes it possible to reduce the computational load of EVD-based algorithms since the whitening stage required to obtain a unitary mixture before the cumulant-matrix EVD is spared. Taking into account this consideration, Beres *et al.* [16] have proposed to estimate the channel matrix by computing the eigenvectors of a fourth-order cross-cumulant matrix. This method can be considered as a particular case of JADE with a (2×2) matrix $\mathbf{M}$ with elements $m_{11} = 1$, $m_{12} = m_{21} = m_{22} = 0$ (or $m_{22} = 1$, $m_{12} = m_{21} = m_{11} = 0$) in Equation (9). The performance of this criterion has been improved in Reference [19] by considering a linear combination of two fourth-order cumulant matrices, which corresponds to $m_{11} = -m_{22} = 1/\sqrt{2}$, $m_{12} = m_{21} = 0$.

4. CHANNEL IDENTIFICATION BASED ON MAXIMIZING THE EIGENVALUE SPREAD

The performance of EVD-based methods depends on the difference between the eigenvalues (eigenvalue spread) of the matrix to be diagonalized because, as already mentioned, the eigenvectors associated with equal eigenvalues cannot be determined up to a unitary transformation. Since the matrix channel cannot be identified when the eigenvalue spread is close to zero, we propose to estimate the channel matrix by diagonalizing the cumulant matrix given in Equation (9) using the matrix $\mathbf{M}$ that maximizes its eigenvalue spread. In particular for the (2×1) Alamouti scheme, the eigenvalue spread $L(\mathbf{M})$ is defined here as the difference between the two eigenvalues of $\mathbf{Q}_{\mathbf{x}}^{(4)}(\mathbf{M})$ given in Equation (11).

Considering that the channel matrix verified the orthogonality property given in Equation (5), we can define a normalized matrix $\hat{\mathbf{H}} = \frac{\mathbf{H}}{\|\mathbf{h}\|}$, so that $\hat{\mathbf{H}}\hat{\mathbf{H}}^{\mathrm{H}} = \mathbf{I}_2$. As a consequence, Equation (11) can be rewritten as follows

$$\begin{aligned}\mathbf{Q}_{\mathbf{x}}^{(4)}(\mathbf{M}) &= \mathbf{H}\ \Delta(\mathbf{M})\ \mathbf{H}^{\mathrm{H}} \\ &= \hat{\mathbf{H}}\Delta(\mathbf{M})\hat{\mathbf{H}}^{\mathrm{H}}\|\mathbf{h}\|^2 = \hat{\mathbf{H}}\hat{\Delta}(\mathbf{M})\hat{\mathbf{H}}^{\mathrm{H}}\end{aligned} \tag{13}$$

[†] In fact, expression (11) is valid for any mixing matrix $\mathbf{H}$, even if it is not unitary.

where $\hat{\Delta}(\mathbf{M})_{ii} = \|\mathbf{h}\|^2 \mathbf{h}_i^H \mathbf{M} \mathbf{h}_i \rho$. Hence, the eigenvalue spread of $\mathbf{Q}_{\mathbf{x}}^{(4)}(\mathbf{M})$ is given by

$$\begin{aligned} L(\mathbf{M}) &= |[\Delta(\mathbf{M})]_{11} - [\Delta(\mathbf{M})]_{22}| \\ &= |\rho| \|\mathbf{h}\|^2 |\mathbf{h}_1^{\mathrm{H}} \mathbf{M} \mathbf{h}_1 - \mathbf{h}_2^{\mathrm{H}} \mathbf{M} \mathbf{h}_2|. \end{aligned} \tag{14}$$

To avoid arbitrarily large eigenvalue spread values, we will constrain matrix $\mathbf{M}$ to have unit norm, i.e., $\|\mathbf{M}\|_{\mathrm{FRO}}^2 = |m_{11}|^2 + |m_{12}|^2 + |m_{21}|^2 + |m_{22}|^2 = 1$.

As a result, the optimization criterion can be written as follows:

$$\begin{aligned} \mathbf{M}_{\mathrm{opt}} &= \arg \max_{\|\mathbf{M}\|_{\mathrm{FRO}}^2 = 1} L(\mathbf{M}) \\ &= \arg \max_{\|\mathbf{M}\|_{\mathrm{FRO}}^2 = 1} |\rho| \|\mathbf{h}\|^2 |\mathbf{h}_1^{\mathrm{H}} \mathbf{M} \mathbf{h}_1 - \mathbf{h}_2^{\mathrm{H}} \mathbf{M} \mathbf{h}_2|. \end{aligned} \tag{15}$$

The expression above can be also written as

$$\mathbf{m}_{\mathrm{opt}} = \arg \max_{\|\mathbf{m}\|^2 = 1} |\rho| \|\mathbf{h}\|^2 |\tilde{\mathbf{h}}^{\mathrm{H}} \mathbf{m}| \tag{16}$$

where

$$\tilde{\mathbf{h}} = \begin{bmatrix} |h_1|^2 - |h_2|^2 \\ 2h_1^* h_2^* \\ 2h_1 h_2 \\ |h_2|^2 - |h_1|^2 \end{bmatrix}, \quad \mathbf{m} = \begin{bmatrix} m_{11} \\ m_{21} \\ m_{12} \\ m_{22} \end{bmatrix}. \tag{17}$$

According to the Cauchy–Schwarz inequality, the inner product $\tilde{\mathbf{h}}^{\mathrm{H}} \mathbf{m}$ in Equation (16) is maximized when $\mathbf{m}$ has the direction and sense of $\tilde{\mathbf{h}}$. As a result, the normalized vector $\mathbf{m}_{\mathrm{opt}}$ has the form

$$\mathbf{m} = \begin{bmatrix} m_{11} \\ m_{21} \\ m_{12} \\ m_{22} \end{bmatrix} = \frac{1}{\sqrt{2 + 2|\gamma|^2}} \begin{bmatrix} 1 \\ \gamma^* \\ \gamma \\ -1 \end{bmatrix} \quad \text{where}$$

$$\gamma = \frac{2h_1 h_2}{|h_1|^2 - |h_2|^2}. \tag{18}$$

Equivalently, the optimum matrix is given by

$$\mathbf{M}_{\mathrm{opt}} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \frac{1}{\sqrt{2 + 2|\gamma|^2}} \begin{bmatrix} 1 & \gamma \\ \gamma^* & -1 \end{bmatrix}. \tag{19}$$

Note that $\|\mathbf{M}_{\mathrm{opt}}\|_{\mathrm{FRO}}^2 = 1$. As a result, the eigenvalue spread of $\mathbf{Q}_{\mathbf{x}}^{(4)}(\mathbf{M})$ is maximized when the parameter γ is computed using Equation (18), which depends on the channel coefficients h_1 and h_2.

4.1. Sub-optimal approach

We will now propose a simplified approach which consists in diagonalizing the cumulant matrix given in Equation (9) with the highest eigenvalue spread considering only two matrices

$$\mathbf{M}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad \mathbf{M}_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}. \tag{20}$$

These matrices verify $\|\mathbf{M}_1\|_{\mathrm{FRO}}^2 = \|\mathbf{M}_2\|_{\mathrm{FRO}}^2 = 1$. The eigenvalue spreads obtained directly by evaluating these matrices in Equation (14) are

$$L(\mathbf{M}_1) = |\rho| \|\mathbf{h}\|^2 ||h_1|^2 - |h_2|^2| \tag{21}$$

$$L(\mathbf{M}_2) = |\rho| \|\mathbf{h}\|^2 \, 2|h_1 h_2|. \tag{22}$$

The matrix $\mathbf{M}$ that maximizes the eigenvalue spread can then be selected using the following criterion:

$$\frac{|L(\mathbf{M}_2)|}{|L(\mathbf{M}_1)|} = \frac{2|h_1||h_2|}{||h_1|^2 - |h_2|^2|} = \frac{2|h_1 h_2|}{||h_1|^2 - |h_2|^2|} \underset{\mathbf{M}_2}{\overset{\mathbf{M}_1}{\lessgtr}} 1. \tag{23}$$

It is interesting to note that the decision criterion above depends on the absolute value of parameter γ defined in Equation (18), i.e., the matrix $\mathbf{M}_1$ or $\mathbf{M}_2$ must be selected by using the rule

$$|\gamma| \underset{\mathbf{M}_2}{\overset{\mathbf{M}_1}{\lessgtr}} 1. \tag{24}$$

4.2. Comparison among eigenvalue spreads

A way to measure the improvement obtained by using the optimal and suboptimal approaches consists in measuring the probability of the eigenvalue spread of the matrix to be diagonalized being close to zero. The best criterion has the lowest probability.

In order to compare the eigenvalue spread obtained with different matrices $\mathbf{M}$, we have evaluated Equation (15) considering that the channel coefficients have a Rayleigh distribution. Subsequently, we have computed the cumulative probability distribution (cpd) corresponding to each value of $L(\mathbf{M})$. Figure 2(a) plots the cpd for $L(\mathbf{M}_1)$, $L(\mathbf{M}_2)$, $L(\mathbf{M}_{\mathrm{opt}})$ and the suboptimal approach which computes the maximum between $L(\mathbf{M}_1)$ and $L(\mathbf{M}_2)$. The channels have been normalized to avoid scale influence. It is apparent that $L(\mathbf{M}_1)$ has the highest probability of taking values close to zero. Figure 2(b) zooms the part corresponding to the eigenvalue spread smaller than 0.15. Note that the cpd of

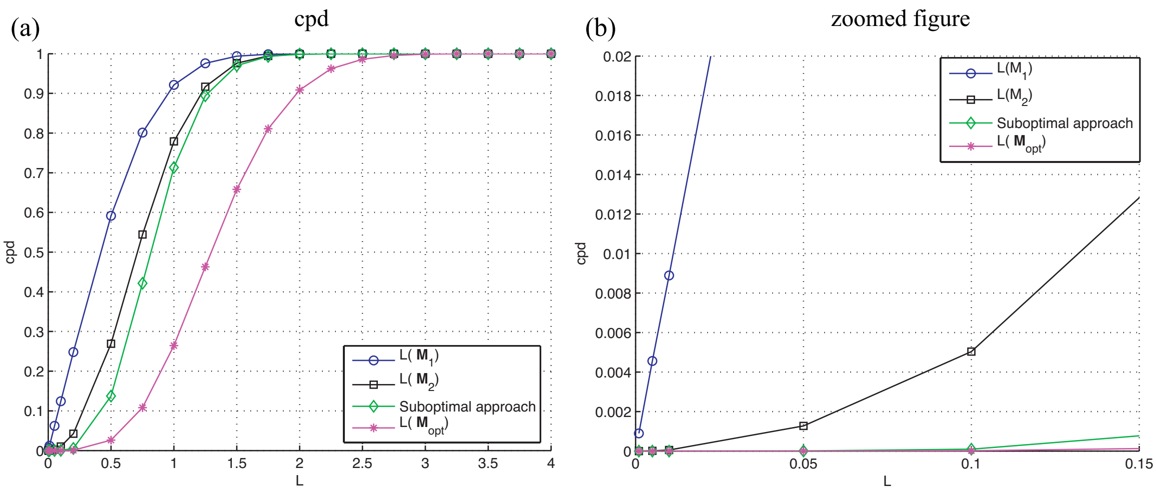

Figure 2. Cumulative probability distribution for different matrices $\mathbf{M}$: $L(\mathbf{M}_1)$, $L(\mathbf{M}_2)$, $\max(L(\mathbf{M}_1), L(\mathbf{M}_2))$, and $L(\mathbf{M}_{\text{opt}})$.

the suboptimal approach coincides with the cpd of the optimal one for $L(\mathbf{M}) \le 0.1$ and, therefore, it is reasonable to think that both approaches will provide similar performance to estimate the channel matrix.

4.3. Blind estimation of parameter γ

The matrix $\mathbf{M}$ obtained for the approaches presented above depend on parameter γ, which is a function of the channel coefficients h_1 and h_2. Considering the signal model in Equation (3) and the linearity property of cumulants given in Equation (10), we can express the following fourth-order cross-cumulants as functions of the channel coefficients

$$\begin{aligned} \text{cum}(x_1, x_1^*, x_1, x_2^*) &= \text{cum}(h_1 s_1 + h_2 s_2 + n_1, h_1^* s_1^* + h_2^* s_2^* \\ &\quad + n_1^*, h_1 s_1 + h_2 s_2 + n_1, h_2 s_1^* \\ &\quad - h_1 s_2^* + n_2) = |h_1|^2 h_1 h_2 \\ &\quad \times \text{cum}(s_1, s_1^*, s_1, s_1^*) - |h_2|^2 h_1 h_2 \\ &\quad \times \text{cum}(s_2, s_2^*, s_2, s_2^*) \\ &= (|h_1|^2 - |h_2|^2) h_1 h_2 \rho \qquad (25) \\ \text{cum}(x_1, x_2^*, x_1, x_2^*) &= 2 h_1^2 h_2^2 \rho. \qquad (26) \end{aligned}$$

Using these definitions, it is straightforward to obtain

$$\begin{aligned} \gamma &= \frac{\text{cum}(x_1, x_2^*, x_1, x_2^*)}{\text{cum}(x_1, x_1^*, x_1, x_2^*)} \\ &= \frac{2 h_1^2 h_2^2 \rho}{(|h_1|^2 - |h_2|^2) h_1 h_2 \rho} = \frac{2 h_1 h_2}{|h_1|^2 - |h_2|^2}. \qquad (27) \end{aligned}$$

Note that the suboptimal approach only needs to know the module of parameter γ that can be estimated by computing the absolute value of Equation (27). An alternative way to obtain $|\gamma|$ is to use the fourth-order cross-cumulant in Equation (25) and

$$\text{cum}(x_1, x_1^*, x_2, x_2^*) = 2|h_1|^2 |h_2|^2 \rho. \qquad (28)$$

In this case, we have

$$\begin{aligned} |\gamma| &= \frac{\text{cum}(x_1, x_1^*, x_2, x_2^*)}{|\text{cum}(x_1, x_1^*, x_1, x_2^*)|} \\ &= \frac{2|h_1|^2 |h_2|^2 \rho}{||h_1|^2 - |h_2|^2||h_1\, h_2|\rho} = \frac{2|h_1 h_2|}{||h_1|^2 - |h_2|^2|}. \end{aligned} \qquad (29)$$

Simulation results presented in the next section show that this second way for estimating $|\gamma|$ is more adequate because the error in the estimation of $\text{cum}(x_1, x_2^*, x_1, x_2^*)$ in larger than for $\text{cum}(x_1, x_1^*, x_2, x_2^*)$.

5. EXPERIMENTAL RESULTS

This section reports several numerical experiments carried out to evaluate and compare the performance of the blind channel estimation algorithms studied in this paper. The experiments have been performed on QPSK source symbols coded with the Alamouti scheme. The channel is assumed to remain constant during the transmission of a block of K symbols. The second- and fourth-order statistics have been calculated by sample averaging over the symbols of a block. The channels have been generated from a Rayleigh distribution. Performance has been evaluated in terms of the SER (Symbol Error Rate) by averaging 100 000 independent simulations.

In order to compare the two methods proposed above to estimate the module of parameter γ, Figure 3 shows the normalized error in the estimation

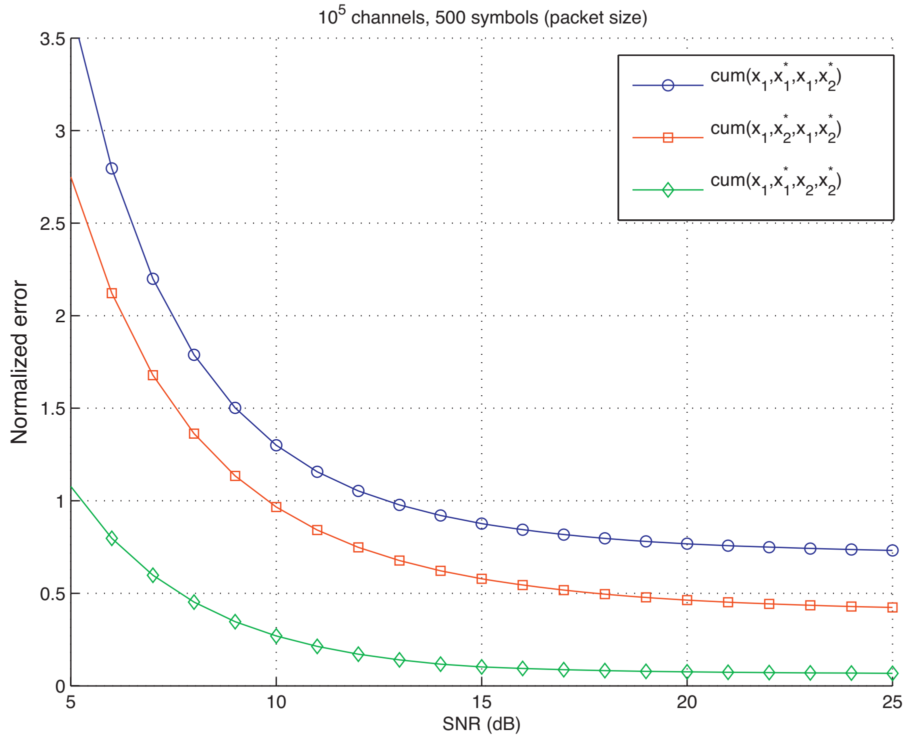

Figure 3. Normalized error between the theoretical and the estimated value of the fourth-order cross-cumulants $\mathrm{cum}(x_1, x_1^*, x_1, x_2^*)$, $\mathrm{cum}(x_1, x_2^*, x_1, x_2^*)$, and $\mathrm{cum}(x_1, x_1^*, x_2, x_2^*)$.

of the fourth-order cross-cumulants $\mathrm{cum}(x_1, x_1^*, x_1, x_2^*)$, $\mathrm{cum}(x_1, x_2^*, x_1, x_2^*)$ and $\mathrm{cum}(x_1, x_1^*, x_2, x_2^*)$. The normalized error has been computed using the following expression:

$$\mathrm{error} = \frac{|\mathrm{cum}(x_i, x_j^*, x_k, x_l^*) - \hat{\mathrm{cum}}(x_i, x_j^*, x_k, x_l^*)|}{|\mathrm{cum}(x_i, x_j^*, x_k, x_l^*)|} \tag{30}$$

where $\mathrm{cum}(x_i, x_j^*, x_k, x_l^*)$ represents the theoretical value, computed directly from the channel realization using Equations (26), (25), and (28), and $\hat{\mathrm{cum}}(x_i, x_j^*, x_k, x_l^*)$ is the estimated value, obtained by sample averaging over $K = 500$ symbols. It is apparent that the error in the estimation of $\mathrm{cum}(x_1, x_1^*, x_2, x_2^*)$ is considerably smaller than the error obtained for $\mathrm{cum}(x_1, x_2^*, x_1, x_2^*)$. This means that the best form to estimate $|\gamma|$ consists in estimating the fourth-order cross-cumulants used in Equation (29) instead of considering the module of Equation (27), as can be observed in Figure 4.

Figure 5 plots the performance of the proposed approach for a packet size of $K = 500$ symbols. The proposed approaches match the Perfect CSI when the theoretical value of γ is used. Note also that both the optimal and the suboptimal approaches present a loss of performance for high SNRs when the parameter γ is estimated using Equation (27). This undesirable situation does not appear when the module of γ is computing using Equation (29).

Figure 6 compares these results with other blind approaches:

- The SOS-based approach proposed by Via *et al.* [17].
- The method proposed by Beres *et al.* [16] corresponding to diagonalize the fourth-order cross-cumulant matrix associated with $\mathbf{M}_1$.
- Joint diagonalization of the fourth-order cumulant matrices corresponding to $\mathbf{M}_1$ and $\mathbf{M}_2$ using the procedure described in Appendix A. This procedure is an optimization of JADE and provides the same performance with a fraction of the computational cost.

Equation (29) has been used to estimated $|\gamma|$ in the suboptimal approach while Equation (27) has been employed for the optimal approach since it needs to compute both the module and the phase. In this figure, we see that the results obtained with the method proposed by Via *et al.*, the suboptimal method and joint diagonalization are very close to the Perfect CSI while the worst result corresponds to the method proposed by Beres *et al.*. This is reasonable because the fourth-order cross-cumulant used in the method of Beres *et al.* corresponds to the eigenvalue spread $L(\mathbf{M}_1)$ which has a high probability of taking values close to zero, as can be seen in Figure 2.

Figure 7 shows performance in terms of the number symbols where the channel remains constant (packet size) at an SNR of 15 dB. Note that the joint diagonalization procedure

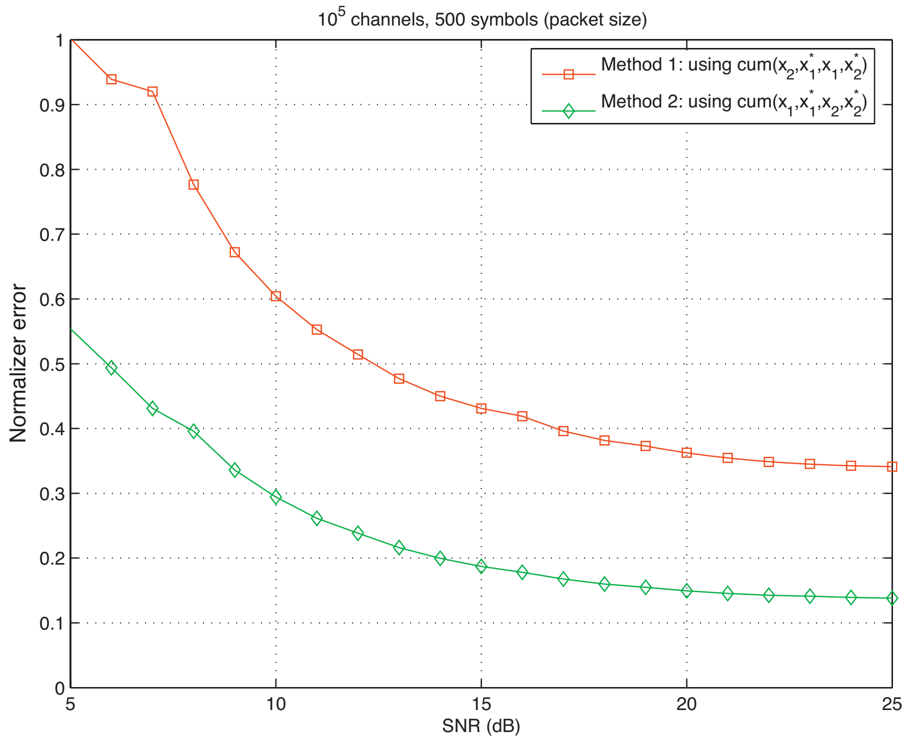

Figure 4. Normalized error in the estimation of $|\gamma|$ using $|\gamma| = \frac{|\mathrm{cum}(x_1, x_2^*, x_1, x_2^*)|}{|\mathrm{cum}(x_1, x_1^*, x_1, x_2^*)|}$ and $|\gamma| = \frac{\mathrm{cum}(x_1, x_1^*, x_2, x_2^*)}{|\mathrm{cum}(x_1, x_1^*, x_1, x_2^*)|}$.

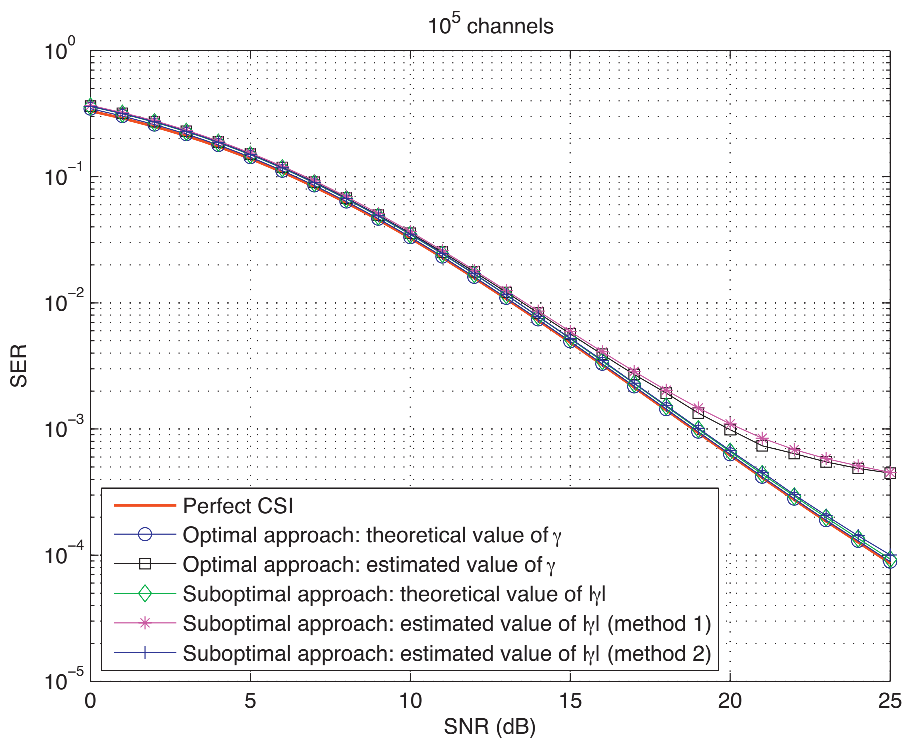

Figure 5. Proposed approach performance: SER *versus* SNR for Rayleigh channels for a block size of 500 symbols.

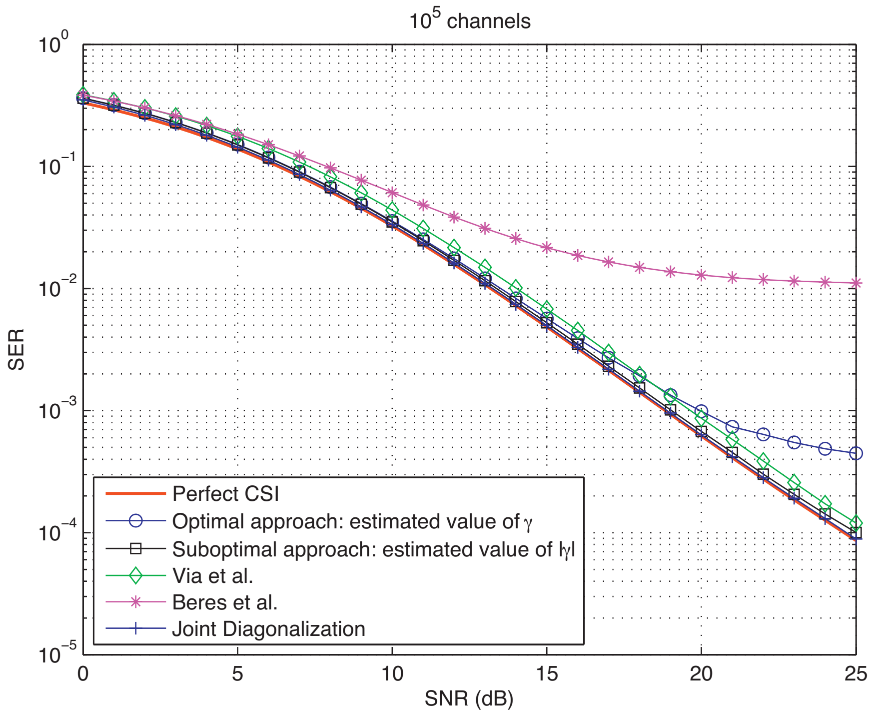

Figure 6. Comparison among EVD-based approaches: SER *versus* SNR for Rayleigh channels for a block size of 500 symbols.

and the suboptimal approach overcome the other methods and achieve an adequate error probability for small packets. This result allows us to conjecture that the suboptimal approach will be also adequate for time-varying channels. As further work we will investigate this topic and the form of implementing our technique using an adaptive method like that proposed in Reference [24].

The decoding complexity of methods based on cumulant-matrix diagonalization depends on two parameters: the number of cumulants to be computed and the size of

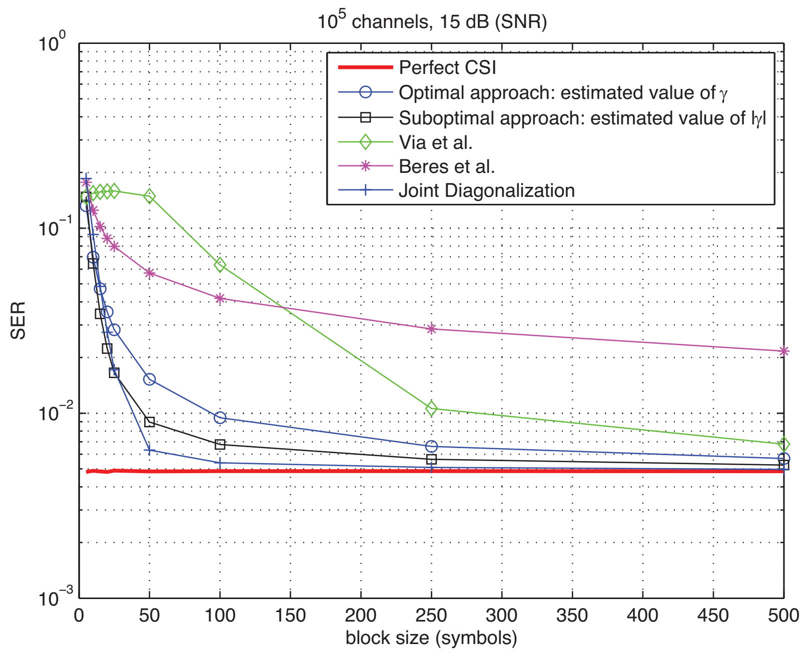

Figure 7. Comparison among EVD-based approaches: SER *versus* packet size for Rayleigh channels at an SNR of 15 dB.

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Table I. Computational load corresponding to compute second- and fourth-order cumulants, and matrix diagonalization.

	Sums 2 flop	Multiplications 6 flop	Squared root 8 flop	Flop
Second-order statistic (SOS)	$K-1$	$K+1$	0	$8K+4$
Fourth-order statistic (FOS)	$7K-4$	$9K+4$	0	$68K+16$
Compute eigenvectors	6	11	3	102
Joint diagonalization	21	24	1	$194+\text{flop(EVD)}$

Table II. Computational load of the compared approaches.

Approach	Number of cumulant	Size of the matrix to be diagonalized	flop
Via *et al.*	3 SOS	2×2	$24K+114$
Beres *et al.*	3 FOS	2×2	$204K+150$
Joint diagonalization	5 FOS	3×3	$340K+\text{flop(EVD)}+274$
$\mathbf{M}_{\text{opt}}$	6 FOS	2×2	$408K+198$
Suboptimal approach	3,7 FOS	2×2	$251.2K+161.2$

the matrix to be diagonalized. Table I shows the number of operations (sums and multiplications) and the FLoating-point OPerations (flop) associated with computing second-order and fourth-order cumulants for a block of K points. We consider that all the operations are performed with complex-valued numbers: a sum corresponds to two flop, a multiplication corresponds to six flop and a square root to eight flop. This table also shows the operations (and flop) related with the procedure used to diagonalize the cumulant matrices. It should be remembered that all approaches compared in Figures 6 and 7, except joint diagonalization, compute the eigenvectors of a (2×2) matrix using expressions in Appendix B. The term flop(EVD) denotes the number of flop needed to compute the eigenvalues of a (3×3) matrix, which is a significant quantity

Table II summarizes the parameters corresponding to the compared algorithms. In order to reduce the computational load, we have used the properties of the cumulants to compute only those that are different. For instance, note that $cum(x_1, x_2^*, x_1, x_2^*) = cum^*(x_2, x_1^*, x_2, x_1^*)$. In the case of the suboptimal approach, the number of fourth-order cross cumulants to be computed depends on the decision criterion (24). In 100 000 independent simulations, we have obtained that $\mathbf{M}_1$ (three different fourth-order cross-cumulants) is used 30% of times and $\mathbf{M}_2$ (four different fourth-order cross-cumulants) is computed 70% of times, which corresponds to computing an average number of 3.7 fourth-order cross-cumulants. Taking into account these results and those presented above, we conclude that the suboptimal approach provides excellent performance in terms of SER and computational load.

6. CONCLUSION

Channel estimation in (2×1) Alamouti's space–time block coded systems can be performed blindly from the eigendecomposition (or diagonalization) of matrices composed of the receiving antenna output fourth-order cumulants. The main assumption behind this approach is the statistical independence between the source symbol substreams. However, the performance of these methods is considerably degraded when the eigenvalue spread of the matrix to diagonalize is close to zero.

In this paper, we have obtained in closed-form the cumulant matrix with maximum eigenvalue spread. Our analysis shows that the optimum matrix depends on a parameter γ whose value must be computed taking into account the correct value of the channel coefficients. Simulation results verify that the performance of this approach matches the optimal performance. We have also determined a simple form of estimating parameter γ by computing the fourth-order cross-cumulants of the observations but, unfortunately, a loss of performance for high SNR has been observed due to finite-sample estimation errors.

Another contribution of this paper has been to propose a suboptimal approach to select the matrix of highest eigenvalue spread from a set of only two cumulant matrices. This approach presents a satisfactory performance compared with the optimal approach and with other blind techniques, like JADE. Furthermore, the suboptimal approach has important advantages with respect to JADE for real implementation in FPGAs and DSPs: it computes fewer cross-cumulant matrices and diagonalizes a single (2×2) matrix. Moreover, parameter γ is only used as a threshold in the suboptimal approach and, therefore, it is less sensitive to estimation errors.

APPENDIX A: JOINT DIAGONALIZATION PROCEDURE

The procedure used by JADE to simultaneous diagonalization of several fourth-order cross cumulant matrices is an extension of the Jacobi technique: a joint diagonalization criterion is iteratively optimized under plane rotations. We have optimized the original code presented in Reference

[25] by considering the special structure of the Alamouti's system.

Let $\mathbf{C}$ be a (2×4) defined as

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \end{bmatrix}. \tag{A1}$$

For instance, the first step consists in obtaining the following (3×2) matrix:

$$\mathbf{G} = \begin{bmatrix} c_{11} - c_{22} & c_{13} - c_{24} \\ c_{12} & c_{14} \\ c_{21} & c_{23} \end{bmatrix}. \tag{A2}$$

From this matrix, we obtain $\mathbf{F} = \mathbf{G}\mathbf{G}^{\mathrm{H}}$ and a (3×3) symmetric matrix given by

$$\mathbf{E} = \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix} \tag{A3}$$

where

$$e_{11} = f_{11} \tag{A4}$$

$$e_{12} = e_{21} = \mathrm{real}(f_{12} + f_{13}) \tag{A5}$$

$$e_{13} = e_{31} = \mathrm{imag}(-f_{12} + f_{13}) \tag{A6}$$

$$e_{22} = \mathrm{real}(f_{22} + 2f_{23} + f_{33}) \tag{A7}$$

$$e_{23} = e_{32} = \mathrm{imag}(f_{22} + 2f_{23} + f_{33}) \tag{A8}$$

$$e_{33} = \mathrm{real}(f_{22} - 2f_{23} + f_{33}). \tag{A9}$$

The eigenvalues are computed and, then, the eigenvector $\mathbf{u}_1$ corresponding to the largest eigenvalue is obtained. If $u_{11} < 0$ then we assign $u_1 = -u_1$. Finally, the channel matrix is estimated as

$$\mathbf{H} = \begin{bmatrix} c & -s^* \\ s & c \end{bmatrix} \tag{A10}$$

where $c = \sqrt{\frac{1+u_{11}}{2}}$ and $s = \frac{u_{12} - \sqrt{-1}u_{13}}{2c}$.

To compute the eigenvalues of the (3×3) matrix $\mathbf{E}$ has a significant computational load in comparison with the (2×2) case. A way to obtain the eigenvalues consists in computing the roots of the following polynomial:

$$\det(\mathbf{E} - \lambda\mathbf{I}_3) = -\lambda^3 + \lambda^2 \mathrm{tr}(\mathbf{E}) + \lambda\frac{1}{2}\left(\mathrm{tr}(\mathbf{E}\mathbf{E}) - \mathrm{tr}^2(\mathbf{E})\right) + \det(\mathbf{E}) \tag{A11}$$

where $\mathbf{I}_3$ is the (3×3) identity matrix. The roots can be found using methods for solving cubic equations, like Cardano's procedure or the Lagragian method. The eigenvector corresponding to the largest eigenvalue, λ_1, can then be computed by solving the following system of equations:

$$(\mathbf{E} - \lambda_1\mathbf{I}_3)\,\mathbf{u}_1 = \mathbf{0}. \tag{A12}$$

APPENDIX B: EIGENVALUE DECOMPOSITION OF A (2 × 2) MATRIX

In this appendix we present a closed form of computing the eigenvectors for a 2×2 matrix. Let $\mathbf{C}$ be a (2×2) matrix defined as follows:

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}. \tag{B1}$$

Its eigenvalues are the roots of the following polynomial:

$$++\det(\mathbf{C} - \lambda\mathbf{I}_2) = \lambda^2 - (c_{11} + c_{22})\lambda + c_{11}c_{22} - c_{12}c_{21} \\ = \lambda^2 - \lambda t + d$$

where $t = c_{11} + c_{22}$ and $d = c_{21}c_{12} - c_{11}c_{22}$. The roots are

$$\lambda_{1,2} = \frac{t \pm \sqrt{t^2 + 4d}}{2} \tag{B2}$$

where $p = (c_{11} - c_{22})/2$ and $p = \sqrt{p^2 + c_{12}c_{21}}$.

The eigenvectors can then be computed as

$$\mathbf{U}' = \begin{bmatrix} \mathbf{u}'_1 & \mathbf{u}'_2 \end{bmatrix} = \begin{bmatrix} \frac{c_{12}}{\lambda_1 - c_{11}} & \frac{c_{12}}{\lambda_2 - c_{11}} \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{c_{12}}{-p+q} & \frac{c_{12}}{-p-q} \\ 1 & 1 \end{bmatrix}. \tag{B3}$$

Finally, the normalized eigenvectors are given by

$$\mathbf{U} = \begin{bmatrix} \frac{\mathbf{u}'_1}{\|\mathbf{u}'_1\|_2} & \frac{\mathbf{u}'_2}{\|\mathbf{u}'_2\|_2} \end{bmatrix} \tag{B4}$$

where

$$\|\mathbf{u}'_1\|_2 = \sqrt{\left(\frac{c_{12}}{-p+q}\right)^2 + 1}, \quad \|\mathbf{u}'_2\|_2 = \sqrt{\left(\frac{c_{12}}{-p-q}\right)^2 + 1}. \tag{B5}$$

Note that the method to estimate the channel matrix in the Alamouti's system only requires to compute the eigenvectors. Therefore, it is only needed to compute Equations (B3), (B4), and (B5).

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