Interference Cancelation and Information Processing in Communication Systems

Geodesic Learning

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Introduction

Manifolds?

- "Curved spaces"
- Example: sphere (dim $= 2$) in $\mathbb{R}^3$
- In general: $\mathcal{M} \subset \mathbb{R}^D$, with $d := \dim \mathcal{M} \leq D$
- Several notions of calculus can be inserted: derivative, gradient, integral, volume, etc.

Geodesics?

- Shortest path conecting two point of $\mathcal{M}$
- Analogous to the lines in $\mathbb{R}^D$

Problema

- We do not know the manifold $\mathcal{M}$
- But we do know a set of samples that we assume that belong to $\mathcal{M}$
- We want to estimate the geodesic between any two given points

Proposal

- Fixed points: $x_0 = a$, $x_{N+1} = b$
- Chose $\{x_1, \ldots, x_N\}$ that minimizes:

$$J(\{x_i\}) = \underbrace{\sum_{i=1}^{N} d(\mathcal{M}, x_i)^2}_{(a)} + \lambda \underbrace{\sum_{i=1}^{N+1} \|x_i - x_{i-1}\|^2}_{(b)}.$$

- (a): make $\{x_i\}$ "gets closer" to the manifold $\mathcal{M}$
- (b): length of the path connecting the points $\{x_i\}_{i=0}^{N+1}$

Proposal (cont.)

- samples: $\{y_1, \ldots, y_M\}$
- $d(\mathcal{M}, x_i)$ is approximated by:

$$\frac{1}{K} \sum_{y_j \in \mathrm{nn}_K(x_i)} \|y_j - x_i\|^2$$

where $\mathrm{nn}_K(x_i)$ is the set of the K points in $\{y_i\}$ which are the closest ones to x_i

- $\sum_{i=1}^{N+1} \|x_i - x_{i-1}\|^2$ is "smoothed" by

$$\sum_{i,j=0}^{N+1} w_{ij} \|x_i - x_j\|^2,$$

where $w_{ij} = \frac{1}{\#\mathrm{an}_L(i)}$, e $\mathrm{an}_L(i)$ is the set of the L adjacent pointsde x_i in $\{x_i\}$

Algorithm

Table: Geodesic learning algorithm.

Initialize:

$$a = x_0, x_1, \ldots, x_N, x_{N+1} = b$$

Repeat:

For $i = 1, \ldots, N$:

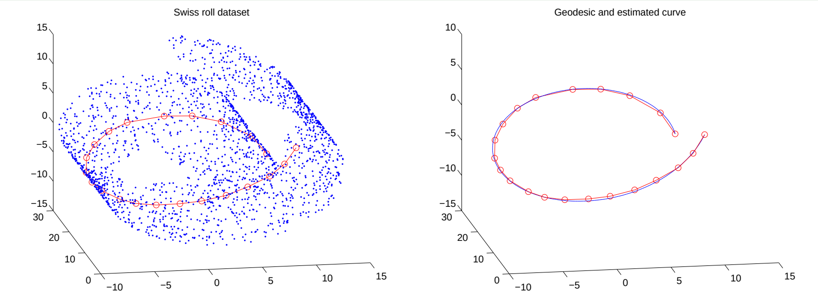
$$x_i \leftarrow \frac{1}{\#\mathrm{an}_L(i)} \left(\sum_{x_j \in \mathrm{an}_L(i)} x_j \right)$$

Find $\mathrm{nn}_K(x_i)$

$$x_i \leftarrow \frac{1}{K} \left(\sum_{y_j \in \mathrm{nn}_K(x_i)} y_j \right)$$

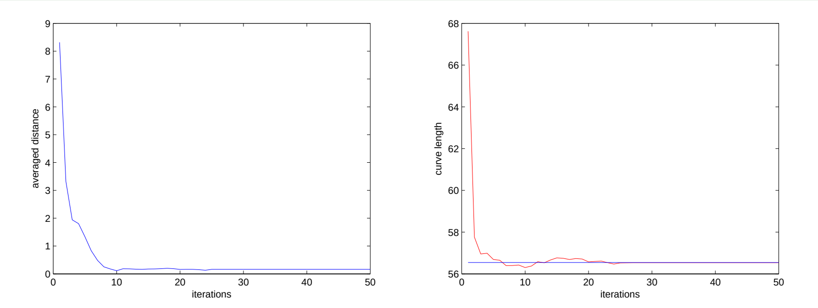
Until $\{x_i\}$ converges

Results



Swiss roll dataset
Geodesic and estimated curve
15
10
5
0
−5
−10
−15
30
20
10
0
−10
−5
0
5
10
15

Results (cont.)



Results (cont.)