Training Sequence vs. Cyclic Prefix
A new look on Single Carrier Communication

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Abstract—
Frequency domain equalization has gained a lot of attention in the last decade, mainly pushed by the OFDM (Orthogonal Frequency Division Multiplexing) communication scheme. It has been adopted, in various flavours, for, among others, Wireless Lans and xDSL. The technique is indeed leading to low complexity implementation, at the cost of a Cyclic Prefix, used to avoid Inter Block Interference (IBI). The idea has been adapted to single carrier communication, almost "as is", yielding the same computational advantages and avoiding the high crest factor of OFDM. We will name this scheme CP-SC (Cyclic Prefix - Single Carrier).

We take a new look at this single carrier scheme, and, with a slight modification, show that the cyclic prefix can be implemented as a training sequence, and hence play two important roles : avoid IBI and help in synchronisation and channel estimation. The latter topic is of utter importance in fast fading situations (e.g. mobile). This new Training Aided Frequency Domain Equalized Single Carrier (we will name it TASC) scheme offers these advantages at the expense of only a small fraction of a dB (in terms of E_b/N_o).

All these arguments make TASC an interesting candidate in situations where multipath and fast fading are present, while in other situations it has hardly no drawback compared to CP-SC.

I. Introduction

One of the major advantages of OFDM is it's capability of using low complexity frequency domain equalization (one complex multiplication per data symbol). This is done at the expense of a "Cyclic Prefix", which is able to cope with time dispersive channels, as long as the length of their impulse response is shorter than the cyclic prefix.

The same approach has been adapted to single carrier communications [1], using also a cyclic prefix to absorb the channel. Advantages of SC vs OFDM are that, for SC, the effect of a notch in the frequency response of the channel will not be catastrophic (as it is in uncoded OFDM) and that it's crest factor is low (Constant Modulus is even achievable by using MSK or Constant Modulus Trellis Coded Modulation [2]). On the other hand, it uses one FFT more at the receiver side.

The overhead induced by the cyclic prefix can be used in a more efficient way if its content is better known. Indeed, the CP is hardly useful for channel estimation, and is sometimes insufficient for proper synchronization.

We propose to modify this scheme by using a Training Sequence instead of a Cyclic Prefix. This training sequence will play the same role as the cyclic prefix, but at the same time will serve as a pilot symbol for synchronisation (carrier synchronisation and time offset estimation) and channel estimation. The big advantage of this scheme is that pilot symbols are not needed anymore to track synchronisation and channel variations. Furthermore, there is seldom any net data rate penalty, some gain (a fraction of a dB) can even be achieved in practical situations. More important, the modification that we introduce will yield exactly the same performance as CP-SC, when using the same FFT-size and the same CP-TS size.

We will further explain how to modify the framing of the data and data processing to see a training sequence as a cyclic prefix and how to use it for synchronisation and channel estimation. These arguments will show that the proposed TASC is a good candidate for frequency domain equalized single carrier communication schemes.

To assess our claims, we propose two synchronisation algorithms wich can be seen as extensions of existing algorithms for OFDM. The first one is a direct ad-hoc extension of a Maximum Likelihood frequency offset estimation based on the Cyclic Prefix (and hence non data-aided). The second one proposes a Maximum Likelihood joint estimation of frequency offset and frame timing. It can be seen as a generalisation of [3].

II. Cyclic Prefix as a Training Sequence

A. Frame structure

The single carrier counterpart of OFDM consists in sending data frames where the start and end of the frames contain the same data, viz. the Cyclic Prefix. By doing this, the convolution of the data with the channel becomes a cyclic convolution, and frequency domain equalization is possible with only one multiplication per data symbol (or one tap per subcarrier in the OFDM terminology).

The basic idea of TASC is to transform the Cyclic Prefix in a Training Sequence(TS). This TS should be viewed as a Cyclic Prefix by the FFT device and be always the same and hence, not include useful data. The modification in the framing is depicted in figure 1, where the original CP-SC scheme is drawn for comparison. The basic difference is that, instead of having to throw away the Cyclic Prefix, we always process the Training Sequence, hence, there is no gap anymore between two FFT's. Note that in practical situations, to allow small timing synchronisation errors, the FFT is taken in the middle of the TS. Moreover, as the TS is always persent on both edges of the data block, the transformation from linear convolution to cyclic

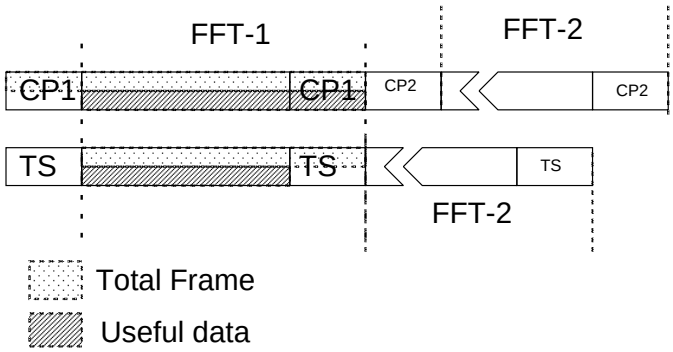

Fig. 1. Single Carrier with Cyclic Prefix and Training Sequence

convolution is kept, and the performance of the original CP-SC is also kept.

Noting respectively N_c, N_p and N_l as the total number of carriers, the length of the CP and the number of pilot carriers, the ratio of the useful data vs the total data is $\frac{N_c-N_p}{N_c}$ for TASC and $\frac{N_c-N_l}{N_c+N_p}$ for CP-SC, leading to a gain of $\frac{N_c^2-N_p^2}{N_c^2-N_cN_l}$ for the TASC scheme. Obviously, if $N_cN_l > N_p^2$, there is a gain in net data rate (and loss if not). As an example, for $N_l = 4, N_p = 16$ and $N_c = 64$, there is no modification in the data rate, for $N_l = 4, N_p = 16$ and $N_c = 256$, the gain is .05 dB and for $N_l = 8, N_p = 16$ and $N_c = 256$, the gain is .12 dB.

The global Transmit/Receive chain becomes as in figure 2.

B. Some general comments

Before diving into some sample algorithms taking advantage of the short training sequence that is provided, let's see what we can expect a priori.

The TS sequence does not contain data. Hence, it can be optimized to get appropriate properties (e.g. autocorrelation) and it's symbols could even be chosen from a separate alphabet. This avoids the accidental presence of the TS sequence in the useful data, which makes the TASC akin to PSAM (Pilot-symbol-assisted modulation). A scheme inspired by [4] could for example be adopted for Frame Synchronisation.

On a synchronisation point of view, the TASC acquisition will essentially be the same as for the CP-SC : as the TS sequence is rather short, we can probably not rely on it for this task. For the tracking algorithms, Data-Aided (DA) algorithms are known to perform better than their Non Data-Aided counterparts (NDA) (see [5], chapter 6). They avoid Decision Directed algorithms and alleviate the problem of feeding the decisions back, which would mean a delay of one frame.

For the channel estimation, the concept of Semi-Blind identification [6] (i.e. using both the TS sequence and the statistical properties of the signals) can be applied directly here, with possible extensions to the multiuser case. Of course, these methods are most useful when the channel is varying rather rapidly, like in mobile communications.

The extensions to the multiuser case are also easier, as the TS sequences can differentiate the users, which is useful as well for the synchronisation as for the channel estimation.

III. Carrier Synchronisation (tracking)

Carrier synchronisation can be performed in a Maximum Likelihood manner described by [7]. Just as a quick reminder, let the received signal samples be denoted as y_k transmitted at baud rate $1/T$. The signal is sampled at the baud rate. If there is a frequency offset of Δf, for the samples y_k belonging to the training sequence, we have :

$$y_{k+N} = y_k e^{\imath 2\pi \Delta f N T} \tag{1}$$

Considering we have two subsequent sequences of N_s symbols, the two emitted sequences having their first N_Σ symbols equal, the ML estimator of Δf is

$$\widehat{\Delta f} = \frac{1}{2\pi(N_s - N_\Sigma)T} \arg\left(\sum_{i=0}^{N_\Sigma-1} y_{i+k+N_s} y_{i+k}^*\right) \tag{2}$$

This is a scheme used for OFDM [7] or CP-SC [8], where the tracking is performed by computing the above autocorrelation on the Cyclic Prefix. The maximum frequency deviation allowed for CP-SC is $\pm 1/(2N_cT)$. For high SNR, the variance of the estimation error in additive white gaussian noise is :

$$\sigma_{\Delta f}^2 = \frac{1}{N_\Sigma (2\pi(N_sT))^2} \frac{1}{\mathrm{SNR}} \tag{3}$$

By transforming the CP in a TS, we can use the autocorrelation of two non-subsequent symbols, separated by N_{sy} frames, and the estimator is similar to (2) :

$$\widehat{\Delta f} = \frac{1}{2\pi N_c N_{\mathrm{sy}} T} \arg\left(\sum_{i=0}^{N_p-1} y_{i+k+N_{\mathrm{sy}}.N_c} y_{i+k}^*\right) \tag{4}$$

By doing this, the maximum frequency deviation is decreased (it becomes $\pm 1/(2(N_{\mathrm{sy}}N_c - N_p)T)$). On the other hand, the variance of the estimation error becomes, for $N_{\mathrm{sy}} > 1$:

$$\sigma_{\Delta f}^2 = \frac{1}{N_p (2\pi(N_{\mathrm{sy}}N_c - N_p)T)^2} \frac{1}{\mathrm{SNR}} \tag{5}$$

Figure 3 shows the result of a simulation where QPSK symbols were emitted in frames of 64 symbols, with training sequences of length 16 and the whole range of admissible frequency offsets. Simulations are performed on an AWGN channe with an SNR of 10 dB. The simulation points follow the theoretical curve. Other simulations show that this is correct for other frame lengths, TS lengths and SNR. The extension to the multipath case is rather easy [7] and performance enhancement is the same.

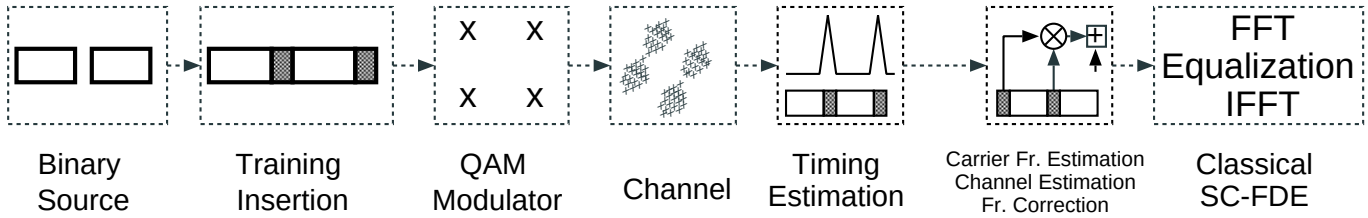

Fig. 2. TASC transmit/receive chain

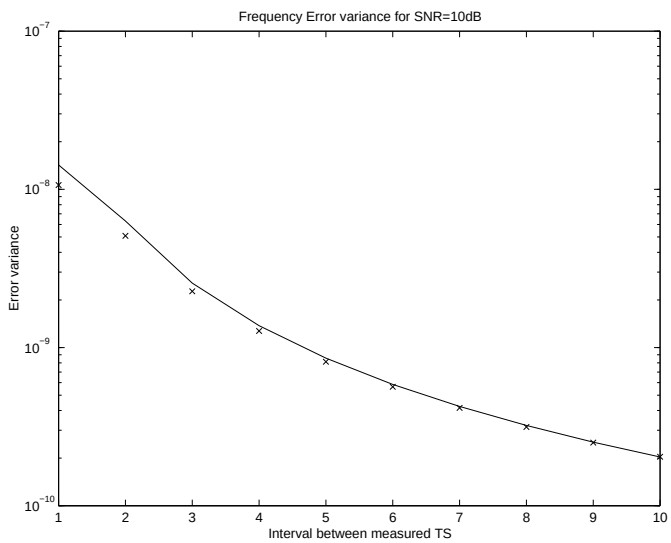

Fig. 3. Verification of the error variance (5)

This simple scheme allows us to obtain very accurate frequency offset correction, for slow varying offset, while maintaining the possibility to track fast variation with less precision as in the CP-SC. Note that using estimates for different N_{sy} allows us to get a vector of estimates and unwrapping is possible, yielding an estimate who's ambiguity is the ambiguity for $N_{\text{sy}} = 1$. Better performance can also be obtained by averaging if necessary. Note that, like for the original algorithm, this frequency synchronisation can be used in fading channels.

IV. Joint Time/Frequency Synchronisation

For time synchronisation, like for it's frequency counterpart, we will base ourselves on an observation time incorporating more than two TS, but without a priori knowledge of the TS itself (like here above). The observation interval and the timing parameter τ are sketched in figure 4.

A. The Log Likelihood function

The autocorrelation sequence can be written as :

$$\begin{aligned}\mathrm{E}\{y(k)y^*(k+m)\} = \\ \sigma_x^2 + \sigma_n^2\delta_{m0} + \textstyle\sum_k \sigma_x^2 e^{\imath 2\pi\Delta f m/N}\delta_{m(kN)}\end{aligned} \tag{6}$$

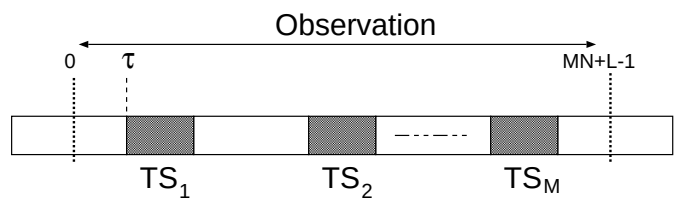

Fig. 4. Structure of the observations

For ease of notations, we will introduce the observation vector $\mathbf{y} = [y(0), \cdots y(MN+L-1)]^T$ and a subsampled observation vector as $\mathbf{y}_s(k) = [y(k), y(k+N), \cdots y(k+MN)]^T$.

Hence, the log-likelihood of the observations can be written as

$$\begin{aligned}\Lambda(\tau,\Delta f) &= \log f(\mathbf{y}|\tau,\Delta f) \\ &= \log \prod_{k\in TS_1} f(\mathbf{y}_s(k)) \prod_{k\notin \cup TS_i} f(y(k)) \\ &= \log \prod_{k\in TS_1} \frac{f(\mathbf{y}_s(k))}{\prod_{l=1}^{M} f(\mathbf{y}_s(k+lN))} \prod_k f(y(k))\end{aligned} \tag{7}$$

where dependence on the parameters has been skipped for clarity. The last product can be skipped, as $f(y(k))$ does not depend on τ and Δf. The probability density function of the subsampled observations can be written as

$$f(\mathbf{y}_s(k)) = \frac{1}{(\pi \det C_{ss})^M} e^{-\mathbf{y}_s^H C_{ss}^{-1}\mathbf{y}_s} \tag{8}$$

where $C_{ss} = \sigma_n^2 I + \sigma_x^2 W.W^H$, σ_n^2 and σ_x^2 respectively denote the noise and signal power and $W = [1\ e^{-\imath 2\pi N_c \Delta f} \cdots e^{-\imath 2\pi N_c (M-1)\Delta f}]^H$. With the matrix inversion lemma, we can compute

$$C_{ss}^{-1} = \sigma_n^{-2} I - \frac{\sigma_x^2}{(M\sigma_x^2 + \sigma_n^2)\sigma_n^2} W W^H$$

and $\det C_{ss} = (M\sigma_x^2 + \sigma_n^2)\sigma_n^{2(M-1)}$ which completes the definiton of the pdf. After some algebraic calculations (see appendix), we

find that

$$\Lambda(\mathbf{y}|\tau,\Delta f) \doteq \sum_{k=\tau}^{\tau+N_p-1} -M\mathbf{y}_s(k)^H\mathbf{y}_s(k)\frac{\sigma_x^2}{\sigma_x^2+\sigma_n^2} + \mathbf{y}_s(k)^H(WW^H)\mathbf{y}_s(k) \quad (9)$$

where $\doteq$ means that only the relevant terms are included.

B. Quadratic form of the ML estimation

The right-hand side of (9) consists clearly of two terms, the first depending only on τ, while the second depends on τ and Δf.

Assume τ is known, than, we have to maximize $\sum_{k=\tau}^{\tau+N_p-1} \mathbf{y}_s(k)^H(WW^H)\mathbf{y}_s(k)$, which can be written as

$$\max_{\Delta f}\{W^H\mathcal{Y}\mathcal{Y}^HW\} \quad (10)$$

where

$$\mathcal{Y} = \begin{bmatrix} y(\tau) & \cdots & y(\tau+N_p-1) \\ y(\tau+N_c) & \cdots & y(\tau+N_c+N_p-1) \\ \vdots & \ddots & \vdots \\ y(\tau+(M-1)N_c) & \cdots & y(\tau+(M-1)N_c+N_p-1) \end{bmatrix}$$

In the absence of noise, all lines of $\mathcal{Y}$ are proportional and $\mathcal{Y}\mathcal{Y}^H$ is of rank one. The solution of (10) is the eigenvector of $\mathcal{Y}\mathcal{Y}^H$ corresponding to it's maximum eigenvalue (maximum eigenvector : $V_{\max}$).

A closer look at $W^H\mathcal{Y}\mathcal{Y}^HW$ shows that this can be written as

$$\sum_i\sum_j |z_{ij}|\cos(\varepsilon(i+j)+\text{phase}z_{ij})$$

where $\varepsilon = 2\pi\Delta f N_c$ and z_{ij} is the $\{i,j\}$ element of matrix $\mathcal{Y}\mathcal{Y}^H$. This results from the fact that $W^H\mathcal{Y}\mathcal{Y}^HW$ is real. Hence, denoting $\mathbf{1}$ as a vector of ones and $|\mathcal{Y}\mathcal{Y}^H|$ the matrix built with the absolute values of $\mathcal{Y}\mathcal{Y}^H$, we can perform the maximization in a two step procedure :

1. $\widehat{\tau}_{\text{ML}} = \arg\max_{\tau} \text{trace}\{\mathcal{Y}\mathcal{Y}^H\} - M\frac{\sigma_x^2}{\sigma_x^2+\sigma_n^2}\mathbf{1}^H|\mathcal{Y}\mathcal{Y}^H|\mathbf{1}$
2. $W(\widehat{\Delta f}_{\text{ML}}) = V_{\max}\{\mathcal{Y}(\widehat{\tau}_{\text{ML}})\mathcal{Y}(\widehat{\tau}_{\text{ML}})^H\}$

and $\widehat{\Delta f}_{\text{ML}}$ can be extracted from the averaged and scaled phases of $W(\widehat{\Delta f}_{\text{ML}})$.

Note that the rank one property of $\mathcal{Y}\mathcal{Y}^H$ is an interesting property for the tracking.

On a complexity point of view, even if the eigenvalue decomposition can appear frightening, the simulations show that we get most of the benefits for small values of M (4 to 6), hence, the eigenvalue decomposition has only to be performed on small $M \times M$ matrices. Furthermore, the structure of the matrix is such that it can be updated iteratively, as well as it's decomposition, if bigger values of M are used.

C. Simulations

We have performed simulations with 1000 frames consisting of $N_c = 64$ QPSK symbols, among which there are $N_p = 8$ training symbols. The algorithm has been run for SNR's from 0 to 20 dB and using $M = 2:10$ training sequences in the observation interval. The performances in figures below have to be compared to the upper curve (for $M = 2$), which corresponds to the ML solution for the CP-SC solution. The benefit of the modified scheme is evident for short observation intervals at low SNR's. It really extends the framing synchronisation capabilities to the low SNR's and offers a good improvement on the frequency synchronisation. Note however that the ad-hoc solution hereabove performs better than the ML solution, as it takes less intermediate correlation products into account.

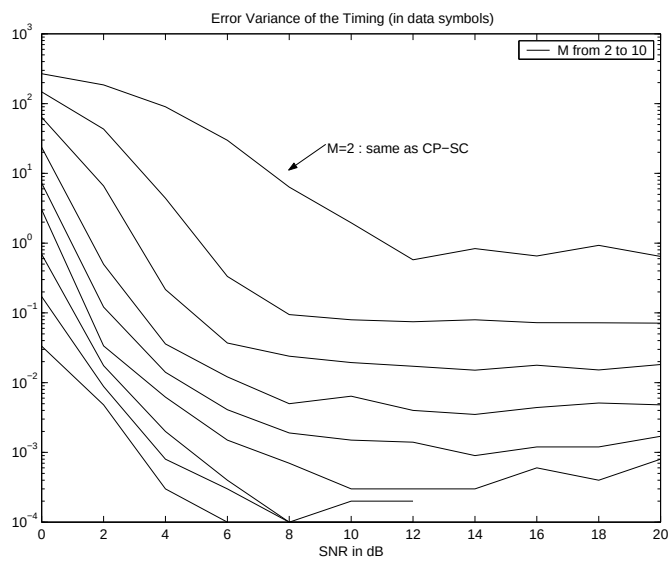

Fig. 5. Performance of the Frame Synchronisation

V. Conclusions

We have introduced a new Training-Aided Single Carrier (TASC) scheme taking advantage of the Cyclic Prefix nature of a short Training Sequence (TS) . This allows to perform frequency domain equalization, like for the Single Carrier with Cyclic Prefix (CP-SC) scheme. The modification of the CP in a TS allows to improve the performance of the synchronisation either by using the knowledge of the training sequence, or by extending the observation time.

To this end, we have introduced an ad-hoc extension of a Maximum Likelihood carrier synchronisation algorithm which

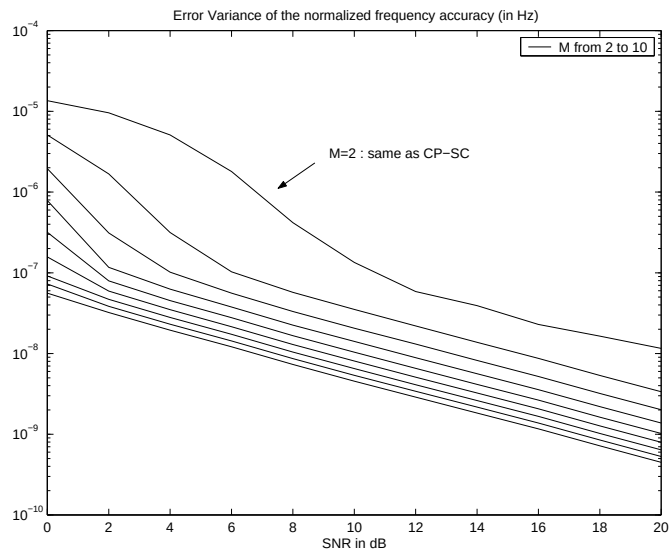

Fig. 6. Performance of the frequency Synchronisation

shows the benefits we can await from the training sequence compared to the use of a classical Cyclic Prefix like in the CP-SC. Furthermore, we have derived the Maximum-Likelihood joint estimation of frequency offset and frame timing, based on a model relying only on the cyclic repetition of the Training Sequence, for an AWGN channel.

Further incorporation of the knowledge of the training sequence itself will lead to another bunch of synchronisation algorithms and should also benefit from the extensive literature on sequence design for synchronisation (e.g., but not restricting to it, for spread-spectrum).

Further research will allow to use this TS to perform semi-blind channel identification, which is useful in fast fading channels (like in mobile communications).

These considerations make TASC an interesting candidate in situations where multipath and fast fading are present, while in other situations it will perform almost like CP-SC.

APPENDIX

Starting from equation (7), for a given k, we get

$$\Lambda(\mathbf{y}(k)|\tau,\Delta f) = \log \frac{f(\mathbf{y}_s(k))}{\prod_l f(y(k+lN))} =$$

$$\log\{(\sigma_x^2+\sigma_n^2)^M(\det C_{ss})^{-M}$$

$$\exp\left\{-\mathbf{y}_s^H C_{ss}^{-1}\mathbf{y}_s + \frac{\sum_l |y(k+lN)|^2}{\sigma_x^2+\sigma_n^2}\right\}\}$$

Skipping the term which do not depend on the parameters :

$$\begin{aligned}
&\Lambda(\mathbf{y}(k)|\tau,\Delta f)\\
&\doteq -\mathbf{y}_s^H C_{ss}^{-1}\mathbf{y}_s + \sum_l \frac{|y(k+lN)|^2}{\sigma_x^2+\sigma_n^2}\\
&\doteq \frac{-\mathbf{y}_s(k)^H\mathbf{y}_s(k)}{\sigma_n^2} + \frac{\sigma_x^2\mathbf{y}_s(k)^H WW^H\mathbf{y}_s(k)}{(M\sigma_x^2+\sigma_n^2)\sigma_n^2}\\
&\quad + \sum_l \frac{|y(k+lN)|^2}{\sigma_x^2+\sigma_n^2}\\
&\doteq \frac{-M\sigma_x^2\mathbf{y}_s(k)^H\mathbf{y}_s(k)}{\sigma_x^2+\sigma_n^2} + \mathbf{y}_s(k)^H(WW^H)\mathbf{y}_s(k)
\end{aligned}$$

REFERENCES

[1] H. Sari, G. Karam, and I. Jeanclaude. "Frequency-domain equalization of mobile radio and terrestrial broadcast channels,". In *GLOBECOM*, pages 1–5. 1994.

[2] H. Sari, G. Karam, V. Paxal, and K. Maalej. "Trellis-coded constant-envelope modulations with linear receivers,". *IEEE Trans. on Communications*, 44(10):1298–1307, October 1996.

[3] J.J. van de Beek, M. Sandell, and P.O. Börjesson. "ML Estimation of Time and Frequency Offset in OFDM Systems,". *IEEE Trans. on Signal Processing*, 45(7):1800–1805, July 1997.

[4] J. A. Gansman, M. P. Fitz, and J. V. Krogmeier. "Optimum and SUb-optimum Frame Synchronization for Pilot-Symbol-Assisted Modulation,". *IEEE Trans. on Communications*, 45(10):1327–1337, October 1997.

[5] H. Meyr, M. Moeneclaey, and S. A. Fletchel. *Digital Communication Receivers : Synchronization, Channel Estimation and Signal Processing*. John Wiley & Sons, 1997.

[6] E. de Carvalho and D.T.M. Slock. "Cramer-Rao bounds for semi-blind, blind and training sequence based channel estimation,". In *First IEEE Signal Processing Workshop on Signal Processing Advances in Wireless Communications*, pages 129–132. 1997.

[7] P. H. Moose. "A technique for orthogonal frequency division multiplexing frequency offset correction,". *IEEE Trans. on Communications*, 42(10):2904–2914, October 1994.

[8] A. Czylwik. "Low overhead pilot-aided synchronization for single carrier modulation with frequency domain equalization,". In *GLOBECOM*, volume 4, pages 2068–73. 1998.