VISUAL PREDICTIVE CONTROL

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Abstract: In this paper, we present an original application of the model predictive control. The visual servoing problem is addressed with a MPC strategy. The minimization of the difference between the desired and the measured visual features is performed in the image plane over a prediction horizon and takes into account the unavoidable constraints on robotic systems. Simulations and experiments on a wheeled mobile robot with an embedded omnidirectional camera illustrate the efficiency and the real-time applicability of the visual predictive control.

Keywords: Visual servoing, model predictive control, real-time application.

1. INTRODUCTION

Visual based control, also named visual servoing, is a control law using visual informations in the feedback loop. The visual servoing presents an alternative solution to motion control of autonomous manipulators, mobile robots,...
In the last few years, the visual servoing is the object of intensive research and the applications are widespread in different fields:

- aerial field: a common application is the tracking of line (road or electric line). We also find visual servoing for military applications (Unmanned Aerial Vehicles -UAV-) and for visual landing;
- medical field: to give aid to laparoscopic surgery (Krupa *et al.*, 2003)
- robotic field: robot motion in hostile environments.

Initially developed with perspective cameras, the visual servoing is now generalized to catadioptric cameras which allow larger fields of view (Abdelkader *et al.*, 2005). The key point of visual servoing is the interaction matrix, linking the robot velocities to the image observations. This matrix can be difficult to obtain and its inverse can lead to singularities.
To address these both problems, a numerical method for estimating the inverse of the interaction matrix has recently been proposed (Lapresté *et al.*, 2004). A large number of points is used for the learning stage and the knowledge of the global optimum is required.
In this context, an alternative approach to visual servoing based on a MPC strategy has been proposed (Allibert *et al.*, 2006). The aim of visual servoing is written as a minimization of a cost function over a prediction horizon in the image plane. The optimization problem can easily take into account constraints, unavoidable on robotic systems. This approach does not require the explicit formulation of the interaction matrix.
The objective of this paper is not to provide new theoretical results on MPC, but to contribute to original applications of MPC technique on fast systems.
This paper is organized as follows. In section 2, we present the visual servoing: the principle, the implementation stages and the drawbacks. The omnidirectional vision is also addressed. In section

3, we present the visual predictive control (VPC), application of the MPC strategy to image plane. Simulations and experimental results on a wheeled mobile robot illustrate the capabilities and the efficiency of the VPC.

2. OMNIDIRECTIONAL VISUAL SERVOING

Visual Servoing (VS) concerns several fields of research including vision, robotics and automatic control. It can be useful for a wide range of applications and can be used to control different dynamic systems as manipulators arms, mobile robots (Vidal *et al.*, 2003),(Carvalho *et al.*, 2000), drone, etc. The aim of visual servoing is to control a robot using the information provided by a vision system. A tutorial in visual servoing can be found in (Hutchinson *et al.*, 1996). The fundamental classification of visual servoing distinguishes three approaches: the position-based (3D) control, the image-based (2D) control and the hybrid control (2D1/2, mix of 2D and 3D approaches) according to the design of the control scheme. The considered approach in this paper is the image-based control structure depicted figure 1.
In image-based visual servo control, the error signal is directly defined in terms of image feature parameters. An image feature is any structural feature than can be extracted from an image. Typically, it will correspond to the projection of physical feature of an object onto the camera image plane (e.g. a point, a line, a circle,...).

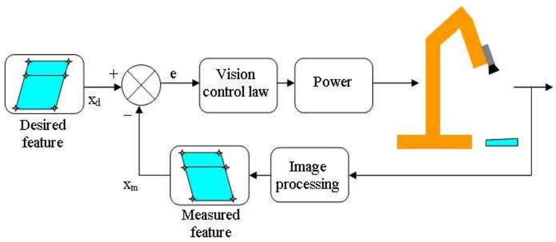

Fig. 1. Image-based control scheme

In classical approach, the main steps of the image-based control implementation are: the modelling of camera, the determination of the interaction matrix and the design of the control law.

2.1 *Omnidirectional vision*

Many applications in computational vision require a large field of view, independent of the robot position. Unfortunately, conventional systems are limited to a region around the observed object. One effective way to enhance the field of view is to combine convex mirror and a conventional camera. Mirror shape can be spherical, conic, parabolic or hyperbolic. The general approach of combining mirrors with conventional imaging systems is referred to as catadioptric image formation or omnidirectional vision. These systems can provide an image with 360° field of view at video rate.

Fig. 2. Omnidirectional image

Recent works on modelling of catadioptric cameras have been reported in (Geyer and Daniilidis, 2001). The authors show that in the case of unique viewpoint, the projection of a world point in the image (Fig. 3) is given by :

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \lambda X \\ \lambda Y \\ 1 \end{pmatrix} \quad (1)$$

with:

$$\lambda = \frac{\xi + m}{Z + \xi\sqrt{X^2 + Y^2 + Z^2}} \quad (2)$$

- (X,Y,Z) are the coordinates of a world point (Pw) in the mirror frame;
- (u,v) are the coordinates in pixels of the point (Pw) in the image frame (Pi);
- u_0,v_0,α_u,α_v are the camera intrinsic parameters;
- ξ and m are the mirror intrinsic parameters.

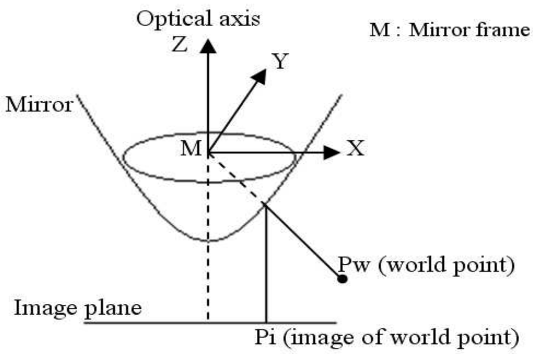

Fig. 3. Point projection: parabolic mirror and orthographic camera

2.2 *Interaction matrix*

The aim of 2D visual servoing is to control the camera pose such that the measured visual features x_m reach the desired features x_d. The camera pose is usually defined by six parameters: three for the position Ω and three for the rotation T. If the objects that the camera observes are assumed to be motionless, the relationship between the value of the visual features $x(t)$ and the camera pose $\theta(t) = [\Omega(t)T(t)]$ can be written as:

$$x(t) = f(\theta(t)) \tag{3}$$

where f is a nonlinear function. Visual servoing requires the reverse: the computation of θ given x as input.
A local estimate, by differentiating (3) is usually used. The relationship between the velocity of the visual features $\dot{x}(t)$ in the image plane and the velocity in the camera space (known in robotics literature as a velocity screw) is then:

$$\dot{x}(t) = \frac{\partial f}{\partial \theta}(t).\dot{\theta}(t) = J.\dot{\theta}(t) \tag{4}$$

where J is called interaction matrix or image jacobian matrix. In the case of perspective cameras, the determination of the interaction matrix has been addressed for different features: point, line, circle, cylinder, etc... In the case of omnidirectional cameras, only point and line features have been studied (Abdelkader *et al.*, 2005),(Barreto *et al.*, 2002).

2.3 Control law

The interaction matrix is the keystone of classical visual servoing. Once the matrix determined, the control objective is to ensure an exponential decay of the error signal, e(t), difference between the desired features x_d and the measured ones x_m. Most of time, a stabilizing control law ($\lambda > 0$) is used:

$$\dot{e}(t) = -\lambda e(t) \tag{5}$$

Combining (4) with equation (5), we obtain:

$$\dot{\theta}(t) = -\lambda.J_i^{-1}(t).e(t) \tag{6}$$

The robot velocity screw is function of the error signal in the image frame. Its calculus requires the inversion of the interaction matrix which can be difficult and lead to singularities.

2.4 Implementation difficulties of classical visual servoing

The interaction matrix given in (4) is function of ρ, called the depth, distance from the camera to the object. In the case of omnidirectional camera, ρ is given by:

$$\rho = \sqrt{X^2 + Y^2 + Z^2} \tag{7}$$

In practice, ρ is an unknown parameter. Two practical ways to obtain this value are :

- the use of external sensors as ultrasound,three dimensional reconstruction (additional camera in stereo approach),... to estimate ρ;
- the use of an approximated value at the desired camera position (the final position is given).

One of the major drawbacks of image-based control is the existence of singularities that lead to failures of the control law. Two kinds of singularities exist (Chaumette, 1998):

- Representation singularities: the interaction matrix becomes singular when we have a degeneration of visual features. For example, if the optical center of the camera is coplanar with two stationing points;
- Isolated singularities: locally, due to the number of visual features, the interaction matrix is not invertible and the synthesis of the control law is not possible.

A new method has been recently proposed in (Lapresté *et al.*, 2004) in order to avoid these problems. The authors propose a method to estimate the inverse jacobian matrix without computing the direct jacobian matrix. Simulations have been performed and have shown that the results obtained using this jacobian are satisfactory. However, this learning method needs a lot of points and an a priori knowledge of the global optimum.
In the next section, we propose an alternative and original approach based on MPC strategy for visual servoing task.

3. THE VISUAL PREDICTIVE CONTROL

Most of the visual servoing applications is manipulator or mobile robot, always subject to mechanical constraints (joint bounds). In 2D servoing, visual features should stay in the image, inducing visibility constraints, function of the image size.
The visual servoing problem is to minimize an error signal, difference between desired and measured features. Predictive control strategy is well-adapted to address both problems: the constraint handling and the minimization.
Extended to visual servoing, the predictive control law, denoted visual predictive control (VPC), is formulated into a constrained optimization problem in the image plane. The VPC objective is to determine a sequence of Nc future controls (robot commands) by the minimization of a cost function J over a finite prediction horizon Np.

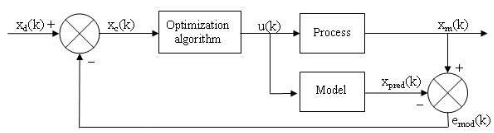

Fig. 4. IMC structure

Combining to the IMC structure (Fig. 4), the mathematical formulation is:

$$\min_{\widetilde{U}} J = \sum_{j=k+1}^{k+Np} [x_c(j) - x_{pred}(j)]^T Q \, [x_c(j) - x_{pred}(j)] \tag{8}$$

with:
$x_{pred}(j)$ the predicted features from the model (robot+camera), $x_c(j) = x_d(j) - e_{mod}(j)$ with $x_d(j)$ the desired features and $e_{mod}(j) = x_m(j) - x_{pred}(j)$ represents modelling errors.
$x_m(j)$ the measured features from image processing, $Q > 0$ a symmetric matrix.
Nc and Np are respectively the prediction horizon and the control horizon.
$\widetilde{U} = \{u_k, u_{k+1}, ..., u_{k+Np-1}\}$, the sequence of Nc control inputs subject to constraints:

$$u_{min} \leq u_k \leq u_{max}.$$

The constrained optimization problem (8) is solved at each sampling time. Only the first control input of the optimal control sequence is applied to the robot. At the next sampling time, the measurements are updated and the procedure starts again.
The model (robot+camera) plays a crucial role in terms of control performance. For real-time application, the computational time (needed to the image processing, the model resolution and the optimization resolution) must be reduced as short as possible. Due to the increase of computer power, an application such that VPC, is now possible. The Levenberg Marquadt algorithm, known for its efficiency and its robustness, can be used. However, numerous optimization routines are available in software libraries.

4. APPLICATION

The Visual Predictive Control has been validated by simulations and experiments. The platform is a wheeled mobile robot where a catadioptric camera, looking at the upwards from the ground, is embedded (Fig. 5).
The mobile robot from the K-team firm (Koala) is equipped with two rigid driving wheels.
The nonholonomic constraints on the motion of the mobile robot arise from rolling-without-slipping of the mobile platform permit to write the discrete kinematic model:

$$\begin{aligned} x(k+1) &= x(k) + Te.V_r.\cos(\theta(k) + a); \\ y(k+1) &= y(k) + Te.V_r.\sin(\theta(k) + a); \\ \theta(k+1) &= \theta(k) + Te.W_r; \end{aligned} \tag{9}$$

with a=Vr(Te/2), Te the sampling time. Vr and Wr are respectively the heading speed and the angular velocity of the unicycle.
The robot posture (position and orientation) is given by the vector (x,y,θ).

Fig. 5. Koala mobile robot and its environement

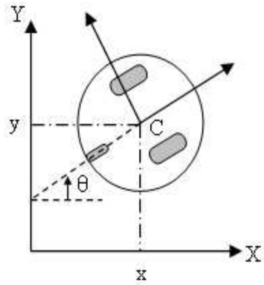

Fig. 6. Posture definition

Thanks to omnidirectional vision, the different targets can be placed all over the work space and so, no visual constraints are necessary. The image processing task is easy in our case. The different colored targets are placed in a white space.

Combining (1) and (9), the nonlinear model (robot + camera) used to predict the feature over the prediction horizon Np for one point like feature, is given by:

$$\mathrm{x_{pred}}(k+1) = \begin{pmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} \lambda X(k+1) \\ \lambda Y(k+1) \\ 1 \end{pmatrix} \tag{10}$$

with:

$$\begin{aligned} \lambda &= \frac{\xi + m}{Z(k+1) + \xi\sqrt{X(k+1)^2 + Y(k+1)^2 + Z(k+1)^2}}; \\ X(k+1) &= X_o - (x(k) + Te.V_r.\cos(\theta(k) + \frac{Te}{2} W_r)); \\ Y(k+1) &= Y_o - (y(k) + Te.V_r.\sin(\theta(k) + \frac{Te}{2} W_r)); \\ Z(k+1) &= Z(k) = Z_o. \end{aligned} \tag{11}$$

where (X_o, Y_o,Z_o) are constants and represent the initial distance between the vertices of the square and the camera.
Four simulations are given, corresponding to four different initial mobile robot positions. Each simulation is chosen so that the movement between the initial and final configurations requires ma-

noeuvres.
For each simulations, are represented:

- The initial positions (defined by '+') and desired positions (defined by 'o') of the visual feature;
- The feature trajectories in the image;
- The initial configuration (in green) and the final configuration of the robot in the XY space (rear motion is symbolized by the color black and forward motion by the color blue);
- The heading speed Vr and the angular velocity Wr of the unicycle;
- The signal error between the measured and desired features in the image.

As shown in figures 7-10, the signal error is well regulated to zero for each different robot postures. The projection of the visual features in the omnidirectional image at the final position of mobile robot corresponds to the desired features. For the well-known difficult configuration (180° rotation around the optical axis (Chaumette, 1998)), simulation (Fig. 9) and experimental results (available videos) show that the VPC converges without difficulties. The control horizon Nc is a relevant parameter for the control performance. Owing to the nonholonomic constraints, the mobile robot can not move sideways with one command: a manoeuvre (forward and reverse gear) is then necessary. The control horizon allows to deal with this problem by proposing Nc different controls at time k similar to a manoeuvre. The computation of the sequence can be compared to an implicit trajectory planification.
The robustness of VPC has been tested for camera modelling error of 20% in figure 10.
Remark. The hardware used is a PC Intel Pentium 4, 512 MB RAM with the Matlab software. The computational time is about 50 ms.

5. CONCLUSION ET PERSPECTIVES

The visual predictive control presented in this paper is both an original application of model predictive control and an efficient alternative approach to visual servoing. The minimization is performed in the image plane and can take into account mechanical and visual constraints. The model implicitly plays the crucial role of the interaction matrix.
Simulations and experiments have demonstrated both the efficiency and the real-time applicability of the VPC on a fast system (a wheeled mobile robot). Combining to the IMC structure, the robustness and the accuracy of the VPC have been addressed. We also pointed out the influence of the control horizon Nc on the control performance.
In regard to the needed computational time for the image processing and the optimization resolutions, the increase of computer power offers a large field of investigation for visual predictive control.

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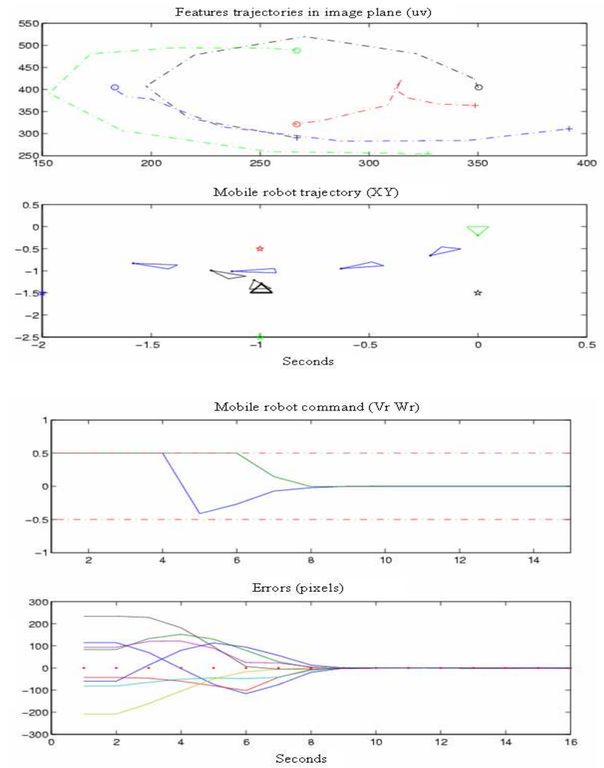

Fig. 7. Simulation 1 (Nc=5, Np=8)

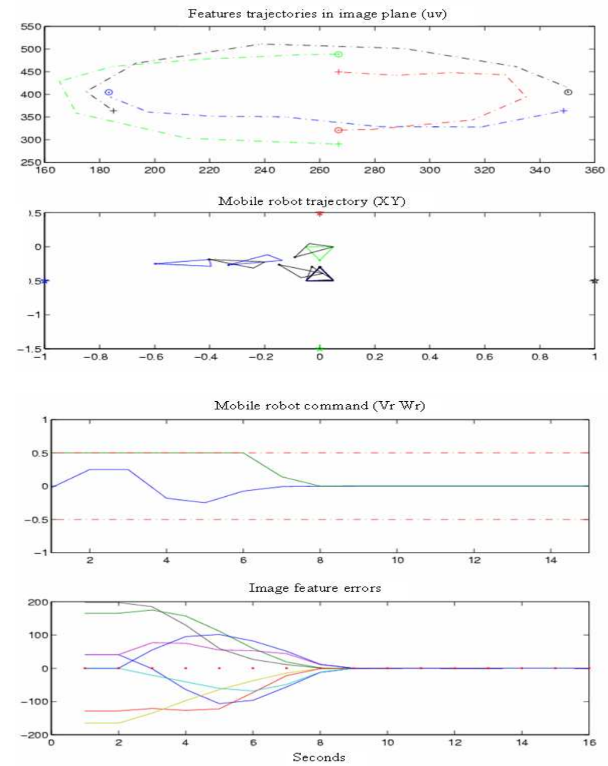

Fig. 9. Simulation 3 (Nc=5, Np=8)

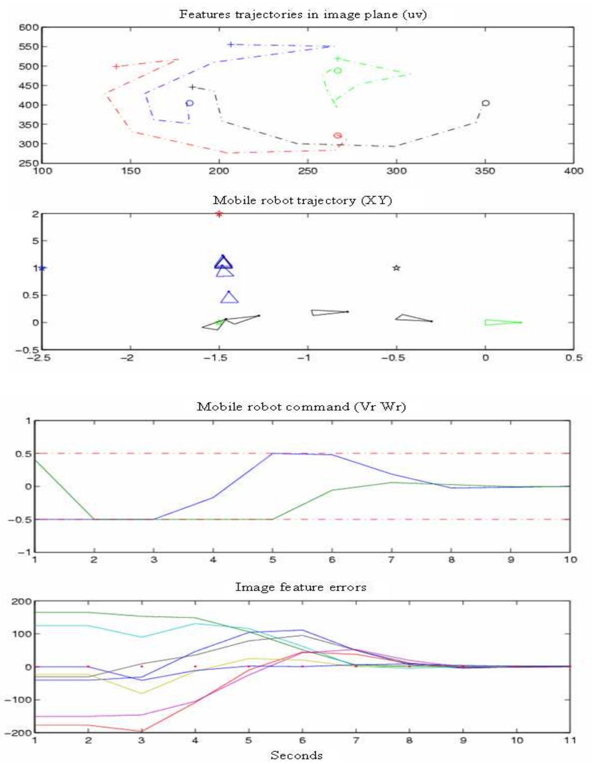

Fig. 8. Simulation 2 (Nc=5, Np=8)

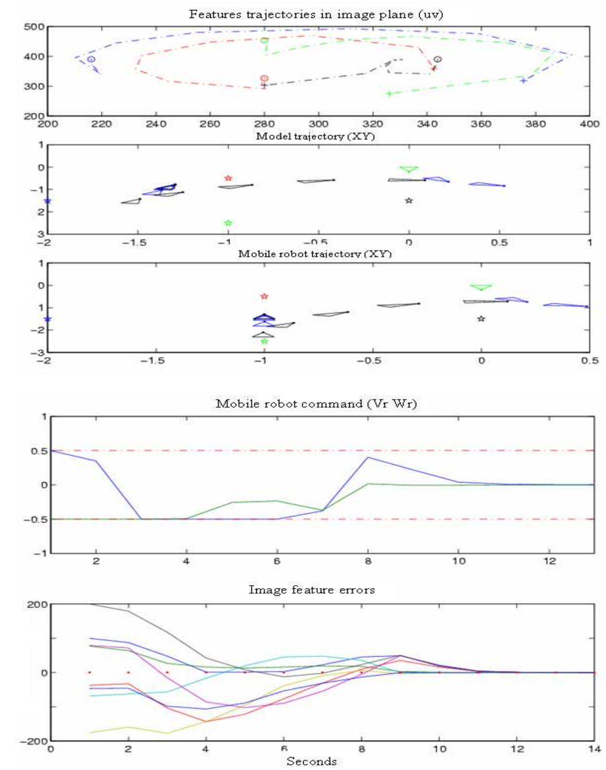

Fig. 10. Simulation 4 (Nc=5, Np=8)