Compensation of IQ imbalance in OFDM systems.

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Abstract— **Today a lot of attention is spent on developing inexpensive OFDM receivers. Especially, zero-IF receivers are very appealing, because they avoid costly IF filters. However, this implies IQ demodulation at RF, which therefore cannot be done digitally and thus introduces IQ mismatch. Unfortunately, OFDM is very sensitive to receiver IQ imbalance. Therefore, we developed a new compensation scheme to combat the IQ imbalance at baseband. In this paper, we describe the algorithm and present the performance results. Our compensation scheme eliminates the IQ imbalance almost perfectly. This leads to tremendous improvements, especially in multi-path channels (up to 10 dB performance gain), and enables low-cost zero-IF receivers.**

I. Introduction

OFDM is a widely recognized and standardized modulation technique [1], [2]. Unfortunately, OFDM is also sensitive to non-idealities in the receiver front-end [3]. This leads either to heavy front-end specifications and thus an expensive front-end or large performance degradations. IQ imbalance has been identified as a key front-end effect for OFDM systems. In this paper, we investigate the performance degradation due to receiver IQ imbalance and introduce a compensation scheme to combat the IQ imbalance effect.

In section II, we explain the model we have used for IQ imbalance and analyze the effect of IQ imbalance on OFDM. In section III, we derive a compensation scheme and show the impact on the performance for an AWGN channel. Section IV follows the same steps for multi-path channels and section V deals with channel estimation under IQ imbalance degradation. All findings are verified through simulations. Finally, section VI summarizes the conclusions of our work and comments on future work.

II. IQ imbalance Effect

A. Model

IQ imbalance arises when a front-end component doesn't respect the power balance or the orthogonality between the I and Q branch. We can therefore characterize this effect by 2 parameters: the amplitude imbalance ϵ, and the phase mismatch $\Delta\phi$. The complex baseband equation for the IQ imbalance effect on the ideal signal x is given by [4] as

$$\begin{aligned} y &= (1+\epsilon)\cos\Delta\phi\Re\{x\} - \jmath(1-\epsilon)\sin\Delta\phi\Re\{x\} \\ &+ \jmath(1-\epsilon)\cos\Delta\phi\Im\{x\} - (1+\epsilon)\sin\Delta\phi\Im\{x\} \quad (1) \\ &= (\cos\Delta\phi - \jmath\epsilon\sin\Delta\phi)\cdot x \\ &\quad -(\epsilon\cos\Delta\phi - \jmath\sin\Delta\phi)\cdot x^* \quad (2) \\ &= \alpha\cdot x + \beta\cdot x^* \quad (3) \end{aligned}$$

with y the signal with IQ imbalance, $\Re()$ denotes the real part, $\Im()$ the imaginary and $()^*$ the complex conjugate and

$$\alpha = \cos\Delta\phi + \jmath\epsilon\sin\Delta\phi \quad (4)$$

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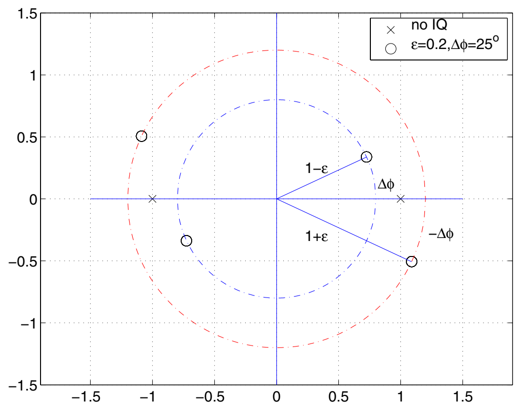

Fig. 1. The effect of IQ imbalance on OFDM-BPSK constellation.

$$\beta = \epsilon\cos\Delta\phi - \jmath\sin\Delta\phi \quad (5)$$

If no IQ imbalance is present, then $\alpha = 1$ and $\beta = 0$ and then (3) reduces to $y = x$.

The same derivation can be found in [5]. Throughout the rest of the paper, the term IQ parameters refers to α and β for calculations ; for simulation parameters ϵ and $\Delta\phi$.

B. IQ imbalance and OFDM

We analyze the effect of the IQ imbalance on OFDM transmission. We first consider the noise-free AWGN case. If $\mathbf{d_t}$ the transmitted OFDM symbol (in the frequency domain), then $IFFT(\mathbf{d_t})$ is the incoming time domain signal in the receiver. Applying the IQ imbalance (3) and taking the fft leads to

$$\begin{aligned} \mathbf{d_r} &= FFT\{\alpha\cdot IFFT(\mathbf{d_t}) + \beta\cdot[IFFT(\mathbf{d_t})]^*\} \\ &= \alpha\cdot\mathbf{d_t} + \beta\cdot(\mathbf{d_t^*})_m \quad (6) \end{aligned}$$

where bold variables denote a vector, bold caps a matrix and regular font a scalar. $\mathbf{d_r}$ is the (soft) received OFDM symbol and $(\mathbf{d_t})_m$ the transmitted OFDM symbol, mirrored over the carriers: $(\mathbf{d_t})_{m(i)} = (\mathbf{d_t})_{mod(N-i+2,N)}$, with N the number of sub-carriers in the OFDM symbol, $1 \leq i \leq N$ and mod the modulo operation. Carrier 1 is the DC carrier.

From (6), it is clear that the effect of the IQ imbalance is twofold: first, the transmitted constellation is scaled by a complex factor α; secondly, a scaled version (β) of the mirror image of the OFDM symbol is subtracted. For BPSK, this is illustrated in figure 1.

III. IQ imbalance in AWGN

A. Correction

A correction scheme for the IQ imbalance is derived from (6)

$$\mathbf{d_{corr}} = \frac{\mathbf{d_r} - (\mathbf{d_t^*})_m\cdot\beta}{\alpha} \quad (7)$$

If we can estimate α, β and $\mathbf{d_t}$, (7) can be used to correct the distortion due to the IQ imbalance. $\mathbf{d_t}$ is known in case of pilot symbols or by making a hard decisions on the received soft signal $\mathbf{d_r}$. The decision-directed approach will work if the IQ imbalance doesn't degrade the BER too much, i.e. for small IQ parameters. We will comment on this restriction in section V-C. We follow the decision-directed approach.

The effect of IQ imbalance can be described on each non-zero carrier as (6). If we assume the hard decisions $\bar{\mathbf{d_r}}$ to be a good approximation for $\mathbf{d_t}$, then (6) becomes a complex equation in the complex variables α and β. The complex equation is equivalent to 2 scalar equations. The complex variables α and β are in fact determined by 2 scalar variables, ϵ and $\Delta\phi$, through the non-linear equations (5). This means that for each non-zero carrier (6) represents a set of 2 scalar non-linear equations in 2 scalar variables ϵ and $\Delta\phi$. However, solving non-linear equations in a mobile terminal is not trivial. Moreover, approximating $\cos\Delta\phi$ and $\sin\Delta\phi$ by polynomial expansions still needs high-order polynomials to obtain reasonable performance.

Another approach is to consider (6) as a complex equation in the complex variables α and β. If we consider 2 non-zero carriers together, we get a set of 2 linear complex equations, which can be solved easily. This means we ignore certain relations between α and β (5) which could improve the estimation. If we consider carrier i and carrier j, then

$$(d_r)_i = \alpha \cdot (d_t)_i + \beta \cdot (d_t^*)_{m(i)} \quad (8)$$

$$(d_r)_j = \alpha \cdot (d_t)_j + \beta \cdot (d_t^*)_{m(j)} \quad (9)$$

Solving this set of equation yields,

$$\alpha = \frac{(d_r)_i \cdot (d_t^*)_{m(j)} + (d_r)_j \cdot (d_t^*)_{m(i)}}{(d_t)_i \cdot (d_t^*)_{m(j)} - (d_t)_j \cdot (d_t^*)_{m(i)}} \quad (10)$$

$$\beta = \frac{(d_r)_i - \alpha \cdot (d_t)_i}{(d_t^*)_{m(i)}} \quad (11)$$

Each non-zero carrier in the OFDM symbol yields an estimate. We take the average over all values obtained in one symbol and use these averages in (7).

Reference [5] uses the same compensation equation, but handles the estimation of the IQ parameters through LMS filtering.

B. Performance results

As a case study for the IQ compensation scheme we have used a WLAN setup compliant to the standards [1], [2]. Figure 2 shows that we can largely compensate the effect of an IQ imbalance: at a BER of 10^{-4} $\epsilon = 20\%$ and $\Delta\phi = 10^o$ causes an SNR degradation or Implementation Loss (IL) of 1.8 dB ; the exact IQ parameters reduce the IL to 0.25 dB, the estimated values reduces it to 0.4 dB. Note that the estimation is based only on the symbol itself and can be improved by averaging out over 2 or more symbols. Averaging out over several symbols works since the IQ imbalance parameters are static, but doesn't get the degradation below 0.25 dB. This is due to the decision-directed approach for estimating $\mathbf{d_t}$: making some decision errors is inevitable and thus some compensations are wrong. This means we cannot remove the entire IQ degradation (hence the residual degradation), but we do remove most of it.

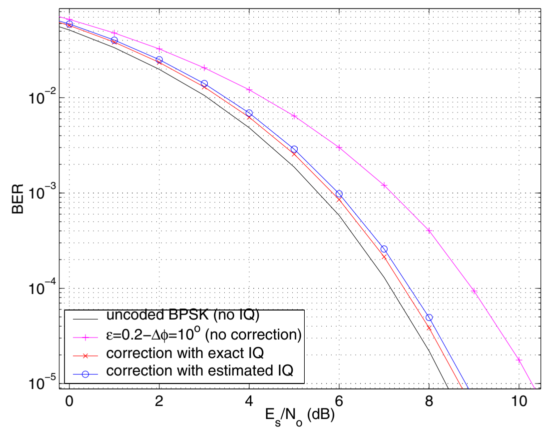

Fig. 2. The effect of IQ imbalance and correction on OFDM-BPSK.

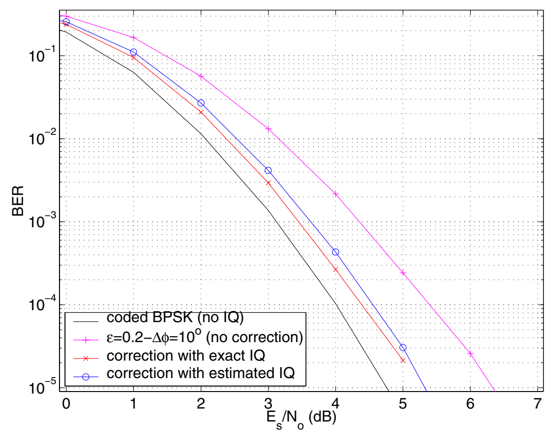

Fig. 3. The effect of IQ imbalance and correction on OFDM-BPSK with coding.

Figure 3 shows the BER curves for OFDM-BPSK including coding (R=3/4 IEEE-802.11a coding). There is a degradation of 1.6 dB at 3.10^{-5} for $\epsilon = 20\%$ and $\Delta\phi = 10^o$; we can correct it except for a residual degradation of 0.5 dB. If the symbol-based estimation for ϵ and $\Delta\phi$ is used, then 0.6 dB degradation remains. Again, this 0.6 dB can be reduced by averaging the estimation of IQ parameters over more than one symbol, but cannot be improved beyond 0.5 dB.

The coded results show that we lose some of the performance enhancement from the correction scheme after coding: compared to the uncoded case there is a smaller degradation before compensation, but the remaining degradation after compensation is larger. To check where this loss comes from we analyze the error distribution before coding. Figure 4 shows the distance between consecutive errors for an AWGN channel at SNR=4dB. OFDM-BPSK with compensated IQ imbalance has the same error distribution as OFDM-BPSK without IQ imbalance, except for small error distances. This is due to the block-based nature of the compensation scheme: if a wrong decision is made, it will result in a wrong compensation, which can cause an extra error within the same block, which is known as error propagation. Indeed, the figure shows that the error distances below 48 output symbols (thus within one block) are more frequent. The

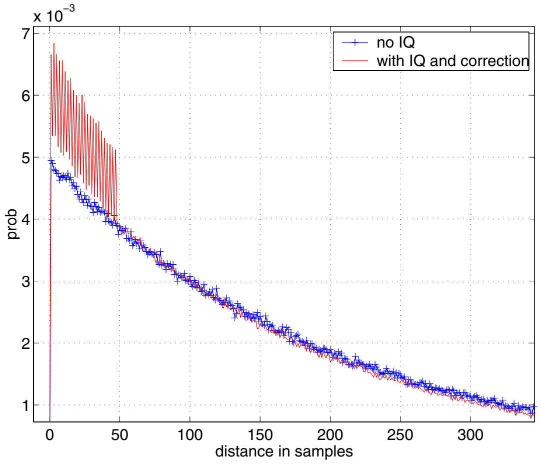

Fig. 4. The separation between consecutive errors for OFDM-BPSK.

compensation scheme introduces only a few extra errors compared to the case without IQ imbalance. However, the few extra errors are introduced in the symbols where there are already errors. This means the errors are more clustered in the corrected IQ imbalance case than in the reference case and therefore the coding gain is lower and thus the IL is higher.

The standardized interleaver doesn't help because it is a symbol-based interleaver. Interleaving over several blocks helps, but introduces extra latency and is not standard compliant.

This error propagation is inherent to the decision-directed scheme. A solution is proposed in section V-C.

IV. MULTI-PATH

A. Analysis

We now analyze the effect of receiver IQ imbalance in a more realistic situation: OFDM is used for its capability to handle a multi-path environment. We first derive the equation for the effect of the IQ imbalance in a multi-path channel for the noise-free case. We start from (6) for the AWGN case and replace $IFFT(\mathbf{d_t})$ with $IFFT(\mathbf{d_t} \cdot \mathbf{c})$ as the incoming signal in the receiver, where $\mathbf{c}$ is the vector with the exact channel coefficients in the frequency domain.

After equalization this leads to

$$\mathbf{d_r} = \mathbf{d_t} \cdot \alpha + \frac{(\mathbf{d_t^*} \cdot \mathbf{c}^*)_\mathbf{m}}{\mathbf{c}} \cdot \beta \tag{12}$$

As for the IQ imbalance in the AWGN scenario (see (6)), the modem receives the transmitted symbol scaled by the same complex factor and an additional degradation which is a function of the mirror image of the transmitted symbol and the IQ imbalance parameters. However, in this multi-path scenario a channel coefficient appears in the denumerator of the degradation term and a different channel coefficient in the numerator. So on some carriers this will increase the additional degradation compared to the AWGN case (see figure 5) and these carriers will dominate the BER performance.

B. Correction

Based on (12), we derive a compensation scheme in the same way as for the AWGN case. We follow a decision-directed approach, using the hard decisions on $\mathbf{d_r}$ as an approximation for $\mathbf{d_t}$. The compensation results in a corrected signal

$$\mathbf{d_{corr}} = \frac{\mathbf{d_r} - \frac{((\bar{\mathbf{d}}_\mathbf{r} \cdot \mathbf{c})^*)_\mathbf{m}}{\mathbf{c}} \beta}{\alpha} \tag{13}$$

Estimation. As for the AWGN case, we could look at (12) for each non-zero carrier separately as an equation in ϵ and $\Delta\phi$, together with (5). However, this would again lead to a set of 2 non-linear equations, which is unsuitable for a modem. Again we have the option of considering 2 non-zero carriers together. This leads to a set of 2 linear equations in α and β, which can be solved easily.

$$\alpha = \frac{(d_r)_i (d_t^*)_{m(j)} {c^*}_{m(j)} c_i + (d_r)_j (d_t^*)_{m(i)} {c^*}_{m(i)} c_j}{(d_t)_i (d_t^*)_{m(j)} {c^*}_{m(j)} c_i - (d_t)_j (d_t^*)_{m(i)} {c^*}_{m(i)} c_j} \tag{14}$$

$$\beta = c_i \frac{(d_r)_i - \alpha \cdot (d_t)_i}{(d_t^*)_{m(i)} {c^*}_{m(i)}} \tag{15}$$

In our WLAN case study there are 52 non-zero carriers, so we will get 26 estimations for α and β. Since some estimations suffer badly from noise enhancement, we need to eliminate them. We can identify these samples by the fact that the denumerator in (14) is small which enhances the noise term in the numerator, implicitly present in d_r.

C. Performance results

Figure 5 shows the simulation results for uncoded OFDM-BPSK with the effect of IQ imbalance. For a BER of 10^{-3} there is a degradation of more than 10 dB. With the exact IQ parameters we can correct this to about 0.1 dB. The estimation using (14-15) reduces the degradation to 0.3 dB.

For the coded case, figure 6 shows a degradation of more than 10 dB at a BER of 10^{-4}, which can be corrected up to 0.2 dB if we know the exact IQ parameters. The estimation using (14-15) reduces the degradation to 0.7 dB.

V. CHANNEL ESTIMATION

A. Analysis

All previous analysis and simulations are performed with perfect Channel State Information (CSI). In practice, channel estimation is performed based on the transmission of a Long Training Symbol (LTS), which is defined in the standards [1], [2]. Naturally, the LTS is also corrupted by IQ imbalance, which will degrade the channel estimation and thus the data and the estimation/compensation of the IQ imbalance. The effect of IQ imbalance on channel estimation can be calculated based on (6)

$$\mathbf{h} = \alpha \cdot \mathbf{c} + \beta \cdot \mathbf{lts2} \cdot (\mathbf{c}^*)_\mathbf{m} \tag{16}$$

where $\mathbf{h}$ is the channel estimate calculated from the LTS, $\mathbf{c}$ is the exact channel vector and $\mathbf{lts2} = \mathbf{lts} \cdot (\mathbf{lts})_\mathbf{m}$.

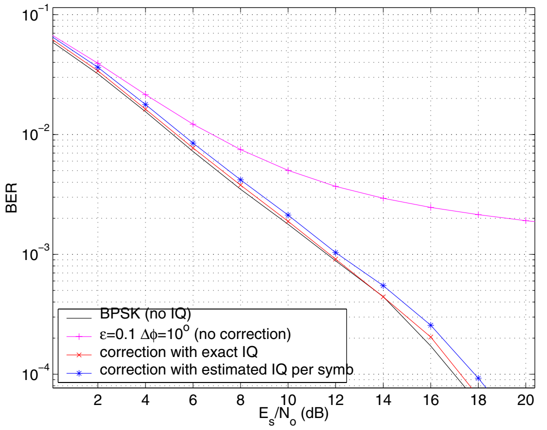

Fig. 5. Effect of IQ imbalance at the receiver and its correction on uncoded OFDM-BPSK.

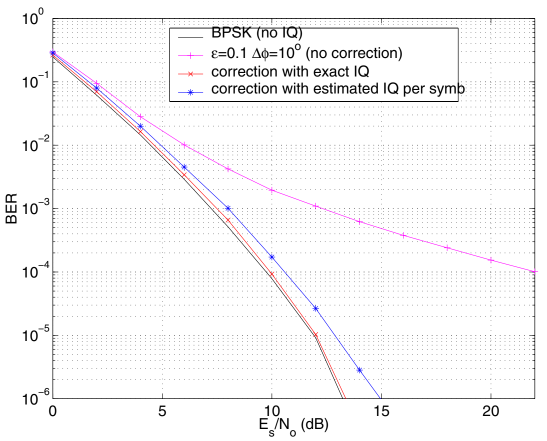

Fig. 6. Effect of IQ imbalance at the receiver and its correction on coded OFDM-BPSK.

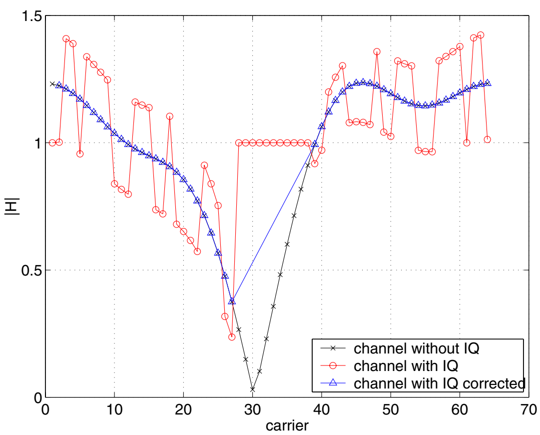

Fig. 7. The effect of IQ imbalance and correction on lts channel estimation.

Figure 7 shows that the IQ imbalance has a large impact on the channel estimate. If we use the estimate as is, we get a terrible performance. So we need to correct the channel estimate.

Solving (16) for $\mathbf{c}$, leads to

$$\mathbf{c} = \frac{\alpha^* \mathbf{h} - \beta(\mathbf{lts2} \cdot \mathbf{h}^*)_\mathbf{m}}{|\alpha|^2 - |\beta|^2} \tag{17}$$

So we can eliminate the IQ imbalance from the channel estimation if we know α and β.

B. Estimation of the IQ parameters

From figure 7 it is clear that the variation between carrier 1 and 2 is small (for the real channel $\mathbf{c}$ as well as for the measured channel $\mathbf{h}$). There is however a rather large difference between h_2 and h_3 (the value of the measured channel on the 2^{nd} and 3^{rd} carrier). This difference depends on ϵ, $\Delta\phi$ and $\mathbf{c}$.

$$h_2 = \alpha \cdot c_2 + \beta \cdot lts2_{64} \cdot c_{64}^* \tag{18}$$
$$h_3 = \alpha \cdot c_3 + \beta \cdot lts2_{63} \cdot c_{63}^* \tag{19}$$

This is a set of 2 equations with 2 unknowns (ϵ and $\Delta\phi$) if we can make appropriate approximations on c_2, c_3, c_{63}, c_{64}. In the appendix, this is explained in detail. The results for the estimation of α and β are

$$\beta_{est} = \frac{h_3 - h_2}{lts2_{64}(h_{64}^* + h_{63}^*)} \tag{20}$$
$$\alpha_{est} = \sqrt{1 - \Im^2\{\beta\}} - \jmath \frac{\Re\{\beta\}\Im\{\beta\}}{\sqrt{1 - \Im^2\{\beta\}}} \tag{21}$$

Because there are 20 transitions in the lts2 sequence, we get 20 estimates of α and β. We average them out and use them in (17).

Figure 7 shows that we can correct the influence of the IQ imbalance on the channel estimate extremely well. Note that carriers 28 to 38 are zero carriers, which means no channel estimate is needed on those carriers.

C. Time domain compensation

Since ϵ and $\Delta\phi$ and thus α and β are static over several symbols, we can use their estimates from the channel correction also for the data correction. This has a major advantage. Since we already have an estimate of the IQ parameters before the data symbols arrive we can compensate the effect in the time domain. This means there is no need to apply the decision-directed frequency domain compensation. Therefore this approach avoids the decision errors, which are inevitable in the decision-directed approach and which introduce some bias or errors in the estimation and compensation. Time domain compensation is thus inherently better than frequency domain compensation.

To get the corrected signal r_{corr} from the time domain signal before the FFT r, we need to solve (3) for the input signal

$$r_{corr} = \frac{\alpha^* \cdot r - \beta \cdot r^*}{|\alpha|^2 - |\beta|^2} \tag{22}$$

We can use the estimates of the IQ parameters obtained from the channel estimation in (22). Figure 8 shows the results for uncoded 64QAM transmission with an IQ imbalance of $\epsilon = 10\%$

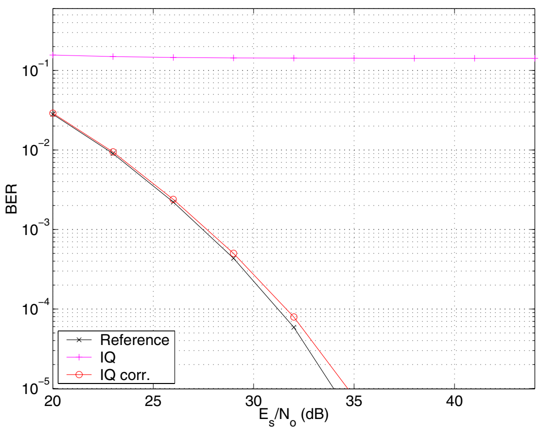

Fig. 8. The correction of IQ imbalance based on LTS channel estimation.

and $\Delta\phi = 10^o$. The time domain compensation scheme gets the degradation at a BER of 10^{-3} down to 0.3 dB. The time domain compensation scheme with the exact knowledge of the IQ parameters fully eliminates the IQ imbalance.

In practice, we can get the remaining degradation even further down, because the standards [1], [2] use 2 consecutive LTS symbols for channel estimation. This means we can average out the estimate of the IQ imbalance parameters and further improve the correction schemes.

Since time domain compensation doesn't depend on any decisions and compensates the IQ imbalance before any decisions are made, time domain compensation will work for any constellation. Naturally, the effect of the residual IQ imbalance after compensation will be larger on 64-QAM than on BPSK.

We can rewrite the equations in section IV to track the residual error after IQ compensation based on the LTS. The decision-directed approach will work since the residual error is small. However, we haven't worked out the equations since the additional gain would be negligible.

VI. Conclusions

IQ imbalance can cause large degradations in an OFDM receiver. Specifically in a multi-path environment, the effect can be dramatic. We have introduced a compensation method to combat the IQ imbalance effect. The channel estimation provides sufficient information to accurately estimate the IQ imbalance. Moreover, time domain compensation outperforms the frequency domain method. This method leads to tremendous performance enhancements. In practical systems, it can improve the performance in the order of 10 dB, even for 64-QAM. This allows the use of less ideal and therefore less expensive components. Since we can compensate most of the IQ imbalance, the IQ mismatch introduced by analog IQ demodulation at RF in zero-IF receivers is greatly alleviated. Hence, our compensation scheme also enables zero-IF architectures.

Future work. Currently, we are investigating the robustness of the IQ compensation scheme against phase noise and CFO and the results are promising. A patent is pending on this scheme. We are working on the implementation of this scheme to incorporate it in a WLAN Zero-IF receiver.

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Appendix

Subtracting equations (19) and (18) leads to

$$h_3 - h_2 = (c_3 - c_2) \cdot \alpha + lts2_{63}(c^*_{63} + c^*_{64})\beta \tag{23}$$

Because the coherency bandwidth of the channel is much larger than the Inter-Carrier-Spacing, we can assume that the variation of two consecutive channel carriers is small. In other words, $c_2 \approx c_3$.

As an approximation for $(c^*_{63} + c^*_{64})$ we can use $(h^*_{63} + h^*_{64})$: if $lts2_{64} = -lts2_{63}$ then $lts2_2 = -lts2_3$ as well and since $c_2 \approx c_3$ and $\alpha = \cos\Delta\phi - \jmath \sin\Delta\phi\epsilon \approx 1$ and $\beta = \epsilon\cos\Delta\phi - \jmath\sin\Delta\phi \approx 0$for reasonable values of ϵ and $\Delta\phi$.

$$\begin{aligned} h^*_{64} + h^*_{63} &= \alpha(c^*_{64} + c^*_{64}) + \beta \cdot lts2_2(c_2 - c_3) \\ &\approx c^*_{63} + c^*_{64} \end{aligned}$$

In other words, h_{63} and h_{64} have about the same deviation from c_{63} and c_{64}, but with opposite signs. Therefore the h-sum is a very good approximation of the c-sum. Using both approximations in (23) leads to

$$h_3 - h_2 = lts2_{63} \cdot (h^*_{63} + h^*_{64}) \cdot \beta \tag{24}$$

$$\Rightarrow \beta = \frac{h_3 - h_2}{lts2_{63}(h^*_{63} + h^*_{64})} \tag{25}$$

From (5) we derive

$$\Re\{\alpha\} = \cos\Delta\phi \tag{26}$$

$$\Im\{\alpha\} = \epsilon\sin\Delta\phi \tag{27}$$

$$\Re\{\beta\} = \epsilon\cos\Delta\phi \tag{28}$$

$$\Im\{\beta\} = -\sin\Delta\phi \tag{29}$$

This means

$$\Re\{\alpha\}\Im\{\alpha\} = -\Re\{\beta\}\Im\{\beta\} \tag{30}$$

$$\Im^2\{\beta\} + \Re^2\{\alpha\} = 1 \tag{31}$$

Solving these equations for $\Re\{\alpha\}$ and $\Im\{\alpha\}$ leads to

$$\Re\{\alpha\} = \sqrt{1 - \Im^2\{\beta\}} \tag{32}$$

$$\Im\{\alpha\} = -\frac{\Re\{\beta\}\Im\{\beta\}}{\sqrt{1 - \Im^2\{\beta\}}} \tag{33}$$