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# T WAVE CANCELLATION FOR THE P-R ESTIMATION DURING EXERCISE AND RECOVERY

*Aline Cabasson, Olivier Meste*

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# T wave Cancellation for the P-R Estimation during Exercise and Recovery

A. Cabasson and O. Meste

Laboratoire I3S-CNRS-UNSA, Université de Nice Sophia-Antipolis, France

## Abstract

The problem of estimation of the heart periods and consequently the problem of the understanding of the neural activity during exercise and recovery is still interesting since it could lead to innovations in the field of pacemaker's design. This work aims to present a new technique to take into account the influence of the T wave overlapping the P wave during high heart rate. Considering only the decreasing part of the T wave, the main idea is to model the T wave, cancel its influence, and finally estimate the P-R intervals. This leads to improve the estimation techniques of the P-R intervals in exercise and recovery.

## 1 Introduction

Although the P-R intervals are difficult to extract and process especially during exercise tests where T-P fusion occurs during higher heart rates, only few works have been proposed until now for estimating P-R intervals during both exercise and recovery [1]. Among the related works, some methods based on the detection of the maximum of cross correlation function provide globally good results. But the T wave which overlaps the P one especially for high heart rate, is not considered [2], [3]. The cancellation of the T wave during high heart rates should reduce the bias in the estimation of the P-R intervals. Therefore, we propose here a new approach that take into account the influence of the T wave overlapping the P wave. Once the T wave is modelled, we can cancel it and integrate this in our method of time delay estimation with unknown signal based on an iterative Maximum-Likelihood approach which generalizes the well known Woody's method [4], [5]. The first part of the paper is devoted to an extension of the T wave modelling technique using a straight line [5]. We propose a refinement by modelling the overlapping T wave by a  $l^{th}$  order polynomial function or by a piecewise approach. Finally, the results of the investigation with pseudo-real simulations of ECG are presented.

## 2 Method

The technique used to estimate the P-R intervals is based on our Woody's improved method [4], [5]. This method use an iterative Maximum Likelihood Estimator (MLE) to estimate delays corresponding to P-R intervals up to an unknown constant. The aim of our study is to model the T wave in order to estimate efficiently the P-R intervals. Here, we propose two models that will be included in the time delay estimation process.

We consider a model where  $x_i(n)$  represents all the sampled observations (for all  $n$ ) of the considered  $i^{th}$  P-R interval (with  $i = 1..I$ ,  $I$  the number of realizations). Each observation contains  $s_{d_i}(n)$  defined as the template wave delayed by  $d_i$  as  $s_{d_i}(n) = s(n - d_i)$ , plus  $e_i(n)$  an observation's noise. As during exercise tests T-P fusion occurs, we consider the T wave represented by a function  $f(n; \underline{\theta}_i)$  linearly parameterized.

Finally, our model is expressed as:

$$x_i(n) = \alpha_i \cdot s_{d_i}(n) + \alpha_i \cdot f_{d_i}(n; \underline{\theta}_i) + e_i(n) \quad (1)$$

where the variable  $d_i$  is the  $i^{th}$  P-R interval to be estimated up to an unknown constant.

Considering only the decreasing part of the T wave, the main idea is to model the T wave,  $f(\underline{\theta}_i)$  and to cancel its influence before the estimation of the P-R.

### 2.1 Modeling the T wave by a $l^{th}$ order polynomial function

We will assume that the T wave should be described by a regular and smooth function, i.e. a  $l^{th}$  order polynomial function characterized by its coefficients in the vector  $\underline{\theta}_i$ . Thus, the T wave can be modeled by the function:

$$f(\underline{\theta}_i)[n] = \sum_{l=0}^L \theta_i[l] \cdot n^l \quad (2)$$

In a previous work [5], we considered that  $l = 1$ , i.e. we take into account the overlapping of the T wave modeled as a straight line. We tested our method for a  $2^{nd}$  order but we will present in this work the model corresponding to a  $3^{rd}$  order polynomial function:

$$f(\underline{\theta}_i)[n] = \theta_i[3] \cdot n^3 + \theta_i[2] \cdot n^2 + \theta_i[1] \cdot n + \theta_i[0] \quad (3)$$

The main difficulty of the segmentation of the observations is to not take into account the increasing part of the T wave. Thus, it is more convenient to consider the T wave as a  $3^{rd}$  order polynomial function which is decreasing. This constraint is fulfilled introducing constraints on the coefficients  $\underline{\theta}_i$  such as:

$$\begin{cases} \theta_i[3] > 0 \\ \theta_i[1] < 0 \\ 3 \cdot \theta_i[3] \cdot N^2 + 2 \cdot \theta_i[2] \cdot N + \theta_i[1] < 0 \end{cases} \quad (4)$$

with  $N$  the length of our observation's window.

Therefore, we consider our problem of minimization as a linear least squares with linear inequality constraints (problem LSI) [6].

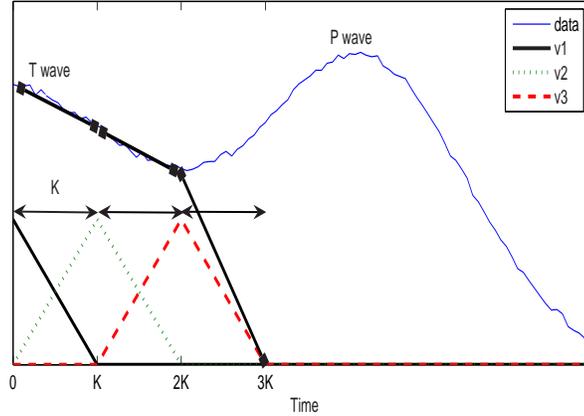


Figure 1: Example of T wave modelled as a decreasing piecewise straight line thanks to a basis composed of 3 vectors.

## 2.2 Modeling the T wave by a decreased piecewise straight line

Considering only the decreasing part of the T wave, the main idea is to model the T wave by a decreased piecewise straight line, see figure 1.

The T wave is considered as a piecewise straight line using a function  $f(n; \underline{\theta}_i)$ , so we model the T wave with a weighted sum of basic vectors  $v_l$  as :

$$f(\underline{\theta}_i)[n] = \sum_{l=1}^L \theta_i[l].v_l[n] \quad (5)$$

We create a vector basis of  $L$  vectors which define  $L$  intervals of width  $K$ .  $L$  and  $K$  are chosen arbitrarily such as the length  $L \times K$  is representative of the width of the decreasing part of the T wave. The nature of the vectors of the basis can be various : gaussian function, cosine, straight line ... On the figure 1, we choose for example straight lines.

In order to be consistent with the observations, some constraints are added:

- In each interval of the basis, a negative slope is imposed,
- In order to keep the continuity of the modelled T wave, the joining points between two consecutive intervals must respect the following configuration : the last point of the  $l^{th}$  interval must be identical to the first point of the  $(l + 1)^{th}$  interval.

The aim is to build a basis of  $L$  vectors. We choose arbitrarily  $L = 3$  as on the figure 1. Therefore, we consider 3 intervals. On each interval of the basis, for  $n \in [k \times K : (k + 1) \times K]$  (with  $k = 0..2$ ), we model the T wave by a segment that is a weighted sum of 2 non zero vectors:

$$\begin{aligned} f[n] &= \theta_1.v_1[n] + \theta_2.v_2[n]; & n \in [0 : K] \\ f[n] &= \theta_2.v_2[n] + \theta_3.v_3[n]; & n \in [K : 2K] \\ f[n] &= \theta_3.v_3[n]; & n \in [2K : 3K] \end{aligned}$$

Moreover we want to model the T wave by decreasing segments so we need to impose the following conditions on each interval of the basis:

$$\begin{aligned} f'[n] &= \theta_1.v'_1[n] + \theta_2.v'_2[n] \leq 0; n \in [0 : K] \\ f'[n] &= \theta_2.v'_2[n] + \theta_3.v'_3[n] \leq 0; n \in [K : 2K] \\ f'[n] &= \theta_3.v'_3[n] \leq 0; n \in [2K : 3K] \end{aligned}$$

In order to obtain a tractable relation for the conditions on the coefficients  $\theta_i$ , we choose arbitrarily the 3 vectors of the basis as :

$$\begin{cases} v'_1[n] < 0; n \in [0 : K] \\ v'_1[n] = -v'_2[n]; n \in [0 : K] \\ v'_2[n] = -v'_3[n]; n \in [K : 2K] \end{cases} \quad (6)$$

Given our choice of basis on the figure 1, these relations imply that the vectors  $v_1$  and  $v_3$  are decreasing respectively on the intervals  $[0 : K]$  and  $[2K : 3K]$ .

Imposing these properties to the basis vectors, we need to check the conditions in the joining points between two consecutive intervals in order to keep the continuity of the modelled T wave. We need the property of continuity for  $n = K$  and  $n = 2K$ .

Thus for example in  $n = K$  we want:

$$\theta_1.v_1[K] + \theta_2.v_2[K] = \theta_2.v_2[K] + \theta_3.v_3[K] \quad (7)$$

However, using (6), on each interval we get the relation :

$$\begin{cases} v_1[n] = -v_2[n] + C1; n \in [0 : K] \\ v_2[n] = -v_3[n] + C2; n \in [K : 2K] \end{cases} \quad (8)$$

where  $C1$  and  $C2$  are constant values.

By replacing in (7), the condition of continuity in  $K$  becomes :

$$(\theta_2 - \theta_1).v_2[K] + \theta_1.C1 = (\theta_3 - \theta_2).v_3[K] + \theta_2.C2 \quad (9)$$

We impose that :

$$\begin{cases} v_1[K] = 0 \\ v_3[K] = 0 \end{cases} \quad (10)$$

which implies given the relations (8):

$$\begin{cases} v_2[K] = C1 \\ v_2[K] = C2 \end{cases} \quad (11)$$

The condition of continuity for  $n = K$  (9) becomes :

$$\theta_2.v_2[K] = (\theta_2 - \theta_3).v_2[K] + \theta_3.C2$$

Thanks to this relation and (11) we have  $C1 = C2$  for all  $\theta_k$ , so the continuity for  $n = K$  is ensured.

Finally, when we build a basis of  $L$  vectors, we can apply these rules :

- The nature of the vectors can be various: gaussian function, cosine, straight line ...,
- The first vector is decreasing on the interval  $[0 : K]$  and is null after,
- The last vector is null for  $n \in [0 : (L-2)K]$  and is decreasing on the interval  $[(L-1)K : LK]$ .

This implies that  $\theta_L$  must be positive in order to keep the decreasing property of the modelled T wave.

Besides, thanks to the hypotheses (6) and (8), the constraints on the  $\theta_i$ 's are :

$$\forall l \in [1 : L - 1], \theta_i[l] > \theta_i[l + 1] > 0 \quad (12)$$

Note that the previous development has been given without lack of generality since it is valid for any number of vectors  $L$  modelling the decreasing part of the T wave.

### 3 Results

In order to measure the performances of our T wave modelling, we introduce pseudo-real simulations of ECG. We compute a simulated ECG with a constant P-R interval added to T and P waves with increasing overlapping intervals. Theoretically, the estimation of the time delay, i.e the P-R interval, should be constant. It is expected the T wave introduces bias with a value depending of the overlapping ratio. Figure 3 shows the bias of the time delay estimator for the 3 models: thick blue line, green line and red dashed line correspond respectively to the T wave modelling by decreasing piecewise straight line, a decreasing  $3^{rd}$  order polynomial function and a decreasing single straight line [5]. The black dotted line corresponds to an estimation of the time delays without modelling. The more the beat number increases, the more the T wave overlaps the P one, leading to an increasing bias. We observe globally that, with or without modelling, the bias is not very important especially when the T wave is no present in the P one. However, when the T wave overlaps the P one, we note that with the approach proposed by piecewise, the bias is lower that with the method presented in [5] where the T wave influence was considered as a decreasing single straight line.

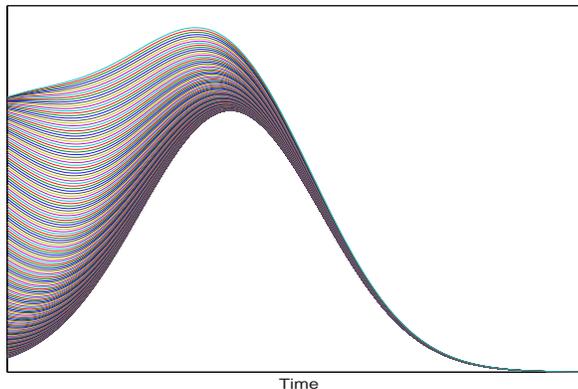


Figure 2: Pseudo-real simulations of the overlapping T wave .

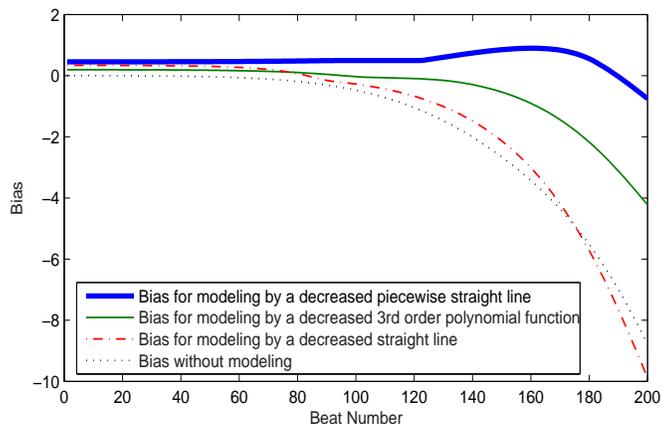


Figure 3: Bias of the time delay estimator for the 3 models: thick blue line, green line and red dashed line correspond respectively to the T wave modelling by decreasing piecewise straight line, a decreasing  $3^{rd}$  order polynomial function and a decreasing single straight line. The black dotted line corresponds to an estimation of the time delays without modelling.

## 4 Conclusion

In a previous work [5], the T wave was modelled using a single straight line. In this report, we propose an extension of this model : the presence of the T wave into the P one is thus modelled by a  $3^{rd}$  order polynomial function or by a decreasing piecewise straight line. Once the T wave is modelled, we include it in our Woody’s improved method for the estimation of the P-R intervals [4], [5]. The most difficult choice of our approach by piecewise is the selection of the vector basis: what kind of vectors is better? straight line, cosinus, gaussian functions ... How much vectors shall we take? What is the optimal width for each vector of the basis? Further studies must be done in order to optimize our method but the difficulty is that each subject is different, and then it is not warranted that it exists an optimal vector basis valid for everyone. The introduction of the decreasing requirement is a need to reduce the variance in the estimation process. The corresponding constraints have a cost: it is assumed that the T wave is observed in its decreasing part. That is met using adapted derivation leads and a matched window. A refinement to a fixed window definition would be to adapt its position using the Bazett correction or any Q-T predictor [7], [8].

Thanks to the computation of these modellings on simulated data, we conclude that our new method exhibits better performances than the previous one according to the bias of the estimators, especially when overlapping ratio is high.

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