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Minimal parameterization of fundamental matrices using motion and camera properties

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Abstract

This paper addresses the optimal recovery of the displacement and projection parameters from uncalibrated monocular video sequences. We study the particular cases of camera and objects displacements and camera projection in order to extract an optimized parameterization of the problem of parameters recovery for each cases.

This work follows previous studies on particular cases of displacement, scene geometry and camera analysis and focuses on the particular forms of fundamental matrices. This paper introduces the idea of using not all particular cases as individual cases but grouping these cases into a tractable number of sets, using properties on fundamental matrices.

Some experiments were performed in order to demonstrate that if several models are correct, the model with the least parameters gives the best estimate, corresponding to the true case. © 2002 Published by Elsevier Science B.V.

Keywords: Fundamental matrix; Particular displacement; Parameters estimation

1. Introduction

This paper deals with video sequences taken by an uncalibrated camera in an unknown environment. Our interest is to estimate as many parameters as possible on the camera and objects motion and the camera projection using a strategy of hypothesis testing.

Many efforts have been made in the Computer Vision community for determining motion and camera parameters from video sequences. Relations between 2D views exist [7] as the fundamental matrix **F**, but, in the general case, we cannot extract all the unknown parameters from this **F** matrix. It is however possible in some particular situations.

This work follows previous work on particular cases of displacement, scene geometry and camera analysis [10,11,18]. It focuses on the particular forms of fundamental matrices.

Several authors have already been interested in particular cases of projection [2,5,9,13,14,16] or displacement [3,4,8,17]. Some of them consider several cases and compare each result, in order to automatically determine which case was performed.

We call by general case the situation where we do not know anything about motion or camera projection. A particular case is when we know (or make the hypothesis) that a parameter is null, constant or known, or related to other parameters. A particular case has fewer parameters and/or simpler equations than the general one.

The motivations for these studies are threefold:

- to eliminate singularities of general equations by considering each case that may conduct to singularity,
- to estimate the parameters with more robustness using a simplified model (an adapted model gives more accuracy than the general one as shown in [18]),

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- to retrieve parameters that cannot be retrieved in the general case because we eliminate some unknowns that are meaningless in the particular case studied.

It is already known that the large number of particular cases prevent examining all the cases linearly. In this paper, we introduce a new way to deal with this amount of cases in three steps. (1) We eliminate, with some simple rules, some redundant cases and some physically impossible cases. (2) We divide the set of cases into two sets, each corresponding to homographic or fundamental relations. (3) We divide again the fundamental cases into sets corresponding to particular forms. We will provide details for each of these steps in the following sections.

2. Stereo framework

In this section, we present the stereo framework and the notations we will use in this paper.

2.1. Rigid displacements

We consider a rigid or piecewise rigid scene. A 3D-point $\mathbf{M} = [X\,Y\,Z1]^{\mathrm{T}}$ is moving onto $\mathbf{M}' = [X'\,Y'\,Z'1]^{\mathrm{T}}$ by a rotation $\mathbf{R}$ followed by a translation $\mathbf{t} = [t_0\,t_1\,t_2]^{\mathrm{T}}$:

$$\mathbf{M}' = \mathbf{RM} + \mathbf{t}.$$

A rotation matrix $\mathbf{R}$ depends only on three parameters $\mathbf{r} = [r_0 r_1 r_2]^{\mathrm{T}}$ related to the rotation angle θ and axis $\mathbf{u}$ by

$$\mathbf{r} = 2\tan\left(\tfrac{1}{2}\theta\right)\mathbf{u} \Leftrightarrow \theta = 2\arctan\left(\tfrac{1}{2}\|\mathbf{r}\|\right).$$

A rigid displacement is then parameterized by six parameters.

We note by $\tilde{\mathbf{r}}$ the antisymmetric matrix representing the cross product $\mathbf{r} \wedge \cdot$:

$$\tilde{\mathbf{r}}\mathbf{x} = \mathbf{r} \wedge \mathbf{x} \quad \forall \mathbf{x}.$$

The rotation matrix $\mathbf{R} = \mathrm{e}^{\mathbf{r}\wedge\cdot} = \mathrm{e}^{\tilde{\mathbf{r}}}$ can be developed as a rational Rodrigues formula [15]

$$\mathbf{R} = \mathbf{I} + \left[\frac{\tilde{\mathbf{r}} + (1/2)\tilde{\mathbf{r}}^2}{1 + \mathbf{r}^{\mathrm{T}} \cdot \mathbf{r}/4}\right].$$

2.2. Camera projection

The most commonly camera model states that a 3D-point $\mathbf{M} = [X\,Y\,Z1]^{\mathrm{T}}$ is projected with a perspective projection onto an image plane on a 2D-point $\mathbf{m} = [uv1]^{\mathrm{T}}$. In the reference frame attached to the camera, the projection equation is

$$Z\mathbf{m} = \underbrace{\begin{pmatrix} \alpha_u & \gamma & u_0 & 0 \\ 0 & \alpha_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\mathbf{A}}\mathbf{M}, \qquad (1)$$

where α_u and α_v represent the horizontal and vertical lengths, u_0 and v_0 correspond to the image of the optical center and γ is the skew factor. Those parameters are the intrinsic parameters and are collected in the projection matrix $\mathbf{A}$.

2.3. Considering two frames

Let I_1 and I_2 denote two images. In the general case, there exists a fundamental relation [7] between points $\mathbf{m}_2$ in I_2 and points $\mathbf{m}_1$ in I_1:

$$\mathbf{m}_2^{\mathrm{T}}\mathbf{F}\mathbf{m}_1 = \mathbf{0},$$

where $\mathbf{F}$ is called the fundamental matrix and is related to the intrinsic and extrinsic parameters by

$$\mathbf{F} = (\tilde{\mathbf{A}}_2 t)\mathbf{A}_2\mathbf{R}\mathbf{A}_1^{-1},$$

where $\mathbf{A}_1$ and $\mathbf{A}_2$ are the projection matrix for the first and second frames, respectively, see (1).

This kind of relationship vanishes if the displacement is a pure rotation or if the scene is planar. The relation between points is homographic:

$$\mathbf{m}_2 = \mathbf{H}\mathbf{m}_1,$$

where $\mathbf{H}$ is called the homographic matrix. Another study on homographic matrices can be found in [11].

3. Deriving all particular cases

In order to study all particular cases of cameras, object displacements and camera projection, we will examine each particular value, considering each parameter at a time. A particular model is obtained by combining several particular values.

3.1. Particular cases of intrinsic parameters

Authors generally make several hypotheses regarding intrinsic parameters. For example, the most general auto-calibration hypothesis states that the intrinsic parameters are constant. They can be known or unknown. However, usually, some parameters are constant while others are not.

- The principal point of coordinates (u_0, v_0) can be fixed and/or known in some cases (e.g. in the image center), thus changing the reference frame, regarding the principal point position.
- The γ parameter is usually assumed to be null or, at least, considered to be a constant value.
- Enciso [6] has experimentally proven that for a large number of cameras α_u/α_v can be considered to be constant even if other intrinsic parameters change. We express this as $f = \alpha_u = \alpha_v$.

Table 1 summarizes, for each intrinsic parameter, the particular cases of interest (constant values are indexed by zero). Subsequently, we will refer to each case by the label given in the first column. For example, `g1` means that the γ parameter is null.

Table 1
Table of particular cases of intrinsic parameters for two frames

g1	$\gamma = 0$	γ constant and null
g2	$\gamma = \gamma_0$	γ constant
g3	$\gamma = \gamma(\tau)$	γ free
s1	$\alpha_u = \alpha_v(\tau)$	α_u/α_v constant and known
s2	$\alpha_u = \alpha_u(\tau)$	α_u free
f1	$\alpha_v = 1$	α_v constant and known
f2	$\alpha_v = f_0$	α_v constant
f3	$\alpha_v = \alpha_v(\tau)$	α_v free
c1	$u_0 = v_0 = 0$	u_0 and v_0 constant and known
c2	$u_0 = u_{0_0}$ and $v_0 = v_{0_0}$	u_0 and v_0 constant
c3	$u_0 = u_0(\tau)$ and $v_0 = v_0(\tau)$	u_0 and v_0 free

3.2. Particular cases of displacement

3.2.1. Discrete motion–continuous motion

In an image sequence, if the displacement between two frames is small, we can approximate the rotation equations by their first order:

$$\mathbf{R} = \mathrm{e}^{\tilde{\mathbf{r}}} = \mathbf{I} + \tilde{\mathbf{r}} + \mathrm{o}(\tilde{\mathbf{r}})$$

which occurs frequently in images sequences except with high speed objects.

If the motion is larger, we can also consider the second order expansion

$$\mathbf{R} = \mathbf{I} + \tilde{\mathbf{r}} + \tfrac{1}{2}\tilde{\mathbf{r}}^2 + \mathrm{o}(\tilde{\mathbf{r}}^2).$$

3.2.2. About extrinsic parameters

The rotation parameters are related to the rotation axis and the rotation angle by $\mathbf{r} = 2\tan(\theta/2)\mathbf{u}$, where $\mathbf{u}$ is a unitary vector giving the direction of the rotation axis.

Some components of $\mathbf{u}$ can be known or null. Some value of θ may yield singularities; $\theta = \pi/4$ and the rotation axis is parallel to the translation vector for a screw displacement.

Some robotic systems give precise values of the robot displacements (angle, axis, translation). Some values may be known (we denote by $\_\theta_0$ a constant and known value of a parameter θ). Other informations regarding parallelism or orthogonality to a known direction or to an other vector may also be available:

- the rotation axis is orthogonal to the translation plane (e.g. planar motion):

$$\mathbf{r} \perp \mathbf{t} \Leftrightarrow \mathbf{r}.\mathbf{t} = 0,$$

- screw displacement:

$$\mathbf{r} \| \mathbf{t} \Leftrightarrow \exists\kappa/\mathbf{r} = \kappa\mathbf{t}.$$

3.2.3. All constraints on motion

All these constraints, also called “atomic particular cases”, have simple expressions that can be easily combined. In this purpose, we use the fact that $\mathbf{u}$ is a unitary vector and that, for monocular systems, the norm of translation cannot be recovered. To parameterize these vectors with only two parameters, we divide each component by a non-zero component. Then, the dot product and scalar product induce linear relations. For example, $t_2 = 1$ and $\mathbf{t} \perp \mathbf{r}$ are equivalent to $t_0u_0 + t_1u_1 + u_2 = 0 \Rightarrow u_2 = -t_0u_0 - t_1u_1$. All cases are collected in Table 2.

3.2.4. Generating all cases

All particular cases, each called a “molecular case”, are generated by combining the atomic cases and solv-

Table 2
Table of particular cases of displacements

u1	$u_0 = u_2 = 0,\ u_1 = 1$	Rotation axis ∥ y-axis	R1	$\mathbf{R} = \mathbf{I}$	Null rotation
u2	$u_0 = 0,\ u_1 = 1$	Rotation axis ⊥ x-axis	R2	$\mathbf{R} = \mathbf{I} + \tilde{\mathbf{r}}$	First order
u3	$u_2 = 0,\ u_1 = 1$	Rotation axis ⊥ z-axis	R3	$\mathbf{R} = \mathbf{I} + \tilde{\mathbf{r}} + \frac{1}{2}\tilde{\mathbf{r}}^2$	Second order
u4	$u_1 = 1$	General case	R4	$\mathbf{R} = \mathbf{I} + (\tilde{\mathbf{r}} + 1/2\tilde{\mathbf{r}}^2)/(1 + \mathbf{r}^{\mathrm{T}}\mathbf{r}/4)$	General case
u5	$u_0 = u_2 = 0,\ u_1 = -1$	Rotation axis ∥ y-axis			
u6	$u_0 = 0,\ u_1 = -1$	Rotation axis ⊥ x-axis	a1	$\theta = \pi/2$	Quarter turn
u7	$u_2 = 0,\ u_1 = -1$	Rotation axis ⊥ z-axis	a2	θ	Free angle
u8	$u_1 = -1$	General case			
u9	$u_0 = u_1 = 0,\ u_2 = 1$	Rotation axis ∥ z-axis	t1	$t_1 = t_2 = 0,\ t_0 = 1$	Translation ∥ x-axis
u10	$u_0 = 0,\ u_2 = 1$	Rotation axis ⊥ x-axis	t2	$t_1 = 0,\ t_0 = 1$	Translation ⊥ y-axis
u11	$u_1 = 0,\ u_2 = 1$	Rotation axis ⊥ y-axis	t3	$t_2 = 0,\ t_0 = 1$	Translation ⊥ z-axis
u12	$u_2 = 1$	General case	t4	$t_0 = 1$	General translation
u13	$u_0 = u_1 = 0,\ u_2 = -1$	Rotation axis ∥ z-axis	t5	$t_0 = t_2 = 0,\ t_1 = 1$	Translation ∥ y-axis
u14	$u_0 = 0,\ u_2 = -1$	Rotation axis ⊥ x-axis	t6	$t_0 = 0,\ t_1 = 1$	Translation ⊥ x-axis
u15	$u_1 = 0,\ u_2 = -1$	Rotation axis ⊥ y-axis	t7	$t_2 = 0,\ t_1 = 1$	Translation ⊥ z-axis
u16	$u_2 = -1$	General case	t8	$t_1 = 1$	General translation
u17	$u_1 = u_2 = 0,\ u_0 = 1$	Rotation axis ∥ x-axis	t9	$t_0 = t_1 = 0,\ t_2 = 1$	Translation ∥ z-axis
u18	$u_1 = 0,\ u_0 = 1$	Rotation axis ⊥ y-axis	t10	$t_0 = 0,\ t_2 = 1$	Translation ⊥ x-axis
u19	$u_2 = 0,\ u_0 = 1$	Rotation axis ⊥ z-axis	t11	$t_1 = 0,\ t_2 = 1$	Translation ⊥ y-axis
u20	$u_0 = 1$	General case	t12	$t_2 = 1$	General translation
u21	$u_1 = u_2 = 0,\ u_0 = -1$	Rotation axis ∥ x-axis			
u22	$u_1 = 0,\ u_0 = -1$	Rotation axis ⊥ y-axis	Z1	$\mathbf{t} \cdot \mathbf{u} = 0$	Translation ⊥ rotation axis
u23	$u_2 = 0,\ u_0 = -1$	Rotation axis ⊥ z-axis	Z2	$\mathbf{t} \wedge \mathbf{u} = 0$	Screw displacement
u24	$u_0 = -1$	General case	Z3		No relation

ing the constraints by a substitution.[1] A molecular case is composed of one case in each family, a family being named by a letter (g, s, f or c for projection as seen in Table 1 and u, R, a, t or Z for motion as seen in Table 2). Thus, a molecular case is identified by the sequence:

g[1–3]f[1–3]s[1–3]c[1–3]R[1–4]a[1–2]
  u[1–24]t[1–12]Z[1–3],

where g[1–3] means "one atomic case among g1, g2 and g3".

3.2.5. *How many cases do we have?*

If we look at the expression of the particular above-mentioned cases, we obtain 6×10^6 particular cases. However, this is not the real number because of the incompatibility of some atomic cases and the redundancy of some constraints. Two different sets of atomic constraints can generate the same simplified model.

It is easy to eliminate incompatible constraints. It is not possible to deal with redundant constraints, because this requires to compare each set of combined constraints with all others in order to determine the similarity. The complexity of this process is $O(n^2)$.

Although we cannot remove redundant cases, we propose an adapted strategy to deal with the large number of cases. The idea of this paper is: (i) to eliminate some of the redundant cases by using some considerations on the atomic cases and (ii) to limit the number of cases by studying the particular forms of the matrices.

3.2.6. *Reducing the number of cases*

Some redundancy are obvious

- In case (R1), one case of axis and angle is considered.
- In cases (R2) and (R3), we do not consider (a1) when θ is equal to $\pi/2$.
- The case (a1) is only considered if $\mathbf{r} \| \mathbf{t}$, (Z2).

[1] This was done using Maple software for symbolic computations.

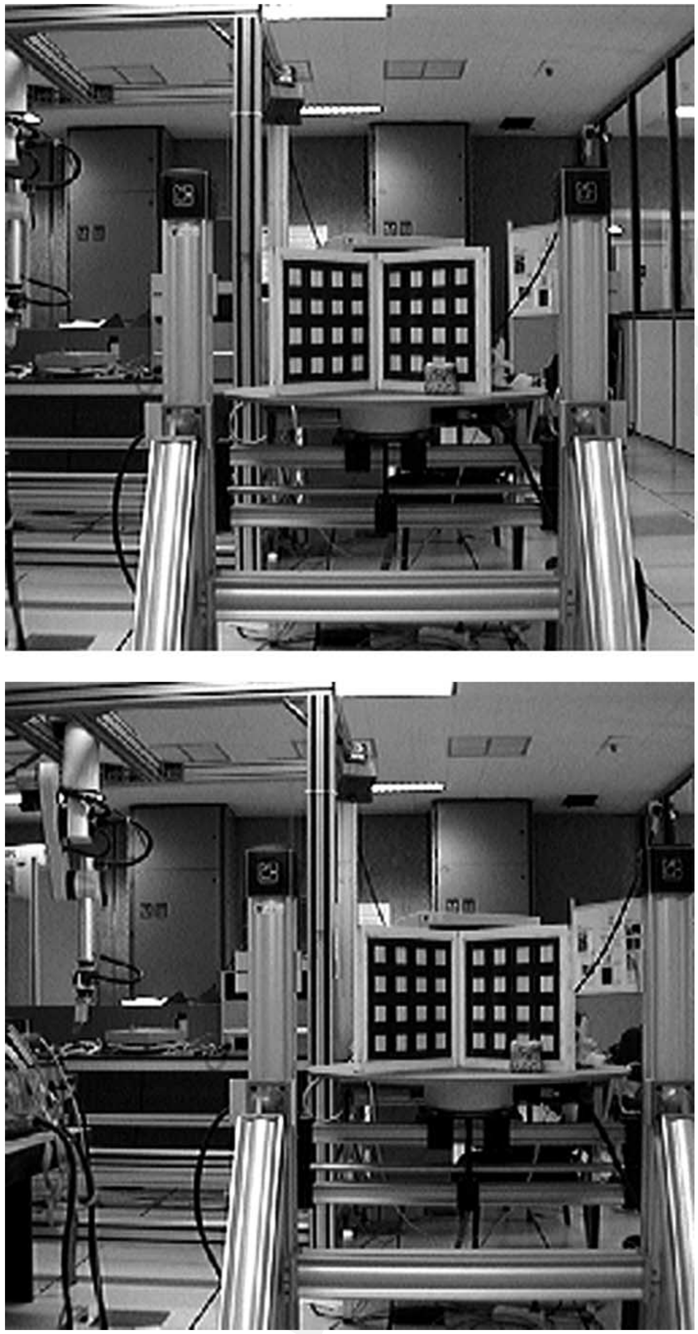

Fig. 1. Images for x-axis translation, small pan rotation and auto-focus.

This reduces the amount of cases of fundamental relations to only 216 756 cases.

4. Forms of fundamental matrices

We have significantly reduced the number of cases but this is not small enough to be computationally tractable. We now split fundamental relations in sets of matrices by forms. The matrix form is determined using simple rules in order to obtain a very simple parameterization. We consider (3×3) matrices having nine parameters (coefficients). If a coefficient is equal to zero, then there is one less parameter. If a coefficient has the same expression or is opposite to another, there is one less parameter again. These operations are very simple and can be rapidly computed in each case. Furthermore, we know that a fundamental matrix is defined up to a scale factor, and that its determinant is fixed to 0 (removing in most cases one parameter). This process reduces the 216 756 cases to only 188 subgroups.

The table in Appendix A shows all the simplified forms obtained, and for each form, an example of case that has generated it. This table will be useful for people who want to implement the algorithm.

5. Experiments

We have recorded several video sequences for which the camera displacement induces a fundamental relation between image points $\mathbf{m}_1$ and $\mathbf{m}_2$. From each particular matrix form, we have estimated the fundamental matrix parameters with the robust least median square method in order to minimize the distance between a 2D point $\mathbf{m}_1$ and its epipolar line $\mathbf{F}\mathbf{m}_2$. To deal with cases with different degrees of freedom, we use an appropriate Akaike criterion [1].

For each recorded video sequence, we have verified that the model with the minimal residual error effectively corresponds to the displacement performed by the robotic system. We present one experiment in Fig. 1 for which the camera has performed a small pan rotation followed by a translation parallel to the x-axis. The auto-focus was also enabled. The case with the minimal residual error corresponds to the fundamental matrix form number **59** in the table given in Appendix A

$$\mathbf{F} = \begin{pmatrix} 0 & 0 & 0 \\ x_0 & x_1 & x_2 \\ 0 & -x_2 & x_3 \end{pmatrix}.$$

This particular form was obtained from cases where the rotation was approximated to its first and second order, the translation is parallel to the x-axis, the rotation axis is orthogonal to the optical axis and the intrinsic parameters are free.

6. Conclusion

In an earlier study on homographic matrices [11], we have shown that it is possible to reduce the amount of particular cases in order to make the case selection computationally feasible. In this paper, we have shown that a similar result can be obtained with fundamental matrices using redundancies. We have experimentally confirmed that our system is able to automatically select the case corresponding to the performed displacement.

The applications are twofold: (i) an incremental reconstruction of the scene and (ii) the segmentation of objects moving with different displacements or with different geometric properties in video sequences.

This work has also been extended to motion estimation of human head inside MRI scanner, improving the registration of fMRI volumes [12].

Appendix A. Table of particular forms of fundamental matrices

We denote by $\mathbf{n}^{\text{o}}$ the form number, by $\mathbf{p}$ the number of parameters (we have not taken into account the fact that the fundamental matrix is defined up to a scale factor and that $\det \mathbf{F} = 0$ but we do so in our implementation) and by $\mathbf{n}$ the number of molecular cases that have generated a form.

n°	**p**	Simplified form of fundamental matrix									For example generated by	**n**
1	1	[0	0	0	0	0	x_6	0	$-x_6$	0]	g1f1s1c1t1R1u24Z3a2	24
2	1	[0	0	x_3	0	0	0	$-x_3$	0	0]	g1f1s1c1t5R1u24Z3a2	4
3	1	[0	x_2	0	$-x_2$	0	0	0	0	0]	g1f1s1c1t9R1u24Z3a2	5
4	2	[0	0	0	0	0	x_6	0	$-x_6$	x_9]	g1f1s1c3t1R1u24Z3a2	12
5	2	[0	0	0	0	0	x_6	0	x_8	0]	g1f3s1c1t1R1u24Z3a2	6
6	2	[0	0	0	0	0	x_6	x_7	$-x_6$	0]	g1f1s1c1t1R2u13Z2a2	16
7	2	[0	0	0	0	x_5	x_6	0	$-x_6$	x_5]	g1f1s1c1t1R2u17Z1a2	396
8	2	[0	0	0	x_4	0	x_6	0	$-x_6$	0]	g1f1s1c1t1R2u1Z2a2	16
9	2	[0	0	x_3	0	0	0	$-x_3$	0	x_9]	g1f1s1c3t5R1u24Z3a2	2
10	2	[0	0	x_3	0	0	0	$-x_3$	x_8	0]	g1f1s1c1t5R2u13Z2a2	8
11	2	[0	0	x_3	0	0	0	x_7	0	0]	g1f1s2c1t5R1u24Z3a2	4
12	2	[0	0	x_3	0	0	x_6	$-x_3$	$-x_6$	0]	g1f1s1c1t3R1u24Z3a2	17
13	2	[0	x_2	0	$-x_2$	0	x_6	0	$-x_6$	0]	g1f1s1c1t11R1u24Z3a2	8
14	2	[0	x_2	0	$-x_2$	0	x_6	0	0	0]	g1f1s1c1t9R2u1Z2a2	24
15	2	[0	x_2	0	$-x_2$	x_5	0	0	0	0]	g2f3s1c1t9R1u24Z3a2	4
16	2	[0	x_2	0	x_4	0	0	0	0	0]	g1f1s2c1t9R1u24Z3a2	3
17	2	[0	x_2	x_3	$-x_2$	0	0	$-x_3$	0	0]	g1f1s1c1t10R1u24Z3a2	4
18	2	[0	x_2	x_3	$-x_2$	0	0	0	0	0]	g1f1s1c1t9R2u17Z2a2	12
19	2	[0	x_2	x_3	0	0	0	$-x_3$	0	0]	g1f1s1c1t5R2u17Z2a2	8
20	2	[x_1	0	x_3	0	0	0	$-x_3$	0	x_1]	g1f1s1c1t5R2u1Z1a2	66
21	2	[x_1	x_2	0	$-x_2$	x_1	0	0	0	0]	g1f1s1c1t10R2u11Z1a2	198
22	3	[0	0	0	0	0	x_6	0	x_8	x_9]	g1f3s1c2t1R1u24Z3a2	12
23	3	[0	0	0	0	0	x_6	x_7	$-x_6$	x_9]	g1f1s1c2t1R2u13Z2a2	32
24	3	[0	0	0	0	0	x_6	x_7	x_8	0]	g1f1s1c1t1R3u13Z2a2	200
25	3	[0	0	0	0	x_5	x_6	0	$-x_6$	x_9]	g1f2s1c1t1R2u17Z1a2	396
26	3	[0	0	0	0	x_5	x_6	x_7	$-x_6$	x_5]	g1f1s1c1t1R2u11Z2a2	16
27	3	[0	0	0	x_4	0	x_6	0	x_8	0]	g1f1s1c1t1R3u1Z2a2	56
28	3	[0	0	0	x_4	0	x_6	x_7	$-x_6$	0]	g1f1s1c1t1R2u10Z2a2	32
29	3	[0	0	0	x_4	x_5	x_6	0	$-x_6$	0]	g2f1s1c1t1R2u1Z2a2	32
30	3	[0	0	0	x_4	x_5	x_6	0	$-x_6$	x_5]	g1f1s1c1t1R2u19Z2a2	16
31	3	[0	0	x_3	0	0	0	$-x_3$	x_8	x_9]	g1f1s1c2t5R2u13Z2a2	16
32	3	[0	0	x_3	0	0	0	x_7	0	x_9]	g1f1s2c2t5R1u24Z3a2	8
33	3	[0	0	x_3	0	0	0	x_7	x_8	0]	g1f1s1c1t5R3u13Z2a2	64
34	3	[0	0	x_3	0	0	x_6	$-x_3$	$-x_6$	x_9]	g1f1s1c3t3R1u24Z3a2	13
35	3	[0	0	x_3	0	0	x_6	$-x_3$	x_8	0]	g2f1s1c1t5R2u13Z2a2	22
36	3	[0	0	x_3	0	0	x_6	x_7	$-x_6$	0]	g1f1s2c1t3R1u24Z3a2	4
37	3	[0	x_2	0	$-x_2$	0	x_6	0	x_8	0]	g1f3s1c1t11R1u24Z3a2	2
38	3	[0	x_2	0	$-x_2$	x_5	x_6	0	$-x_6$	0]	g3f1s1c1t11R1u24Z3a2	4
39	3	[0	x_2	0	$-x_2$	x_5	x_6	0	0	0]	g2f3s1c1t9R2u1Z2a2	12
40	3	[0	x_2	0	x_4	0	x_6	0	$-x_6$	0]	g1f1s2c1t11R1u24Z3a2	4
41	3	[0	x_2	0	x_4	0	x_6	0	0	0]	g1f1s1c1t9R3u1Z2a2	60
42	3	[0	x_2	0	x_4	x_5	0	0	0	0]	g2f1s2c1t9R1u24Z3a2	6
43	3	[0	x_2	x_3	$-x_2$	0	0	x_7	0	0]	g1f3s1c1t10R1u24Z3a2	2
44	3	[0	x_2	x_3	$-x_2$	0	x_6	$-x_3$	$-x_6$	0]	g1f1s1c1t12R1u24Z3a2	40
45	3	[0	x_2	x_3	$-x_2$	0	x_6	0	0	0]	g1f1s1c1t9R2u19Z2a2	60
46	3	[0	x_2	x_3	0	0	0	$-x_3$	x_8	0]	g1f1s1c1t5R2u11Z2a2	16

Appendix A (*Continued*)

n°	p	Simplified form of fundamental matrix	For example generated by	n
47	3	$[0\quad x_2\quad x_3\quad 0\quad 0\quad 0\quad x_7\quad 0\quad 0]$	g1f1s1c1t5R3u17Z2a2	64
48	3	$[0\quad x_2\quad x_3\quad x_4\quad 0\quad 0\quad 0\quad 0\quad 0]$	g1f1s1c1t9R3u17Z2a2	60
49	3	$[x_1\quad 0\quad x_3\quad 0\quad 0\quad 0\quad -x_3\quad 0\quad x_9]$	g1f2s1c1t5R2u1Z1a2	66
50	3	$[x_1\quad 0\quad x_3\quad 0\quad 0\quad 0\quad -x_3\quad x_8\quad x_1]$	g1f1s1c1t5R2u10Z2a2	8
51	3	$[x_1\quad x_2\quad 0\quad -x_2\quad x_1\quad x_6\quad 0\quad 0\quad 0]$	g1f1s1c1t9R2u10Z2a2	24
52	3	$[x_1\quad x_2\quad x_3\quad -x_2\quad x_1\quad 0\quad 0\quad 0\quad 0]$	g1f1s1c1t9R2u11Z2a2	24
53	3	$[x_1\quad x_2\quad x_3\quad 0\quad 0\quad 0\quad -x_3\quad 0\quad x_1]$	g1f1s1c1t5R2u19Z2a2	8
54	4	$[0\quad 0\quad 0\quad 0\quad 0\quad x_6\quad x_7\quad x_8\quad x_9]$	g1f1s1c2t1R3u13Z2a2	400
55	4	$[0\quad 0\quad 0\quad 0\quad x_5\quad x_6\quad 0\quad x_8\quad x_9]$	g1f1s1c2t1R2u17Z1a2	2772
56	4	$[0\quad 0\quad 0\quad 0\quad x_5\quad x_6\quad x_7\quad -x_6\quad x_9]$	g1f2s1c1t1R2u11Z2a2	16
57	4	$[0\quad 0\quad 0\quad 0\quad x_5\quad x_6\quad x_7\quad x_8\quad x_5]$	g2f1s1c1t1R2u11Z2a2	32
58	4	$[0\quad 0\quad 0\quad x_4\quad 0\quad x_6\quad x_7\quad x_8\quad 0]$	g1f3s1c1t1R2u10Z2a2	16
59	4	$[0\quad 0\quad 0\quad x_4\quad x_5\quad x_6\quad 0\quad -x_6\quad x_9]$	g1f2s1c1t1R2u19Z2a2	80
60	4	$[0\quad 0\quad 0\quad x_4\quad x_5\quad x_6\quad 0\quad x_8\quad 0]$	g2f1s1c1t1R3u1Z2a2	112
61	4	$[0\quad 0\quad 0\quad x_4\quad x_5\quad x_6\quad x_7\quad -x_6\quad x_5]$	g1f1s1c1t1R2u12Z2a2	24
62	4	$[0\quad 0\quad x_3\quad 0\quad 0\quad 0\quad x_7\quad x_8\quad x_9]$	g1f1s1c2t5R3u13Z2a2	128
63	4	$[0\quad 0\quad x_3\quad 0\quad 0\quad x_6\quad -x_3\quad x_8\quad x_9]$	g2f1s1c2t5R2u13Z2a2	44
64	4	$[0\quad 0\quad x_3\quad 0\quad 0\quad x_6\quad x_7\quad -x_6\quad x_9]$	g1f1s2c2t3R1u24Z3a2	8
65	4	$[0\quad 0\quad x_3\quad 0\quad 0\quad x_6\quad x_7\quad x_8\quad 0]$	g1f1s1c1t3R2u13Z2a2	588
66	4	$[0\quad x_2\quad 0\quad -x_2\quad x_5\quad x_6\quad 0\quad x_8\quad 0]$	g2f3s1c1t11R1u24Z3a2	4
67	4	$[0\quad x_2\quad 0\quad x_4\quad 0\quad x_6\quad 0\quad x_8\quad 0]$	g1f1s1c1t11R2u1Z2a2	146
68	4	$[0\quad x_2\quad 0\quad x_4\quad x_5\quad x_6\quad 0\quad -x_6\quad 0]$	g2f1s2c1t11R1u24Z3a2	8
69	4	$[0\quad x_2\quad 0\quad x_4\quad x_5\quad x_6\quad 0\quad 0\quad 0]$	g2f1s1c1t9R3u1Z2a2	120
70	4	$[0\quad x_2\quad x_3\quad -x_2\quad 0\quad x_6\quad -x_3\quad -x_6\quad x_9]$	g1f3s1c2t9R1u24Z3a2	9
71	4	$[0\quad x_2\quad x_3\quad -x_2\quad x_5\quad x_6\quad 0\quad -x_6\quad x_5]$	g1f1s1c1t11R2u17Z2a2	8
72	4	$[0\quad x_2\quad x_3\quad -x_2\quad x_5\quad x_6\quad 0\quad 0\quad 0]$	g2f3s1c1t9R2u17Z2a2	36
73	4	$[0\quad x_2\quad x_3\quad 0\quad 0\quad 0\quad x_7\quad x_8\quad 0]$	g1f1s2c1t5R2u11Z2a2	32
74	4	$[0\quad x_2\quad x_3\quad 0\quad x_5\quad x_6\quad -x_3\quad -x_6\quad 0]$	g2f1s1c1t5R2u17Z2a2	12
75	4	$[0\quad x_2\quad x_3\quad 0\quad x_5\quad x_6\quad -x_3\quad -x_6\quad x_5]$	g1f1s1c1t3R2u17Z2a2	8
76	4	$[0\quad x_2\quad x_3\quad x_4\quad 0\quad 0\quad x_7\quad 0\quad 0]$	g1f1s1c1t10R2u17Z2a2	150
77	4	$[0\quad x_2\quad x_3\quad x_4\quad 0\quad x_6\quad 0\quad 0\quad 0]$	g1f1s2c1t9R2u19Z2a2	24
78	4	$[x_1\quad 0\quad x_3\quad 0\quad 0\quad 0\quad -x_3\quad x_8\quad x_9]$	g1f2s1c1t5R2u10Z2a2	8
79	4	$[x_1\quad 0\quad x_3\quad 0\quad 0\quad 0\quad x_7\quad 0\quad x_9]$	g1f1s1c2t5R2u1Z1a2	1056
80	4	$[x_1\quad 0\quad x_3\quad x_4\quad 0\quad x_6\quad -x_3\quad -x_6\quad x_1]$	g1f1s1c1t3R2u1Z2a2	8
81	4	$[x_1\quad x_2\quad 0\quad -x_2\quad x_1\quad x_6\quad x_7\quad -x_6\quad 0]$	g1f1s1c1t11R2u13Z2a2	16
82	4	$[x_1\quad x_2\quad 0\quad x_4\quad x_5\quad 0\quad 0\quad 0\quad 0]$	g1f1s2c1t10R2u11Z1a2	990
83	4	$[x_1\quad x_2\quad x_3\quad -x_2\quad 0\quad x_6\quad -x_3\quad 0\quad x_1]$	g1f1s1c1t10R2u1Z2a2	8
84	4	$[x_1\quad x_2\quad x_3\quad -x_2\quad x_1\quad 0\quad -x_3\quad x_8\quad 0]$	g1f1s1c1t10R2u13Z2a2	16
85	4	$[x_1\quad x_2\quad x_3\quad -x_2\quad x_1\quad x_6\quad 0\quad 0\quad 0]$	g1f1s1c1t9R2u12Z2a2	36
86	4	$[x_1\quad x_2\quad x_3\quad 0\quad 0\quad 0\quad -x_3\quad 0\quad x_9]$	g1f2s1c1t5R2u19Z2a2	8
87	4	$[x_1\quad x_2\quad x_3\quad 0\quad 0\quad 0\quad -x_3\quad x_8\quad x_1]$	g1f1s1c1t5R2u12Z2a2	12
88	5	$[0\quad 0\quad 0\quad 0\quad x_5\quad x_6\quad x_7\quad x_8\quad x_9]$	g1f1s1c2t1R2u11Z2a2	368
89	5	$[0\quad 0\quad 0\quad x_4\quad 0\quad x_6\quad x_7\quad x_8\quad x_9]$	g1f1s1c2t1R2u10Z2a2	240
90	5	$[0\quad 0\quad 0\quad x_4\quad x_5\quad x_6\quad 0\quad x_8\quad x_9]$	g1f3s1c1t1R2u19Z2a2	48

Appendix A (*Continued*)

n°	**p**	Simplified form of fundamental matrix	For example generated by	**n**
91	5	$[0\ \ 0\ \ 0\ \ x_4\ \ x_5\ \ x_6\ \ x_7\ \ -x_6\ \ x_9]$	g1f2s1c1t1R2u12Z2a2	24
92	5	$[0\ \ 0\ \ 0\ \ x_4\ \ x_5\ \ x_6\ \ x_7\ \ x_8\ \ -x_5]$	g1f1s1c1t1R3u10Z2a2	32
93	5	$[0\ \ 0\ \ 0\ \ x_4\ \ x_5\ \ x_6\ \ x_7\ \ x_8\ \ 0]$	g2f1s1c1t1R2u10Z2a2	96
94	5	$[0\ \ 0\ \ 0\ \ x_4\ \ x_5\ \ x_6\ \ x_7\ \ x_8\ \ x_5]$	g1f1s1c1t1R3u11Z2a2	64
95	5	$[0\ \ 0\ \ x_3\ \ 0\ \ 0\ \ x_6\ \ x_7\ \ x_8\ \ x_9]$	g1f1s1c2t3R2u13Z2a2	1176
96	5	$[0\ \ x_2\ \ 0\ \ x_4\ \ x_5\ \ x_6\ \ 0\ \ x_8\ \ 0]$	g2f1s1c1t11R2u1Z2a2	292
97	5	$[0\ \ x_2\ \ x_3\ \ -x_2\ \ 0\ \ x_6\ \ -x_3\ \ x_8\ \ x_9]$	g1f1s1c2t9R2u1Z2a2	26
98	5	$[0\ \ x_2\ \ x_3\ \ -x_2\ \ 0\ \ x_6\ \ x_7\ \ -x_6\ \ x_9]$	g1f1s1c2t9R2u17Z2a2	14
99	5	$[0\ \ x_2\ \ x_3\ \ -x_2\ \ 0\ \ x_6\ \ x_7\ \ x_8\ \ 0]$	g1f3s1c1t12R1u24Z3a2	3
100	5	$[0\ \ x_2\ \ x_3\ \ -x_2\ \ x_5\ \ x_6\ \ -x_3\ \ x_8\ \ 0]$	g3f1s1c1t10R1u24Z3a2	10
101	5	$[0\ \ x_2\ \ x_3\ \ -x_2\ \ x_5\ \ x_6\ \ 0\ \ -x_6\ \ x_9]$	g1f2s1c1t11R2u17Z2a2	8
102	5	$[0\ \ x_2\ \ x_3\ \ -x_2\ \ x_5\ \ x_6\ \ 0\ \ x_8\ \ x_5]$	g2f1s1c1t11R2u17Z2a2	12
103	5	$[0\ \ x_2\ \ x_3\ \ 0\ \ 0\ \ 0\ \ x_7\ \ x_8\ \ x_9]$	g1f1s1c2t5R2u11Z2a2	240
104	5	$[0\ \ x_2\ \ x_3\ \ 0\ \ x_5\ \ x_6\ \ -x_3\ \ -x_6\ \ x_9]$	g1f2s1c1t3R2u17Z2a2	32
105	5	$[0\ \ x_2\ \ x_3\ \ 0\ \ x_5\ \ x_6\ \ -x_3\ \ x_8\ \ 0]$	g2f1s1c1t5R2u11Z2a2	36
106	5	$[0\ \ x_2\ \ x_3\ \ 0\ \ x_5\ \ x_6\ \ x_7\ \ -x_6\ \ x_5]$	g1f1s1c1t3R3u17Z2a2	40
107	5	$[0\ \ x_2\ \ x_3\ \ x_4\ \ 0\ \ x_6\ \ x_7\ \ -x_6\ \ 0]$	g1f1s2c1t12R1u24Z3a2	6
108	5	$[0\ \ x_2\ \ x_3\ \ x_4\ \ x_5\ \ x_6\ \ 0\ \ -x_6\ \ x_5]$	g1f1s1c1t11R3u17Z2a2	40
109	5	$[0\ \ x_2\ \ x_3\ \ x_4\ \ x_5\ \ x_6\ \ 0\ \ 0\ \ 0]$	g2f1s1c1t9R3u17Z2a2	168
110	5	$[x_1\ \ 0\ \ x_3\ \ 0\ \ 0\ \ 0\ \ x_7\ \ x_8\ \ x_9]$	g1f1s1c2t5R2u10Z2a2	128
111	5	$[x_1\ \ 0\ \ x_3\ \ x_4\ \ 0\ \ x_6\ \ -x_3\ \ -x_6\ \ x_9]$	g1f2s1c1t3R2u1Z2a2	8
112	5	$[x_1\ \ 0\ \ x_3\ \ x_4\ \ 0\ \ x_6\ \ -x_3\ \ x_8\ \ x_1]$	g1f1s1c1t3R3u1Z2a2	16
113	5	$[x_1\ \ x_2\ \ 0\ \ -x_2\ \ x_1\ \ x_6\ \ x_7\ \ x_8\ \ 0]$	g1f1s1c1t11R3u13Z2a2	56
114	5	$[x_1\ \ x_2\ \ 0\ \ x_4\ \ x_5\ \ x_6\ \ 0\ \ 0\ \ 0]$	g1f1s2c1t9R2u10Z2a2	120
115	5	$[x_1\ \ x_2\ \ x_3\ \ -x_2\ \ 0\ \ x_6\ \ -x_3\ \ 0\ \ x_9]$	g1f2s1c1t10R2u1Z2a2	8
116	5	$[x_1\ \ x_2\ \ x_3\ \ -x_2\ \ x_1\ \ 0\ \ x_7\ \ x_8\ \ 0]$	g1f1s1c1t10R3u13Z2a2	56
117	5	$[x_1\ \ x_2\ \ x_3\ \ 0\ \ 0\ \ 0\ \ -x_3\ \ x_8\ \ x_9]$	g1f2s1c1t5R2u12Z2a2	12
118	5	$[x_1\ \ x_2\ \ x_3\ \ 0\ \ 0\ \ 0\ \ x_7\ \ 0\ \ x_9]$	g1f1s2c1t5R2u19Z2a2	32
119	5	$[x_1\ \ x_2\ \ x_3\ \ 0\ \ 0\ \ 0\ \ x_7\ \ x_8\ \ -x_1]$	g1f1s1c1t5R3u11Z2a2	16
120	5	$[x_1\ \ x_2\ \ x_3\ \ 0\ \ 0\ \ 0\ \ x_7\ \ x_8\ \ x_1]$	g1f1s1c1t5R3u10Z2a2	32
121	5	$[x_1\ \ x_2\ \ x_3\ \ x_2\ \ x_5\ \ x_6\ \ -x_3\ \ -x_6\ \ x_1]$	g2f1s1c1t5R2u1Z1a2	70
122	5	$[x_1\ \ x_2\ \ x_3\ \ x_4\ \ -x_1\ \ x_6\ \ 0\ \ 0\ \ 0]$	g1f1s1c1t9R3u19Z2a2	48
123	5	$[x_1\ \ x_2\ \ x_3\ \ x_4\ \ 0\ \ x_6\ \ -x_3\ \ 0\ \ x_1]$	g1f1s1c1t10R3u1Z2a2	16
124	5	$[x_1\ \ x_2\ \ x_3\ \ x_4\ \ x_1\ \ x_6\ \ 0\ \ 0\ \ 0]$	g1f1s1c1t9R3u10Z2a2	96
125	5	$[x_1\ \ x_2\ \ x_3\ \ x_4\ \ x_5\ \ 0\ \ 0\ \ 0\ \ 0]$	g1f1s2c1t9R2u11Z2a2	24
126	6	$[0\ \ 0\ \ 0\ \ x_4\ \ x_5\ \ x_6\ \ x_7\ \ x_8\ \ x_9]$	g1f1s1c1t1R3u12Z2a2	5160
127	6	$[0\ \ x_2\ \ x_3\ \ -x_2\ \ 0\ \ x_6\ \ x_7\ \ x_8\ \ x_9]$	g1f1s1c2t9R2u19Z2a2	199
128	6	$[0\ \ x_2\ \ x_3\ \ -x_2\ \ x_5\ \ x_6\ \ -x_3\ \ x_8\ \ x_9]$	g2f3s1c2t11R1u24Z3a2	34
129	6	$[0\ \ x_2\ \ x_3\ \ -x_2\ \ x_5\ \ x_6\ \ 0\ \ x_8\ \ x_9]$	g1f3s1c1t11R2u17Z2a2	44
130	6	$[0\ \ x_2\ \ x_3\ \ -x_2\ \ x_5\ \ x_6\ \ x_7\ \ x_8\ \ 0]$	g2f3s1c1t10R1u24Z3a2	10
131	6	$[0\ \ x_2\ \ x_3\ \ 0\ \ x_5\ \ x_6\ \ -x_3\ \ x_8\ \ x_9]$	g3f1s1c1t3R2u17Z2a2	8
132	6	$[0\ \ x_2\ \ x_3\ \ 0\ \ x_5\ \ x_6\ \ x_7\ \ -x_6\ \ x_9]$	g1f2s1c1t3R3u17Z2a2	40
133	6	$[0\ \ x_2\ \ x_3\ \ 0\ \ x_5\ \ x_6\ \ x_7\ \ x_8\ \ 0]$	g2f1s1c1t5R3u17Z2a2	192
134	6	$[0\ \ x_2\ \ x_3\ \ 0\ \ x_5\ \ x_6\ \ x_7\ \ x_8\ \ x_5]$	g1f1s1c1t3R2u11Z2a2	32

Appendix A (*Continued*)

n°	p	Simplified form of fundamental matrix									For example generated by	n
135	6	[0	x_2	x_3	x_4	0	x_6	x_7	x_8	0]	`g1f3s2c1t12R1u24Z3a2`	3
136	6	[0	x_2	x_3	x_4	x_5	x_6	0	$-x_6$	x_9]	`g1f2s1c1t11R3u17Z2a2`	40
137	6	[0	x_2	x_3	x_4	x_5	x_6	0	x_8	x_5]	`g1f1s1c1t11R2u19Z2a2`	32
138	6	[0	x_2	x_3	x_4	x_5	x_6	x_7	$-x_6$	x_5]	`g1f1s1c1t12R2u17Z2a2`	84
139	6	[x_1	0	x_3	x_4	0	x_6	$-x_3$	x_8	x_9]	`g1f2s1c1t3R3u1Z2a2`	16
140	6	[x_1	0	x_3	x_4	0	x_6	x_7	$-x_6$	x_9]	`g1f1s2c1t3R2u1Z2a2`	16
141	6	[x_1	0	x_3	x_4	0	x_6	x_7	$-x_6$	x_9]	`g1f2s2c1t6R2u5Z3a2`	16
142	6	[x_1	x_2	0	x_4	x_1	x_6	x_7	x_8	0]	`g1f1s1c1t11R2u10Z2a2`	48
143	6	[x_1	x_2	0	x_4	x_5	x_6	x_7	$-x_6$	0]	`g1f1s2c1t11R2u13Z2a2`	16
144	6	[x_1	x_2	x_3	$-x_2$	0	x_6	x_7	0	x_9]	`g1f3s1c1t10R2u1Z2a2`	8
145	6	[x_1	x_2	x_3	$-x_2$	x_1	x_6	x_7	x_8	0]	`g1f1s1c1t12R2u13Z2a2`	126
146	6	[x_1	x_2	x_3	$-x_2$	x_5	x_6	$-x_3$	x_6	x_9]	`g1f1s1c1t10R2u10Z1a2`	144
147	6	[x_1	x_2	x_3	$-x_2$	x_5	x_6	x_3	$-x_6$	x_9]	`g1f1s1c1t11R2u11Z1a2`	144
148	6	[x_1	x_2	x_3	0	0	0	x_7	x_8	x_9]	`g1f1s1c1t5R3u12Z2a2`	1536
149	6	[x_1	x_2	x_3	x_2	x_5	x_6	$-x_3$	$-x_6$	x_9]	`g1f1s1c1t3R2u19Z1a2`	358
150	6	[x_1	x_2	x_3	x_2	x_5	x_6	$-x_3$	x_8	x_1]	`g2f1s1c1t5R2u10Z2a2`	12
151	6	[x_1	x_2	x_3	x_4	0	x_6	$-x_3$	0	x_9]	`g1f2s1c1t10R3u1Z2a2`	16
152	6	[x_1	x_2	x_3	x_4	0	x_6	$-x_3$	x_8	x_1]	`g1f1s1c1t12R2u1Z2a2`	42
153	6	[x_1	x_2	x_3	x_4	0	x_6	x_7	0	x_1]	`g1f1s1c1t10R2u19Z2a2`	16
154	6	[x_1	x_2	x_3	x_4	x_1	0	x_7	x_8	0]	`g1f1s1c1t10R2u11Z2a2`	48
155	6	[x_1	x_2	x_3	x_4	x_5	x_6	$-x_3$	$-x_6$	x_1]	`g2f1s1c1t3R2u1Z2a2`	24
156	6	[x_1	x_2	x_3	x_4	x_5	x_6	0	0	0]	`g1f1s1c1t9R3u12Z2a2`	1428
157	7	[0	x_2	x_3	$-x_2$	x_5	x_6	x_7	x_8	x_9]	`g1f1s1c2t11R2u17Z2a2`	270
158	7	[0	x_2	x_3	0	x_5	x_6	x_7	x_8	x_9]	`g1f1s1c2t3R2u11Z2a2`	2480
159	7	[0	x_2	x_3	x_4	0	x_6	x_7	x_8	x_9]	`g1f1s1c2t10R2u17Z2a2`	912
160	7	[0	x_2	x_3	x_4	x_5	x_6	0	x_8	x_9]	`g1f2s1c1t11R2u19Z2a2`	536
161	7	[0	x_2	x_3	x_4	x_5	x_6	x_7	$-x_6$	x_9]	`g1f2s1c1t12R2u17Z2a2`	84
162	7	[0	x_2	x_3	x_4	x_5	x_6	x_7	x_8	0]	`g2f1s1c1t10R2u17Z2a2`	318
163	7	[x_1	0	x_3	x_4	0	x_6	x_7	x_8	x_9]	`g1f1s1c2t3R2u10Z2a2`	640
164	7	[x_1	x_2	0	x_4	x_5	x_6	x_7	x_8	0]	`g1f1s2c1t11R2u10Z2a2`	584
165	7	[x_1	x_2	x_3	$-x_2$	0	x_6	x_7	x_8	x_9]	`g1f1s1c2t10R2u1Z2a2`	48
166	7	[x_1	x_2	x_3	$-x_2$	x_1	x_6	x_7	x_8	x_9]	`g1f1s1c2t10R2u11Z1a2`	1104
167	7	[x_1	x_2	x_3	$-x_2$	x_5	x_6	$-x_3$	x_8	x_9]	`g1f1s1c1t10R2u10Z2a2`	32
168	7	[x_1	x_2	x_3	$-x_2$	x_5	x_6	x_7	$-x_6$	x_9]	`g1f1s1c1t11R2u11Z2a2`	32
169	7	[x_1	x_2	x_3	x_2	x_5	x_6	$-x_3$	x_8	x_9]	`g2f2s1c1t5R2u10Z2a2`	12
170	7	[x_1	x_2	x_3	x_4	0	x_6	$-x_3$	x_8	x_9]	`g1f2s1c1t12R2u1Z2a2`	42
171	7	[x_1	x_2	x_3	x_4	0	x_6	x_7	0	x_9]	`g1f1s2c1t10R2u19Z2a2`	168
172	7	[x_1	x_2	x_3	x_4	x_5	0	x_7	x_8	0]	`g1f1s2c1t10R2u11Z2a2`	120
173	7	[x_1	x_2	x_3	x_4	x_5	x_6	$-x_3$	$-x_6$	x_9]	`g1f1s1c1t3R2u19Z2a2`	104
174	7	[x_1	x_2	x_3	x_4	x_5	x_6	$-x_3$	x_8	0]	`g2f1s1c1t10R2u13Z2a2`	32
175	7	[x_1	x_2	x_3	x_4	x_5	x_6	$-x_3$	x_8	x_1]	`g2f1s1c1t10R2u1Z2a2`	262
176	8	[0	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9]	`g1f1s1c2t11R2u19Z2a2`	5220
177	8	[x_1	x_2	x_3	$-x_2$	x_5	x_6	x_7	x_8	x_9]	`g1f1s1c2t10R2u10Z1a2`	1232
178	8	[x_1	x_2	x_3	x_2	x_5	x_6	x_7	x_8	x_9]	`g1f1s1c2t3R2u19Z1a2`	1564

Appendix A (*Continued*)

n°	p	Simplified form of fundamental matrix	For example generated by	n
179	8	$[x_1\quad x_2\quad x_3\quad x_4\quad -x_1\quad x_6\quad x_7\quad x_8\quad x_9]$	`g1f1s1c2t9R3u19Z2a2`	96
180	8	$[x_1\quad x_2\quad x_3\quad x_4\quad 0\quad x_6\quad x_7\quad x_8\quad x_9]$	`g1f1s1c2t10R2u19Z2a2`	1104
181	8	$[x_1\quad x_2\quad x_3\quad x_4\quad x_1\quad x_6\quad x_7\quad x_8\quad x_9]$	`g1f1s1c2t10R2u11Z2a2`	384
182	8	$[x_1\quad x_2\quad x_3\quad x_4\quad x_5\quad x_6\quad -x_3\quad x_8\quad x_9]$	`g2f1s1c1t10R2u10Z1a2`	774
183	8	$[x_1\quad x_2\quad x_3\quad x_4\quad x_5\quad x_6\quad x_3\quad x_8\quad x_9]$	`g2f1s1c1t11R2u11Z1a2`	288
184	8	$[x_1\quad x_2\quad x_3\quad x_4\quad x_5\quad x_6\quad x_7\quad -x_6\quad x_9]$	`g1f1s2c1t11R2u11Z1a2`	352
185	8	$[x_1\quad x_2\quad x_3\quad x_4\quad x_5\quad x_6\quad x_7\quad x_6\quad x_9]$	`g1f1s2c1t10R2u10Z1a2`	144
186	8	$[x_1\quad x_2\quad x_3\quad x_4\quad x_5\quad x_6\quad x_7\quad x_8\quad -x_1]$	`g2f1s1c1t5R3u11Z2a2`	32
187	8	$[x_1\quad x_2\quad x_3\quad x_4\quad x_5\quad x_6\quad x_7\quad x_8\quad 0]$	`g1f1s2c1t12R2u13Z2a2`	1078
188	8	$[x_1\quad x_2\quad x_3\quad x_4\quad x_5\quad x_6\quad x_7\quad x_8\quad x_1]$	`g2f1s1c1t10R2u19Z2a2`	128

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