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OPTIMALITY STATEMENT OF THE WOODY'S METHOD AND IMPROVEMENT

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RÉSUMÉ :

Même si les signaux d'électrocardiogrammes sont très étudiés, l'analyse des ondes P l'est un peu moins sans doute à cause de la difficulté d'estimer la position de cette onde. Aussi, peu d'études ont été mené sur l'analyse du temps de conduction auriculo-ventriculaire (intervalle PR) en situation d'exercice. Les méthodes habituelles pour estimer les intervalles PR sont basées sur la détection du maximum de la fonction d'intercorrélation. Cette étude propose une nouvelle méthode d'estimation de temps de retards basée sur le Maximum de Vraisemblance et qui généralise la méthode d'estimation de retards de Woody. Ainsi, cette nouvelle approche permet de determiner les intervalles PR en tenant compte de l'onde T qui vient chevaucher l'onde P notamment lors d'un exercice physique où la fréquence cardiaque augmente.

MOTS CLÉS :

estimation de temps de retards, MV, temps de conduction auriculo-ventriculaire, effort

ABSTRACT:

Very little works have been done on the atrioventricular conduction time (PR interval), undoubtedly because this signal is difficult to extract and process, as in exercise tests where T-P fusion occurs during higher heart rates, what makes this problem still interesting. The common approach for the estimation of PR interval during both exercise and recovery is to determine the latency using the detection of the maximum of cross correlation function. This work aims to present a new method of time delay estimation with unknown signal based on an iterative Maximum-Likelihood approach which generalizes the well known Woody's method. This leads to a new approach to determine the PR intervals taking into account the presence of the T wave that is modeled.

KEY WORDS :

time delay estimation, MLE, atrioventricular conduction time, exercise

Optimality Statement of the Woody's method and Improvement

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Abstract

Very little works have been done on PR intervals, undoubtedly because this signal is difficult to extract and process, as in exercise tests where T-P fusion occurs during higher heart rates, what makes this problem still interesting. The common approach for the estimation of PR interval during both exercise and recovery is to determine the latency using the detection of the maximum of cross correlation function. This work aims to present a new method of time delay estimation with unknown signal based on an iterative Maximum-Likelihood approach which generalizes the well known Woody's method. This leads to a new approach to determine the PR intervals taking into account the presence of the T wave that is modeled.

1 Introduction

The analysis of the heart period series is a difficult task especially under graded exercise conditions. Correlation techniques are usually used to estimate the PR by determination of the latency using the detection of the maximum of the cross correlation [1, 2]. Here, we will present a new approach to determine the PR interval taking into account the presence of the T wave which is especially difficult to extract during high rates because it overlaps the P wave. In order to estimate the PR intervals, we will use the Maximum Likelihood Estimator (MLE). The idea behind maximum likelihood parameter estimation is to determine the parameters that maximize the probability of the sample data.

The first part of the paper is devoted to present the well known Woody's method [3] which is used to analyze variable latency signals but which is suboptimal. We will present a new method to determine the PR intervals which formalizes the Woody's one , using an iterative MLE to estimate delays which correspond to PR intervals. We will extend the Woody model because in exercise tests T-P fusion occurs during higher heart rates. It will be based on the modeling of the T wave which overlaps the P wave especially during the exercise, in order to generalized more over.

2 Woody's method

Charles D. Woody [3], presented in 1967 an adaptative filter which allowed identification and analysis of variable latency signals and the basis of detection of latency by correlation. He calculated the cross correlation between each sweep and a template. Hence, the time lag matching the cross correlation maximum corresponds to the latency shift of the given sweep. In this part of report, we will describe this method.

In the model, $x_i(n)$ represents all the sampled observations (for all n) of the considered i^{th} interval to estimate ($i = 1..I$, with I the number of trials). Each observation contains $s_{d_i}(n)$, considered unknown, defined as the reference wave, or the template, delayed by d_i as $s_{d_i}(n) = s(n - d_i)$, plus $e_i(n)$ an observation's noise:

$$x_i(n) = s_{d_i}(n) + e_i(n) \tag{1}$$

The technique derives from iterative correlation and averaging of the data signals. The method can be summarized as follows: given an initial estimate $\widehat{s}(n)$ of the template, and the set of observations $x_i(n)$, the delay d_i for each trial is estimated as:

$$\widehat{d_i} = \arg\max_d \frac{1}{N} \sum_{n=1}^{N} x_i(n)\widehat{s}_d(n) \tag{2}$$

where N is the total number of samples in each trial.

At each step i, the maximum of cross correlation between the template and the i^{th} trial gives the estimation of delay $\widehat{d_i}$. When all the $\widehat{d_i}$ for $i = 1..I$ are estimated, each i^{th} data trial is corrected by his i^{th} delay $\widehat{d_i}$. The average of these aligned data trials gives a new template. Then, a new iteration for i from 1 to I is computed to determine the new $\widehat{d_i}$ until convergence.

However, we observe that Woody do not allows an amplitude variability α_i as Jaśkowski and Verleger [4] who refereed to a more general model in which also the amplitude jitter is allowed :

$$x_i(n) = \alpha_i.s_{d_i}(n) + e_i(n)$$

We can assume that Woody do not take this additional parameter in his model because the template is made from a constant weighted averaging. Besides, this method is suboptimal because the considered signal is included into the average taken as a template in the cross correlation step. So, the cross correlation is biased. Also, the same template is used to estimate all the delays during one iteration ; all the estimated delays are taking into account in averaging process at the end of the iteration.

3 Woody's Method Improvement regards the optimality

Here, we present the theoretical formulation of our improvement in the problem of delay estimation where the amplitude jitter α_i is not considered in order to confirm that the Woody's method is not optimal.

So, we consider the same model as Woody for the observations given by:

$$x_i(n) = s_{d_i}(n) + e_i(n) \tag{3}$$

The noise $e_i(n)$ is a white Gaussian noise with null mean and a variance σ^2. For one observation, with n fixed, we consider the probability as following:

$$p(x_i(n); s(n), d_i) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2\sigma^2}(x_i(n) - s_{d_i}(n))^2\right) \tag{4}$$

By assuming that the noise is white, and consequently independent, all the observations are also independent. Then, for $\mathbf{x}_i = [x_i(1), x_i(2), ..., x_i(N)]^T$.

$$p(\mathbf{x}_i) = \prod_n p(x_i(n)) \tag{5}$$

Thus,

$$p(\mathbf{x}_i; \mathbf{s}, d_i) = \left(\frac{1}{2\pi\sigma^2}\right)^{\frac{N}{2}} \exp\left(-\frac{1}{2\sigma^2}\sum_n (x_i(n) - s_{d_i}(n))^2\right) \tag{6}$$

Then, for all i, the pdf of the processes x_i's, given the delay d_i's and signal vectors $\mathbf{s}$, is:

$$p(\mathbf{X}; \mathbf{s}, \mathbf{d}) = \left(\frac{1}{2\pi\sigma^2}\right)^{\frac{NI}{2}} \exp\left(-\frac{1}{2\sigma^2}\sum_i\sum_n (x_i(n) - s_{d_i}(n))^2\right) \tag{7}$$

where $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_I]$ and $\mathbf{d} = [d_1, d_2, ..., d_I]^T$.

So according to the MLE, the objective is to find $\widehat{\mathbf{s}}$ and $\widehat{\mathbf{d}}$ which maximise the probability of $\mathbf{X}$. Then, the criterion J to be minimized is:

$$J = \frac{1}{2\sigma^2}.\sum_i\sum_n (x_i(n) - s_{d_i}(n))^2 \tag{8}$$

Finally, the aim of the study is then to solve:

$$(\widehat{\mathbf{s}}, \widehat{\mathbf{d}}) = \arg\min_{\mathbf{s}, d_i} J \tag{9}$$

Since the parameters to find $\widehat{\mathbf{s}}$ and $\widehat{\mathbf{d}}$ are intertwined, we compute firstly the derivation of (8) regards $s(n)$, that produces:

$$\widehat{\mathbf{s}} = \frac{1}{I}\sum_k x_{k,-d_k} \tag{10}$$

Then we replace in our expression of the criterion (8) $s(n)$ by its estimate:

$$\begin{aligned}
J &= \frac{1}{2\sigma^2}\sum_i\sum_n (x_i(n) - \frac{1}{I}\sum_k x_{k,d_i-d_k}(n))^2 \\
&= \frac{1}{2\sigma^2}\sum_i\sum_n \Bigg(x_i^2(n) + \frac{1}{I^2}(\sum_k x_{k,d_i-d_k}(n))^2 \\
&\quad -\frac{2}{I}x_i(n)\sum_k x_{k,d_i-d_k(n)}\Bigg) \\
&= \frac{1}{2\sigma^2}\Bigg[\sum_i\sum_n x_i(n)^2 + \frac{1}{I^2}\sum_i\sum_n\left(\sum_k x_{k,d_i-d_k}(n)\right)^2 \\
&\quad -\frac{2}{I}\sum_i\sum_n\left(x_i(n)\sum_k x_{k,d_i-d_k}(n)\right)\Bigg]
\end{aligned}$$

In this last expression, the second term is not function of the delay d_i because when we compute a double integral on signals which are delayed of d_i, it is the same that calculate the double integral of the mean of these signals. Then, this second term is a an approximation:

$$\approx \frac{1}{I}\sum_n \left(\sum_k x_{k,-d_k}(n)\right)^2$$

Then, the criterion is:

$$\begin{aligned} J &= \frac{1}{2\sigma^2}\sum_n \left[\sum_i x_i(n)^2 + \frac{1}{I}\left(\sum_k x_{k,-d_k}(n)\right)^2 \right. \\ &\quad \left. -\frac{2}{I}\sum_i \left(x_i(n)\sum_k x_{k,d_i-d_k}(n)\right)\right] \\ &= \frac{1}{2\sigma^2}\sum_n \left[\sum_i x_i(n)^2 + \frac{1}{I}A - \frac{2}{I}B\right] \end{aligned}$$

Also, the terms A and B are equals because:

$$\begin{aligned} A &= \left(\sum_k x_{k,-d_k}(n)\right)^2 \\ &= \sum_k\sum_l x_{k,-d_k}(n)x_{l,-d_l}(n) \end{aligned}$$

$$\begin{aligned} B &= \sum_i \left(x_i(n)\sum_k x_{k,d_i-d_k}(n)\right) \\ &= \sum_i\sum_k x_i(n)x_{k,d_i-d_k}(n) \\ &\simeq \sum_k\sum_i x_{k,-d_k}(n)x_{i,-d_i}(n) = A \end{aligned}$$

The criterion J is then simplified:

$$J = \frac{1}{2\sigma^2}\sum_n \left[\sum_i x_i(n)^2 - \frac{1}{I}\sum_k\sum_i x_{k,-d_k}(n)x_{i,-d_i}(n)\right] \tag{11}$$

Also, i and k have symmetrical parts, then:

$$\begin{aligned} J &= \frac{1}{2\sigma^2}\sum_n \left[\sum_i x_i(n)^2 - \frac{1}{I}\sum_i x_i(n)^2 \right. \\ &\quad \left. -\frac{2}{I}\sum_i^I\sum_{k>i}^I x_{k,-d_k}(n)x_{i,-d_i}(n)\right] \\ &= \frac{1}{2\sigma^2}\sum_n \left[\left(1-\frac{1}{I}\right)\sum_i x_i(n)^2 \right. \\ &\quad \left. -\frac{2}{I}\sum_i^I\sum_{k>i}^I x_{k,-d_k}(n)x_{i,-d_i}(n)\right] \end{aligned}$$

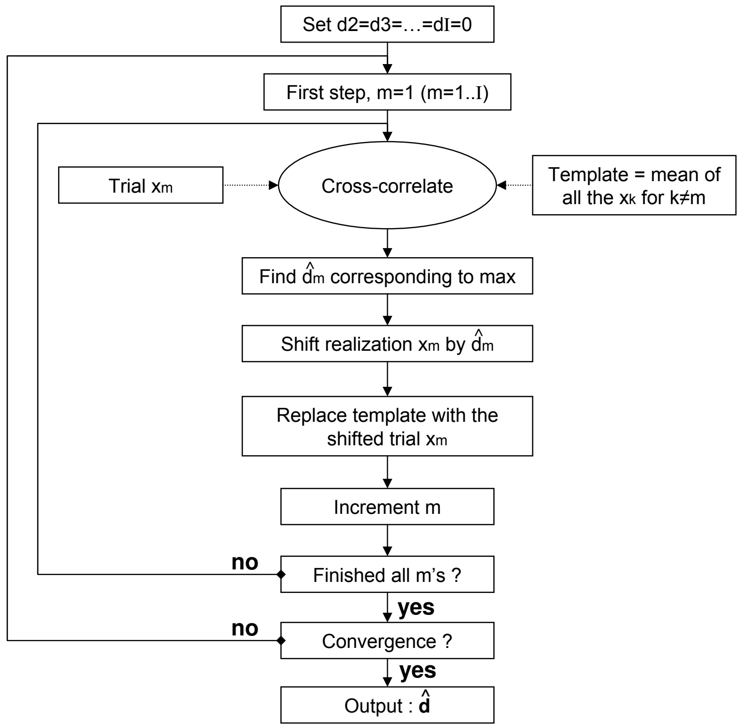

Figure 1: Flow chart of the Woody Improved algorithm.

And finally, as the term $(1 - \frac{1}{I}) \sum_i x_i^2$ is positive, minimize J comes down to maximize the second term in the sum. Then the estimator of the Woody Improved method is:

$$\widehat{d_i} = \arg\max_{d_i} \left[\sum_n \sum_i^I \sum_{k>i}^I x_{k,-d_k}(n) x_{i,-d_i}(n) \right] \tag{12}$$

The solution of (12) is obtained using an iterative scheme described by the flow chart presented in figure 1. The aim is to find $(\widehat{\mathbf{s}}, \widehat{\mathbf{d}})$ in criterion (9) which will be unique if a side condition, such as $\sum_i \widehat{d_i}$ equal to a constant. Arbitrarily, we fix a delays average equal to 0.

Then, using an iterative algorithm, the d_k are fixed for $k \neq i$, we compute the optimum of the criterion J with respect to variable i: we replace d_i by $\widehat{d_i}$ and we reiterate.

Here, we present the first steps of this algorithm.

1^{st} step : $m = 1$
Aim : estimation of the delay $\widehat{d_1}$
Hypothesis: $\widehat{d_2} = \widehat{d_3} = ... = \widehat{d_I} = 0$ Computation of the criterion (12) :

$$J = \sum_n \left[x_{1,-d_1}(n) \sum_{k=2}^{I} x_k(n) + x_2(n) \sum_{k=3}^{I} x_k(n) + ... \right]$$

We observe that only the first term is function of d_1. So, maximize this criterion amounts to make the cross correlation between two signals :

- x_1 delayed of d_1
- the mean of all the x_i for $i \neq 1$

The maximum of this cross correlation gives $\widehat{d_1}$.

2^{nd} step : $m = 2$
Aim : estimation of the delay $\widehat{d_2}$
Hypothesis : $d_1 \rightarrow \widehat{d_1}$ and $\widehat{d_3} = \widehat{d_4} = \ldots = \widehat{d_i} = 0$ Computation of the criterion (12) :

$$\begin{aligned} J &= \sum_n \left[x_{1,-\widehat{d}_1}(n) \left(x_{2,-d_2}(n) + \sum_{k=3}^{I} x_k(n) \right) + x_{2,-d_2}(n) \sum_{k=3}^{I} x_k(n) + x_{3,-d3}(n) \sum_{k=4}^{I} x_k(n) + \ldots \right] \\ &= \sum_n \left[x_{2,-d_2}(n) \left(x_{1,-\widehat{d}_1}(n) + \sum_{k=3}^{I} x_k(n) \right) \right] + \sum_n \left[x_{1,-\widehat{d}_1}(n) \sum_{k=3}^{I} x_k(n) + x_{3,-d3}(n) \sum_{k=4}^{I} x_k(n) + \ldots \right] \end{aligned}$$

As previously, only the term of the criterion is function of the delay d_2 and maximize this criterion amounts to make the cross correlation between two signals :

- x_2 delayed of d_2
- the mean of all the x_i for $i \neq 2$ with x_1 corrected by $\widehat{d_1}$

The maximum of this cross correlation gives $\widehat{d_2}$.

3^{rd} step : $m = 3$ ...

At the end of all steps, we assure the unicity of our estimation thanks to the hypothesis that the average of the delays $\widehat{d_i}$ at each iteration is constant. Arbitrarily, we choose that the average of the delays during the iterations must be equal to the average of the delays estimated during the first iteration.

In conclusion, for each i, in order to determine the delay $\widehat{d_i}$, we maximize the correlation function between the observation x_i and the average of all the x_k for $k \neq i$ with the trials x_k for $k < i$ realigned, corrected by the $\widehat{d_k}$ for $k < i$ already estimated. As for the Woody's method, several iterations are necessary to converge to the optimal solution.

The difference between this Woody Improved method and the Woody's one is that the latter uses the correlation between the trial x_i and the template, the mean of **all** trials, whereas here, the correlation is not biased by the presence of the considered x_i in the template. That is why the Woody method [3] is suboptimal. Also, our Woody Improved method converges faster because the trial x_i is corrected by $\widehat{d_i}$ and the template is updated before the next step. In conclusion, the Woody's method of 1967 is running but it is suboptimal.

4 Woody's method Improvement-Generalization of the model

We remember that our aim is to determine the PR interval on effort ECG but the determination of the P wave is very particulary difficult, especially during the effort and at the beginning of the recovery where the T wave is superposed the P wave. Our idea is then to take into account the presence of the T wave in our model and then it will be easier to estimate the PR interval.

We can describe a model in which $x_i(n)$ represents all the observations (for all n) of the considered i^{th} PR interval. Each observation contains $s_{d_i}(n)$, still considered unknown, defined as the reference wave delayed by d_i as $s_{d_i}(n) = s(n - d_i)$, plus an observation's noise $e_i(n)$, a Gaussian white noise with null mean and a variance of σ^2.

Charles D. Woody [3], presented as we have seen in the first part of this report, a system which is based on iterative correlation-averaging techniques. Later, Pham et al. studied the estimation of variable latencies of noisy signals [5]. Jaśkowski and Verleger [4] refereed to a more general model in which the amplitude variability is also allowed :

$$x_i(n) = \alpha_i.s_{d_i}(n) + e_i(n)$$

However, these two studies, [5, 4], are not really fair regards the optimality of the method since they include frequency a priori in their approach.
As in exercise tests T-P fusion occurs during higher heart rates, we can consider in order to generalized more over that the T wave is represented by a function $f(n;\boldsymbol{\theta}_i)$. We assume that the T wave should be described by a regular and smooth function, i.e. a l^{th} order polynomial function characterized by its coefficients in the vector $\boldsymbol{\theta}_i$.

Finally, our model is expressed like :

$$x_i(n) = \alpha_i.s_{d_i}(n) + \alpha_i.f_{d_i}(n;\boldsymbol{\theta}_i) + e_i(n) \tag{13}$$

where i, $i = 1..I$, is the number of realizations, and the variable d_i is the i^{th} PR interval to be estimated up to an unknown constant.

As previously, it is obvious that if we do not impose constraints on the estimated delays, we will estimate the signal $\mathbf{s}$ with a time-lag. That is why, it is necessary to impose that the average of the estimated delays equal a constant. For example, we choose the average of the delays identical to the average of the estimated delays at the end of the first iteration.

In order to estimate the PR intervals, we use the MLE (Maximum Likelihood Estimation). The idea behind maximum likelihood parameter estimation is to determine the parameters that maximize the probability, the likelihood, of the sample data.
The noise $e_i(n)$ is an iid Gaussian noise with zero mean and a variance of σ^2. So, for one observation, n fixed, we consider the likelihood function:

$$p(x_i(n); s(n), d_i, \boldsymbol{\theta}_i, \alpha_i) = \Gamma.\exp\left(-\frac{1}{2\sigma^2}.(x_i(n) - \alpha_i.f_{d_i}(n;\boldsymbol{\theta}_i) - \alpha_i.s_{d_i}(n))^2\right)$$

For all the samples, i.e. all the n, if the noise is white then all the observations are independent and then:

$$p(\mathbf{x}_i) = \prod_n p(x_i(n))$$

And,

$$p(\mathbf{x}_i; \mathbf{s}, d_i, \boldsymbol{\theta}_i, \alpha_i) = \Upsilon. \exp\left(-\frac{1}{2\sigma^2}.\sum_n (x_i(n) - \alpha_i.f_{d_i}(n; \boldsymbol{\theta}_i)) - \alpha_i.s_{d_i}(n))^2\right)$$

Then, for all records, i.e. all i, the pdf of the processes $\mathbf{x}_i$'s, given the delay d_i's, the signal vector $\mathbf{s}$, the coefficients $\boldsymbol{\theta}_i$ and the parameter of the amplitude jitter α, is:

$$p(\mathbf{X}; \mathbf{s}, \mathbf{d}, \boldsymbol{\theta}_i, \alpha_i) = \Psi. \exp\left(-\frac{1}{2\sigma^2}.\sum_i \sum_n (x_i(n) - \alpha_i.f_{d_i}(n; \boldsymbol{\theta}_i) - \alpha_i.s_{d_i}(n))^2\right)$$

where $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_I]$ and $\mathbf{d} = [d_1 d_2 \ldots, d_I]^T$.

The objective is to estimate the d_i's for all i, in other words to maximise $p(\mathbf{X}; \mathbf{s}, \mathbf{d}, \boldsymbol{\theta}_i, \alpha_i)$. So according the MLE, the criterion J to be minimized is:

$$J = \sum_i \parallel \mathbf{x}_i - \alpha_i.s_{d_i} - \alpha_i.f_{d_i}(\boldsymbol{\theta}_i) \parallel^2 \quad (14)$$

To solve this kind of problem, first we make a change of variables:

$$\mathbf{y}_i = \mathbf{x}_i - \alpha_i.f_{d_i}(\boldsymbol{\theta}_i) \quad (15)$$

So we consider the criterion:

$$J = \sum_i \parallel \mathbf{y}_i - \alpha_i.s_{d_i} \parallel^2 \quad (16)$$

The noise sweeps $e_i(n)$ are modeled as trials of a common zero mean stationary Gaussian process and we assume that e_j and e_k are independent for $j \neq k$. Since the signal $s(n)$ is unknown, we can estimate it by minimization of the criterion (16) with respect to $s(n)$:

$$\widehat{\mathbf{s}} = \frac{1}{I}\sum_k \frac{1}{\alpha_k}\mathbf{y}_{k,-d_k}$$

Substituting this estimation for $\mathbf{s}$ in the equation (16), we obtain:

$$\begin{aligned} J &= \sum_i \parallel \mathbf{y}_i - \alpha_i.\widehat{\mathbf{s}}_{d_i} \parallel^2 \\ &= \sum_i \parallel \mathbf{y}_i - \frac{\alpha_i}{I}\sum_k \frac{1}{\alpha_k}\mathbf{y}_{k,d_i-d_k} \parallel^2 \\ &= \sum_i \parallel \mathbf{y}_i - \frac{\alpha_i}{I}\sum_k \frac{1}{\alpha_k}(\mathbf{x}_{k,d_i-d_k} - \alpha_k.f_{d_i}(\boldsymbol{\theta}_k)) \parallel^2 \end{aligned}$$

Using the equation (15), we obtain:

$$\begin{aligned} J &= \sum_i \parallel \mathbf{x}_i - \alpha_i.f_{d_i}(\boldsymbol{\theta}_i) - \frac{\alpha_i}{I}\sum_k \frac{1}{\alpha_k}.(\mathbf{x}_{k,d_i-d_k} - \alpha_k.f_{d_i}(\boldsymbol{\theta}_k)) \parallel^2 \\ &= \sum_i \parallel \mathbf{x}_i - \frac{\alpha_i}{I}\sum_k \frac{1}{\alpha_k}.\mathbf{x}_{k,d_i-d_k} - \alpha_i.f_{d_i}(\boldsymbol{\theta}_i) + \frac{\alpha_i}{I}\sum_k f_{d_i}(\boldsymbol{\theta}_k) \parallel^2 \end{aligned}$$

As previously mentioned, we can consider that the T wave is represented by a function $f(n;\boldsymbol{\theta}_k)$ which is, for example, a l^{nd} order polynomial characterized by its coefficients in the vector $\boldsymbol{\theta}_k = [\theta_k(0), \theta_k(1), \ldots, \theta_k(L)]^T$:

$$f_{d_i}(n;\boldsymbol{\theta}_k)] = \sum_{l=0}^{L} \theta_k[l].(n-d_i)^l$$

Then, we have:

$$\begin{aligned} \frac{\alpha_i}{I}\sum_k f_{d_i}(n;\boldsymbol{\theta}_k) &= \frac{\alpha_i}{I}\sum_{k=0}^{I}\sum_{l=0}^{L}\theta_k[l].(n-d_i)^l && (17)\\ &= \frac{\alpha_i}{I}\sum_{k=0}^{I}\left[\theta_k[0].1+\theta_k[1].(n-d_i)^1+\theta_k[2].(n-d_i)^2+....\right] && (18)\end{aligned}$$

This is corresponding to the average of the functions $f(n;\boldsymbol{\theta}_k)$, which is delayed of d_i.

Also, in order to assert that the model is identifiable, we add a new non restrictive constraint that is the average of the functions $f(n;\boldsymbol{\theta}_k)$ is zero.

And finally, the criterion to be minimized becomes:

$$J = \sum_i \parallel \mathbf{x}_i - \alpha_i.f_{d_i}(n;\boldsymbol{\theta}_i) - \frac{\alpha_i}{I}\sum_{k=1}^{I}\frac{1}{\alpha_k}\mathbf{x}_{k,d_i-d_k} \parallel^2 \qquad (19)$$

When we develop this criterion (20), we obtain :

$$\begin{aligned} J &= \parallel \mathbf{x}_1 - \alpha_1.f_{d_1}(\boldsymbol{\theta}_1) - \frac{\alpha_1.}{I}\sum_{k=1}^{I}\frac{1}{\alpha_k}\mathbf{x}_{k,d_1-d_k} \parallel^2 \\ &+ \parallel \mathbf{x}_2 - \alpha_2.f_{d_2}(\boldsymbol{\theta}_2) - \frac{\alpha_2.}{I}\sum_{k=1}^{I}\frac{1}{\alpha_k}\mathbf{x}_{k,d_2-d_k} \parallel^2 \\ &+ \parallel \mathbf{x}_3 - \alpha_3.f_{d3}(\boldsymbol{\theta}_3) - \frac{\alpha_3.}{I}\sum_{k=1}^{I}\frac{1}{\alpha_k}\mathbf{x}_{k,d_3-d_k} \parallel^2 + \ldots \end{aligned}$$

$$\begin{aligned} J &= \parallel \mathbf{x}_1 - \alpha_1.f_{d_1}(\boldsymbol{\theta}_1) - \frac{\alpha_1}{I}\left(\frac{1}{\alpha_1}\mathbf{x}_{1,0} + \frac{1}{\alpha_2}\mathbf{x}_{2,d_1-d_2} + \frac{1}{\alpha_3}\mathbf{x}_{3,d_1-d_3} + ...\right) \parallel^2 \\ &+ \parallel \mathbf{x}_2 - \alpha_2.f_{d_2}(\boldsymbol{\theta}_2) - \frac{\alpha_2}{I}\left(\frac{1}{\alpha_1}\mathbf{x}_{1,d_2-d_1} + \frac{1}{\alpha_2}\mathbf{x}_{2,0} + \frac{1}{\alpha_3}\mathbf{x}_{3,d_2-d_3} + ...\right) \parallel^2 \\ &+ \parallel \mathbf{x}_3 - \alpha_3.f_{d3}(\boldsymbol{\theta}_3) - \frac{\alpha_3}{I}\left(\frac{1}{\alpha_1}\mathbf{x}_{1,d_3-d_1} + \frac{1}{\alpha_2}\mathbf{x}_{2,d_3-d_2} + \frac{1}{\alpha_3}\mathbf{x}_{3,0} + ...\right) \parallel^2 + \ldots \end{aligned}$$

We can observe, for example for d_1, that it appears especially in the first term and is present only once in the following terms. Then, we can make the approximation that in the following terms the d_1's influence is negligible; only the 1^{st} term in the criterion is then considered for the first step.

Then, thanks to this approximation, for the i^{th} step, the criterion to be minimized is:

$$J = \| \mathbf{x}_i - \alpha_i . f_{d_i}(\boldsymbol{\theta}_i) - \frac{\alpha_i}{I} \sum_{k=1}^{I} \frac{1}{\alpha_k} \mathbf{x}_{k,d_i - d_k} \|^2 \tag{20}$$

Then, the optimization can use an iterative algorithm. In the first time, we define a reference wave, a template, which is the average of the observations which do not contain T wave considering that all the α_i equal 1. Thanks to the MLE, for the first step (i.e. $i = 1$), we estimate the coefficient $\widehat{\alpha}_1$, the coefficient $\widehat{\theta}_1$ of the polynomial function and the delay $\widehat{d_1}$. We adjust the first observation by substraction of the polynomial function and realign it using the estimated delay. A new template is computed in order to be used in the next steps. If necessary, the process can be iterated depending on the convergence of the algorithm. Thanks to this model, we take into account the overlapping T wave. Then, the PR intervals are produced up to an unknown constant by the estimated delays $\widehat{d}_i$.

5 Conclusion

In this report, it has been demonstrated that the Woody's method [3], is suboptimal and that our approach is faster. The techniques to estimate the PR intervals were based only on the detection of the maximum of cross correlation function [1, 2]. In this study, it has been presented a new method based on an iterative Maximum-Likelihood approach which generalizes the well known Woody's method. Thanks to this new technique, the estimation of PR interval on effort ECG takes into account the presence of the T wave which overlaps the P one at high heart rate.

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