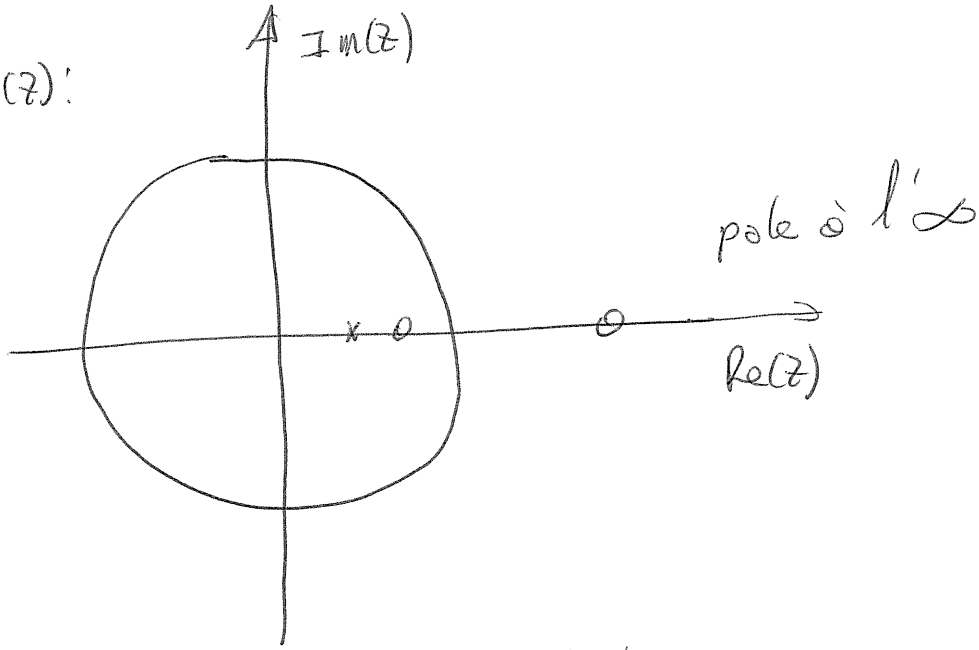
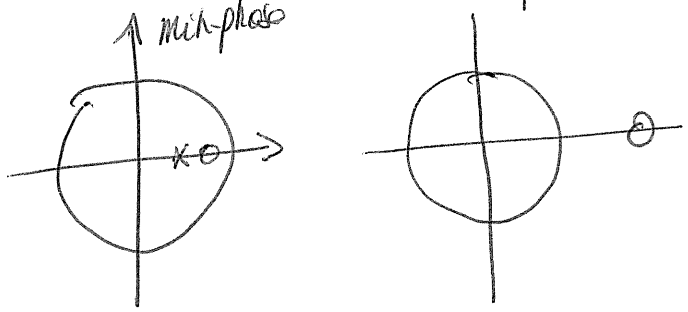
TD2-3-Sol

H(z):

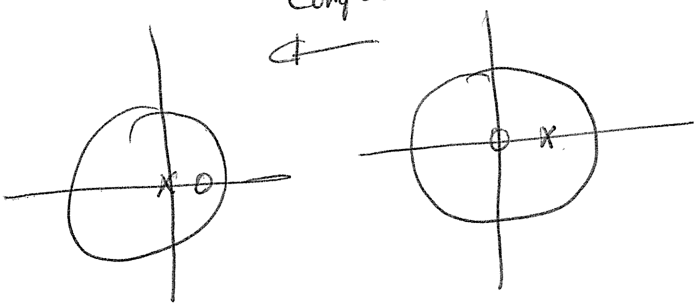


↙ min-phase ↘ passe tout ? pôle à l'∞

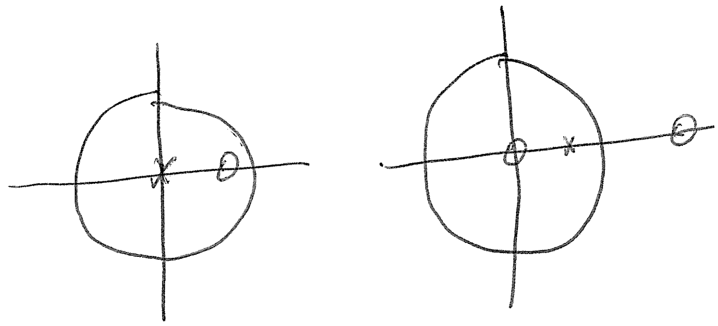


↓ Vrai passe-tout → ajouter
un pôle à $z = \frac{1}{2}$
un zéro à $z = 0$

compenser ←



Résultat ↓



$$\Rightarrow H_1(z) = \frac{z - 3/4}{z} = 1 - \frac{3}{4} z^{-1}$$

$$H_{ap}(z) = \frac{z(z-2)}{z - 1/2} = \frac{1 - 2z^{-1}}{(1 - \frac{1}{2} z^{-1}) z^{-1}}$$

TD 2.4 - Sol

④ ⓐ du principe de décomposition de $H(z)$ en (min-phase + passe-tout)

$$\Rightarrow H(z) = H_{min}(z) \cdot \frac{z^{-1} - z_k^*}{1 - z_k z^{-1}}$$

+ Rajouter un pôle en z_k pour compenser le zéro en z_k

+ remplacer le zéro en z_k par un zéro en $\frac{1}{z_k^*}$

$$= Q(z) \; (z^{-1} - z_k^*)$$

ⓑ

$$H_{min}(z) \left(\frac{1}{1 - z_k z^{-1}} \right) = Q(z)$$

$$\Rightarrow H_{min}(z) = Q(z) - z_k z^{-1} Q(z)$$

$$\Rightarrow h_{min}[n] = q[n] - z_k \, q[n-1]$$

$$H(z) = Q(z) \, (z^{-1} - z_k^*)$$

$$\Rightarrow h[n] = q[n-1] - z_k^* \, q[n]$$

ⓒ

$$\mathcal{E} = \sum |h_{min}[n]|^2 - \sum |h[n]|^2$$

$$=$$

$$= (1 - |z_k|^2) \, |q[m]|^2$$

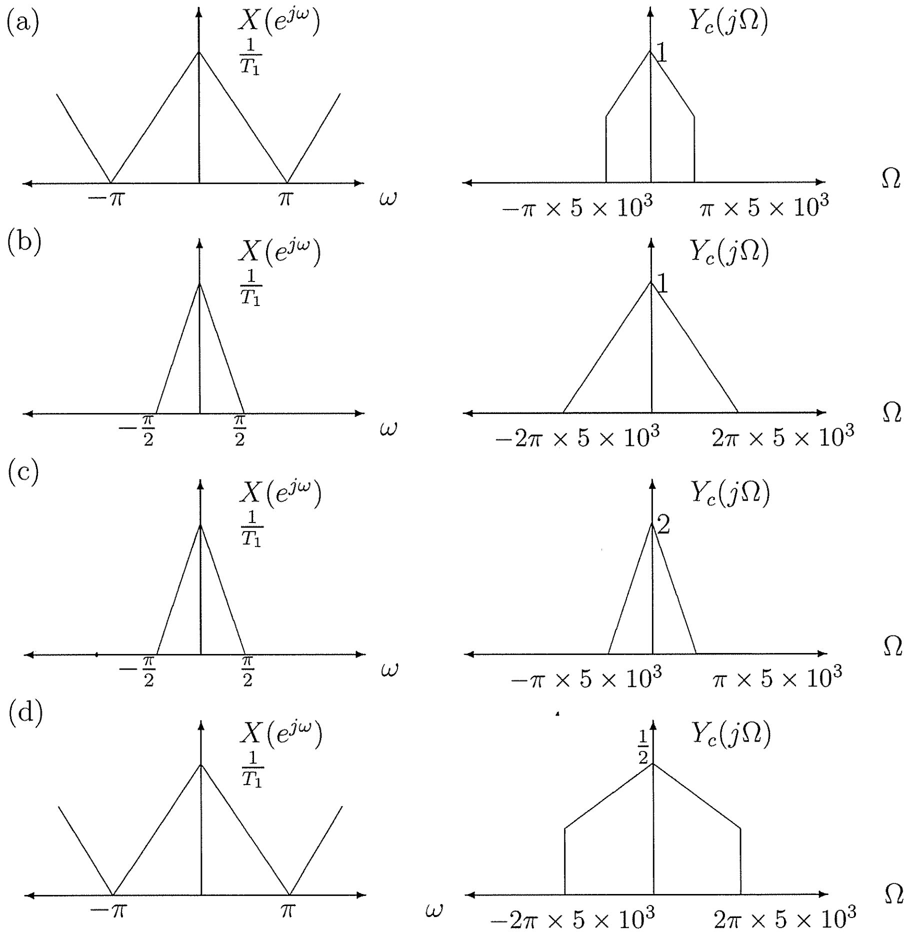
ⓓ

$$|z_k| < 1 \rightarrow (1 - |z_k|^2) \, |q[m]|^2 > 0$$

$$\Rightarrow \sum |h_{min}[n]|^2 > \sum |h[n]|^2$$

TD2_5_Sol

Problem 2.6



$$h[n] = q[n-1] - z_k^* q[n]$$

$$h_{min}[n] = q[n] - z_k q[n-1]$$

(c) Using our answer from part (b):

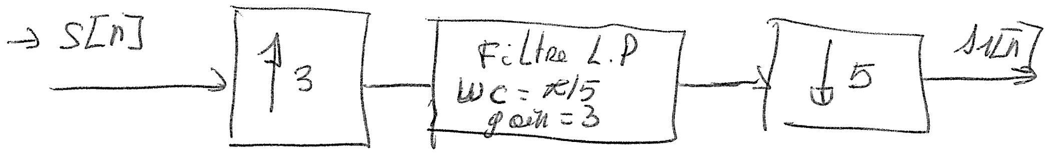
$$\begin{aligned}
\varepsilon &= \sum_{n=0}^{m} |h_{min}[n]|^2 - \sum_{n=0}^{m} |h[n]|^2 \\
&= \sum_{n=0}^{m} |q[n]|^2 - z_k q[n-1] q^*[n] - z_k^* q^*[n-1] q[n] + |z_k|^2 |q[n-1]|^2 \\
&\quad - \sum_{n=0}^{m} |q[n-1]|^2 - z_k q[n-1] q^*[n] - z_k^* q^*[n-1] q[n] + |z_k|^2 |q[n]|^2 \\
&= \sum_{n=0}^{m} (1 - |z_k|^2) |q[n]|^2 - \sum_{n=0}^{m} (1 - |z_k|^2) |q[n-1]|^2 \\
&= (1 - |z_k|^2) |q[m]|^2
\end{aligned}$$

(d) Since $|z_k| < 1$, $(1 - |z_k|^2)$ is positive and therefore $(1 - |z_k|^2)|q[m]|^2 > 0$
Then $\sum_{n=0}^{m} |h_{min}[n]|^2 - \sum_{n=0}^{m} |h[n]|^2 > 0$ thus $\sum_{n=0}^{m} |h_{min}[n]|^2 > \sum_{n=0}^{m} |h[n]|^2$

(6)

note → il n'y a pas eu de repli de spectre car le signal a été filtré avant échantillonage !

⇒ changement de fréquence d'échantillonage d'un rapport $\frac{6}{10} = \frac{3}{5}$

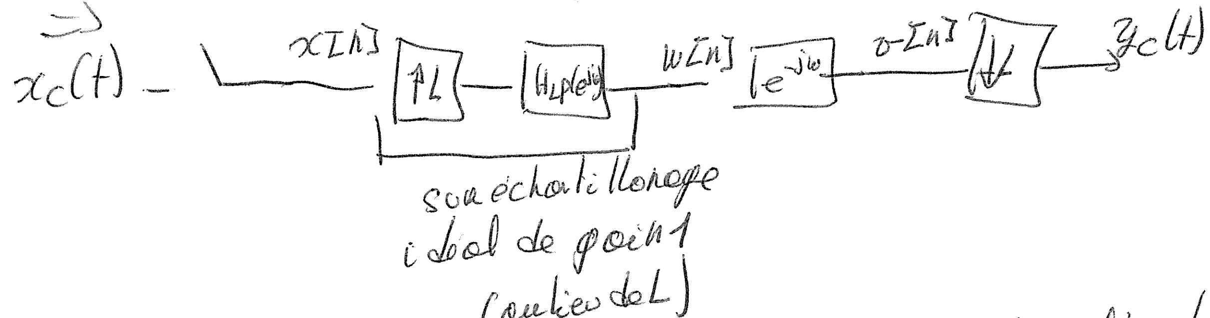


(7)

note → on peut écrire $H(e^{j\omega})$ sous la forme d'un filtre passe-bas idéal + 1 délai

$$H(e^{j\omega}) = H_{LP}(e^{j\omega})e^{-j\omega}$$

$$H_{LP}(e^{j\omega}) = \begin{cases} 1 & |\omega| < \pi/L \\ 0 & \frac{\pi}{L} < |\omega| \leq \pi \end{cases}$$



$x[n] = x_c(nT)$ pas de repli de spectre (hypothèse)

$$w[n] = \frac{1}{L} x_c(nT/L)$$

$$v[n] = \frac{1}{L} x_c((n-1)T/L)$$

$$y[n] = v[nL] = \frac{1}{L} x_c(nT - T/L)$$

$$\Rightarrow y_c(t) = \frac{1}{L} x_c(t - T/L)$$

Problem 2.9

(a) Let's rewrite System A as the cascade of two systems: an S/I (sample to impulse) converter followed by a CT filter. The S/I converter turns the DT signal $x_d[n]$ into a CT impulse train. If we allow the output of the S/I converter to be $x_s(t)$, then we have

$$x_s(t) = \sum_{k=-\infty}^{\infty} x_d[k]\delta(t - kT_1)$$

Then, the output $y_c(t)$ of the CT filter is the convolution of $x_s(t)$ and $h_1(t)$, or

$$y_c(t) = x_s(t) * h_1(t).$$

We see that by combining the above two equations, we get the equation that describes the behavior of System A.

$x_c(t)$ is bandlimited to $\Omega_c = \pi \cdot 10^{-3}$ rad/sec. Thus, we know that we can guarantee the equality of $x_c(t)$ and $y_c(t)$ when T is sufficiently small (*i.e.* no aliasing from the first C/D converter) and System A is an ideal D/C converter with the same sampling period.

We have no aliasing when $\Omega_c T < \pi$, or when $T < 1000$. System A is an ideal D/C converter when $h_1(t)$ is an appropriate sinc function.

Thus, the following conditions work:

$$\begin{aligned} T &= 500. \\ T_1 &= 500. \\ h_1(t) &= \frac{\sin(\pi t/T)}{\pi t/T} \end{aligned}$$

(b) As we saw in the solution to part (a), our choices are not unique. We can choose any T such that $T < 1000$. However, we see that we need $T_1 = T$. We also have a choice regarding $h_1(t)$. Since $X_d(e^{j\omega})$, or the Fourier transform of $x_d[n]$, is zero for

$$\frac{T\pi}{1000} < |\omega| < \pi,$$

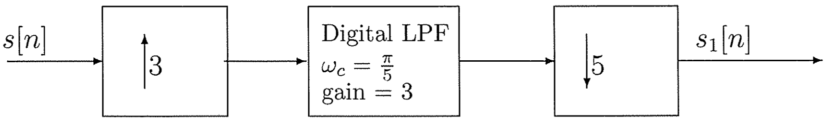
$X_s(j\Omega)$, or the Fourier transform of $x_s(t)$, is zero for

$$\frac{\pi T}{1000T_1} = \frac{\pi}{1000} < |\Omega| < \frac{\pi}{T_1}.$$

Thus, $H_1(j\Omega)$, or the Fourier transform of $h_1(t)$, can be anything in that frequency range (and, by extension, any "copy" of this section of the frequency spectrum). If it is a constant of T for $|\Omega| < \frac{\pi}{1000}$ and zero for $|\Omega| > \frac{\pi}{T}$, then we have $y_c(t) = x_c(t)$.

Problem 2.7

In both systems, the speech was filtered first so that the subsequent sampling results in no aliasing. Therefore, going from $s[n]$ to $s_1[n]$ basically requires changing the sampling rate by a factor of $3kHz/5kHz = 3/5$. This is done with the following system:

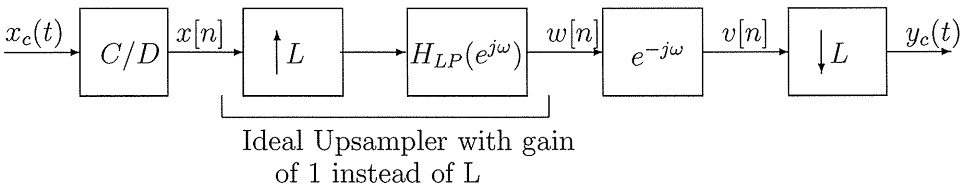


Problem 2.8

Split $H(e^{j\omega})$ into a lowpass and a delay.

$$H(e^{j\omega}) = H_{LP}(e^{j\omega})e^{-j\omega}$$

$$H_{LP}(e^{j\omega}) = \begin{cases} 1, & |\omega| < \frac{\pi}{L} \\ 0, & \frac{\pi}{L} < |\omega| \leq \pi \end{cases}$$



Then we analyze the system as follows:

$$\begin{aligned} x[n] &= x_c(nT) \quad \text{no aliasing assumed} \\ w[n] &= \frac{1}{L}x_c(n\frac{T}{L}) \quad \text{rate change} \\ v[n] &= w[n-1] = \frac{1}{L}x_c\left(n\frac{T}{L} - \frac{T}{L}\right), \quad \text{delay at higher rate} \\ y[n] &= v[nL] = \frac{1}{L}x_c\left(nT - \frac{T}{L}\right) \end{aligned}$$

(c) Since we are interested only in the operations between $x_d[n]$ and $y_d[n]$, we need not worry about aliasing from the first C/D converter in the whole system destroying our hopes for achieving consistent resampling. Thus, there are no absolute restrictions on T and T_1 like we had in parts (a) and (b); they may, however, be related to each other.

In other words, what is going on between $x_d[n]$ and $y_d[n]$? System A is taking each sample of $x_d[n]$ and replacing it with $h_1(t)$ delayed by nT_1 and scaled by $x_d[n]$ at that point. Then, the C/D converter resamples the result.

Let's consider what happens with $x_d[n] = \delta[n - n_0]$ for an integer n_0. Then, $y_c(t) = h_1(t - n_0T_1)$. The sampled version of $y_c(t)$ is

$$\begin{aligned} y_d[n] &= y_c(nT) \\ &= h_1(nT - n_0T_1). \end{aligned}$$

A condition for consistent resampling is thus

$$h_1(nT - n_0T_1) = \delta[n - n_0].$$

Because of the linearity of the mapping from $x_d[n]$ to $y_d[n]$, this is actually the only condition that must be checked. To simplify the condition further, we have

$$\text{evaluating at } n = n_0\text{:} \quad 1 = h_1(n(T - T_1))$$

$$\text{evaluating at } n \neq n_0\text{:} \quad 0 = h_1(nT - n_0T_1))$$

The case of practical significance is to have $T = T_1$, in which case we find that $h_1(t)$ should satisfying an *interpolating condition*: $h_1(0) = 1$ and $h_1(t) = 0$ for all multiples of T. (It doesn't matter what $h_1(t)$ is for other values of t.)