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An Exhaustive Study of Particular Cases Leading to Robust and Accurate Motion Estimation

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For decades, there has been an intensive research effort in the Computer Vision community to deal with video sequences. In this paper, we present a new method for recovering a maximum of information on displacement and projection parameters in monocular video sequences without calibration. This work follows previous studies on particular cases of displacement, scene geometry, and camera analysis and focuses on the particular forms of homographic matrices. It is already known that the number of particular cases involved in a complete study precludes an exhaustive test. To lower the algorithmic complexity, some authors propose to decompose all possible cases in a hierarchical tree data structure but these works are still in development (T. Viéville and D. Lingrand, *Internat. J. Comput. Vision* **31**, 1999, 5–29). In this paper, we propose a new way to deal with the huge number of particular cases: (i) we use simple rules in order to eliminate some redundant cases and some physically impossible cases, and (ii) we divide the cases into subsets corresponding to particular forms determined by simple rules leading to a computationally efficient discrimination method. Finally, some experiments were performed on image sequences acquired either using a robotic system or manually in order to demonstrate that when several models are valid, the model with the fewer parameters gives the best estimation, regarding the free parameters of the problem. The experiments presented in this paper show that even if the selected case is an approximation of reality, the method is still robust. © 2002 Elsevier Science (USA)

Key Words: particular cases; homographies; perspective; paraperspective and orthographic projections.

1. INTRODUCTION

For decades, there has been an intensive research effort in the computer vision community to deal with video sequences. Researchers have been interested in recovering 3D object

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structures, and projection or displacement parameters from such sequences. In the general case, the acquisition device must be considered uncalibrated (for example, in the case of an auto-focus camera). In this paper, we consider uncalibrated monocular video sequences for which we intend to recover as much information as possible on displacement and projection parameters.

The motivations for such studies are threefold: (i) to eliminate singularities of general equations, (ii) to estimate the parameters with more robustness, and (iii) to retrieve parameters that cannot be retrieved in the general case.

The theory states that there exists relations between 2D projected points [9] but the system cannot be solved in the general case since there are more parameters than equations. Furthermore, these equations are degenerate or present singularities in some particular cases. However, we can solve the equations if we know or assume values or relations of some parameters.

In a previous study [26], we have shown that we increase the numerical precision of retrieved parameters by using the set of constraints that gives the smallest residual error given by a criterion (described in the cited paper).

This paper extends previous works [13, 14, 26] on particular displacement cases, scene geometry, and camera analysis. It focuses on the particular forms of fundamental and homographic matrices.

Several authors have already been interested in particular cases of projection [2, 6, 11, 19, 23] or displacement [3, 5, 10, 24, 25]. Some of them consider several particular cases, compare these different parameterizations, and identify which model is consistent with the data.

We will build an exhaustive list of particular cases of projection and displacement, setting some of the parameters to constant and/or known values and using known relations between parameters. This reduces the number of unknowns in the equations and also avoids some singular cases.

It is already known that the huge number of particular cases prevents exhaustive studies [13]. Some attempts in order to reduce the algorithmic complexity are based on tree structures but they are still in development [26]. In this paper, we introduce a new method in order to deal with all cases: (i) we use simple rules in order to eliminate some redundant cases and some physically impossible cases, and (ii) we divide the cases into subsets corresponding to particular forms determined by simple rules leading to a computationally efficient discrimination method. We will provide details for each of these steps in the sections hereafter.

2. STEREO FRAMEWORK

In this section, we describe the equations and the formalism of displacement and projection that allows us to achieve a minimal parameterization of the relations between 2D points into two frames.

In a video sequence, we will consider frames pairwise: two consecutives frames or the first one and the last one. This work could be easily extended to trifocal tensors. Adding some other constraints, the framework could also be extended to sequences, assuming for examples that the translation is constant between consecutives frames, or varies with constant acceleration (see Section 6).

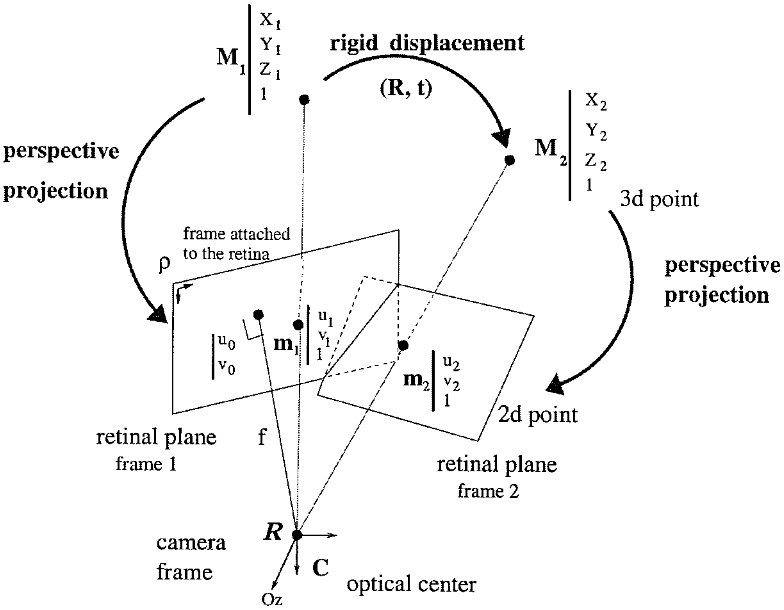

FIG. 1. Stereo framework.

2.1. Rigid Displacements

We will consider a rigid scene or piecewise rigid scene. A 3D-point $\mathbf{M_1} = [X_1\ Y_1\ Z_1\ 1]^T$ is moving onto the point $\mathbf{M_2} = [X_2\ Y_2\ Z_2\ 1]^T$ by a rotation $\mathbf{R}$ and a translation $\mathbf{t} = [t_0\ \ t_1\ \ t_2]^T$: $\mathbf{M_2} = \mathbf{R}\mathbf{M_1} + \mathbf{t}$ as shown in Fig. 1.

A rotation matrix $\mathbf{R}$ depends only on three parameters $\mathbf{r} = [r_0\ \ r_1\ \ r_2]^T$ related to the rotation angle θ and to the rotation axis direction (represented by the unary vector $\mathbf{u}$) by $\mathbf{r} = 2\tan(\frac{\theta}{2})\mathbf{u} \Leftrightarrow \theta = 2\arctan(\frac{\|\mathbf{r}\|}{2})$.

Using the notation of the cross-product

$$\tilde{\mathbf{r}} = \mathbf{r}\wedge = \begin{pmatrix} 0 & -r_2 & r_1 \\ r_2 & 0 & -r_0 \\ -r_1 & r_0 & 0 \end{pmatrix}$$

so that

$$\forall \mathbf{x}, \quad \mathbf{r} \wedge \mathbf{x} = \tilde{\mathbf{r}}\mathbf{x}$$

$\tilde{\mathbf{r}}$ is the antisymmetric matrix representing the cross-product by the $\mathbf{r}$ operator.

The rotation matrix $\mathbf{R} = e^{\tilde{\mathbf{r}}}$ can be developed as a rational Rodrigues formula [20]:

$$\mathbf{R} = \mathbf{I} + \left[\frac{\tilde{\mathbf{r}} + \frac{1}{2}\tilde{\mathbf{r}}^2}{1 + \frac{\mathbf{r}^T \cdot \mathbf{r}}{4}}\right]. \tag{1}$$

2.2. Camera Projection

The most commonly accepted hypothesis states that a 3D point $\mathbf{M}$ is projected with a perspective projection onto an image plane on a 2D point $\mathbf{m} = [u\ \ v\ \ 1]^T$.

2.2.1. The perspective model. Choosing a reference frame attached to the camera, the projection equation is

$$Z\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_u & \gamma & u_0 & 0 \\ 0 & \alpha_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}, \tag{2}$$

where α_u and α_v represent the horizontal and vertical lengths, u_0 and v_0 correspond to the image of the optical center, and γ is the skew factor.

This model can be refined, by taking optical distortions into account [4, 7, 22]. In this paper, we will consider that the needed corrections have been done as a preprocessing step.

Two approximation models of the projection equation (2) have been proposed in the literature: the paraperspective and the orthoperspective projection.

2.2.2. The paraperspective model. The perspective projection model is approximated to its first order with respect to the 3D coordinates [2, 11, 18]. This is equivalent to approximating the perspective projection in two steps (see Fig. 2): (i) a projection parallel to the gaze direction onto an auxiliary plane $\mathbf{P}_a$, which is parallel to the image plane and passes through the scene center $\mathbf{M_0} = [X_0 \; Y_0 \; Z_0]^{\mathrm{T}}$ followed by (ii) a perspective projection onto the image plane. This so-called paraperspective model yields linear equations

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_u & \gamma & \beta_u & u_0 \\ 0 & \alpha_v & \beta_v & v_0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}, \tag{3}$$

where

$$\begin{cases} \beta_u = \alpha_u \frac{X_0}{Y_0} + \gamma \frac{Y_0}{Z_0} \\ \beta_v = \alpha_v \frac{Y_0}{Z_0}. \end{cases}$$

However, its parameters depend on the gaze direction of the scene (β_u and β_v are related to the other intrinsic parameters and to the gaze direction). This equation corresponds to the

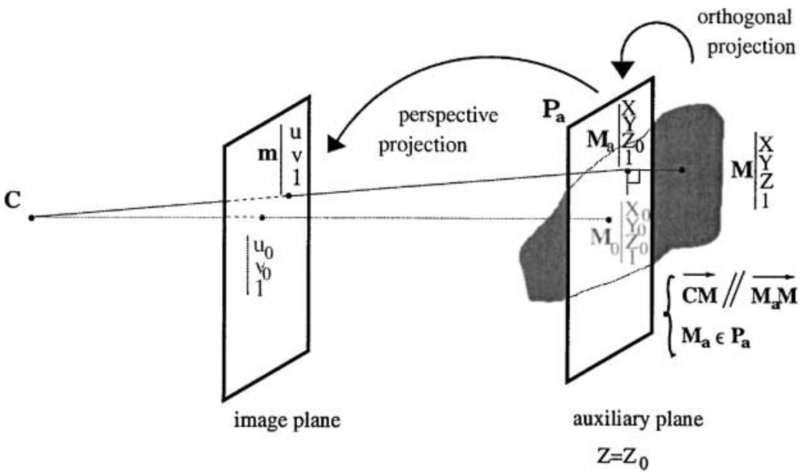

FIG. 2. The paraperspective projection.

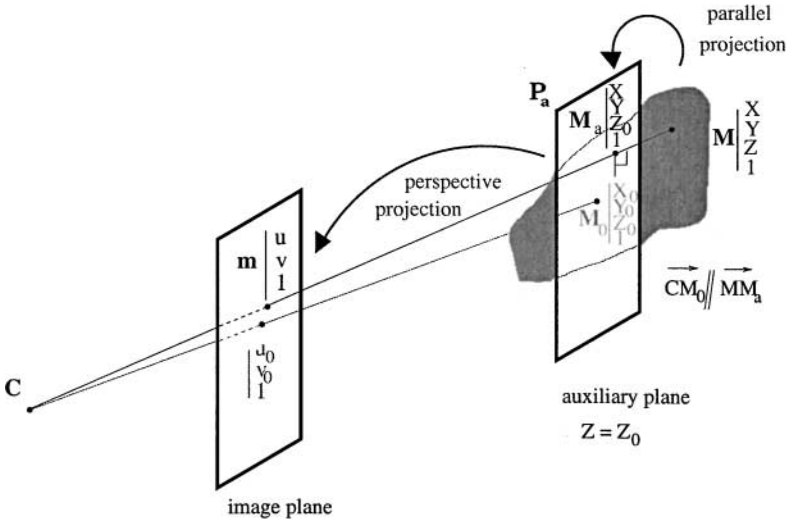

FIG. 3. The orthographic projection.

most general case of paraperspective projection, although more simple expressions have been proposed [19].

2.2.3. The orthographic model. The zero-order development with respect to the 3D depth consists of a rougher approximation. It is also equivalent to another two-step approximation: (i) an orthogonal projection onto the auxiliary plane $\mathbf{P}_a$ followed by (ii) a perspective projection onto the image plane (see Fig. 3). This approximation, called the orthographic model (4), is well adapted to foveal attention and is characterized by linear equations without any new parameters. It is an approximation of the paraperspective model when the observed objects are in the fovea, i.e., close to the optical axis:

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_u & \gamma & 0 & u_0 \\ 0 & \alpha_v & 0 & v_0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}. \tag{4}$$

Those three projection models can be integrated in the expression

$$\kappa \mathbf{m} = \underbrace{\begin{pmatrix} \alpha_u & \gamma & \lambda\beta_u + \mu u_0 & (1-\mu)u_0 \\ 0 & \alpha_v & \lambda\beta_u + \mu v_0 & (1-\mu)u_0 \\ 0 & 0 & \mu & (1-\mu) \end{pmatrix}}_{\mathbf{A}} \mathbf{M} \tag{5}$$

with

Projection case	λ	μ
Perspective projection	1	1
Orthographic projection	0	0
Paraperspective projection	1	0

2.3. General Equations between Two Frames

Let I_1 and I_2 denote two images. In the general case, there exists a fundamental relation between an image point $\mathbf{m_2}$ in I_2 and its corresponding image point $\mathbf{m_1}$ in I_1

$$\begin{cases} \kappa_1 \mathbf{m_1} = \mathbf{A_1 M_1} \\ \kappa_2 \mathbf{m_2} = \mathbf{A_2 M_2} \\ \mathbf{M_2} = \mathbf{R M_1} + \mathbf{t} \end{cases} \Rightarrow \begin{vmatrix} \mathbf{m_1} & \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} & \mathbf{A_1} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} & \mathbf{m_2} & \mathbf{A_2}[\mathbf{R} \parallel \mathbf{t}] \end{vmatrix} = 0,$$

which is a bilinear form in $\mathbf{m_1}$ and $\mathbf{m_2}$ (see [12] for details). This equation can be rewritten in a more common way

$$\mathbf{m_2^T F m_1} = \mathbf{0},$$

where $\mathbf{F}$ is called the fundamental matrix [9].

However, this relation is not defined in some singular cases. For example, it is well known that, in the perspective projection case, if the displacement is a pure rotation, or if the scene is planar, the relation between points is homographic

$$\mathbf{m_2} = \mathbf{H m_1},$$

where $\mathbf{H}$ is called the homographic matrix. In the case of a pure rotation, $\mathbf{H} = \mathbf{H}_\infty = \mathbf{A_2 R A_1^{-1}}$. In the case of a plane with normal $\mathbf{n}$ and distance to the origin d, $\mathbf{H} = \mathbf{A_2}(\mathbf{R} + \frac{\mathbf{t n^T}}{\mathbf{d}})\mathbf{A_1^{-1}}$ which goes to $\mathbf{H}_\infty$ when d goes to ∞. $\mathbf{H}_\infty$ is the homography of the plane at infinity.

Our first new contribution in this paper will be explained in the following two Sections 2.4 and 2.5. It consists of determining in which case of displacement or structure, the relation between corresponding 2D points is homographic when the projection is paraperspective (2.4) or orthographic (2.5).

2.4. Homographic Relation in the Paraperspective Case

In the paraperspective case, we write the projection and displacement equations by extracting the third column from matrix $\mathbf{A}$

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \underbrace{\begin{pmatrix} \alpha_u & \gamma & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{(\mathbf{A})_{-3}} \underbrace{\begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}}_{\underline{\mathbf{M}}} + Z \underbrace{\begin{pmatrix} \beta_u \\ \beta_v \\ 1 \end{pmatrix}}_{(\mathbf{A})_3} = (\mathbf{A})_{-3}\underline{\mathbf{M}} + Z(\mathbf{A})_3,$$

where $(\mathbf{A})_{-3}$ is an invertible square matrix since

$$\det((\mathbf{A})_{-3}) = \alpha_u \alpha_v \neq 0.$$

Thus,

$$\begin{cases} \mathbf{m_1} = (\mathbf{A_1})_{-3}\underline{\mathbf{M}}_\mathbf{1} + Z_1(\mathbf{A_1})_3 \\ \Rightarrow \underline{\mathbf{M}}_\mathbf{1} = ((\mathbf{A_1})_{-3})^{-1}\mathbf{m_1} - Z_1((\mathbf{A_1})_{-3})^{-1}(\mathbf{A_1})_3 \\ \mathbf{m_2} = \mathbf{A_2}\mathbf{M_2} \\ \mathbf{M_2} = [\mathbf{R} \mid \mathbf{t}]\mathbf{M_1}. \end{cases} \tag{6}$$

Let us denote

$$\mathbf{K} = (\mathbf{A_2}[\mathbf{R} \mid \mathbf{t}])_3 - (\mathbf{A_2}[\mathbf{R} \mid \mathbf{t}])_{-3}((\mathbf{A_1})_{-3})^{-1}(\mathbf{A_1})_3$$

and

$$\mathbf{H}_{\infty_{\text{para}}} = (\mathbf{A_2}[\mathbf{R} \mid \mathbf{t}])_{-3}((\mathbf{A_1})_{-3})^{-1}.$$

Equation (6) leads to $\mathbf{m_2} = \mathbf{H}_{\infty_{\text{para}}}\mathbf{m_1} + Z_1\mathbf{K}$.

This relation is homographic if and only if $\mathbf{K} = 0$ or if there exists a (3×3) matrix $\mathbf{H_z}$ such that $Z_1\mathbf{K} = \mathbf{H_z}\,\mathbf{m_1}$. The first condition induces a displacement constraint. It leads to the simple equation $\mathbf{r} = \theta\mathbf{M_0}$, meaning that the rotation axis is parallel to the gaze direction. In that case, the homography is $\mathbf{H}_{\infty_{\text{para}}}$ as defined above. The second condition induces a geometric relation on the 3D point: Z_1 is an affine function of X_1 and Y_1, meaning that the 3D points must belong to a plane $\mathbf{P}$, which cannot contain the optical axis and the gaze direction (see [12] for a demonstration). In that case, the homographic matrix is

$$\mathbf{H}_{\text{para}} = \mathbf{H}_{\infty_{\text{para}}} + [(\mathbf{A_2}[\mathbf{R} \mid \mathbf{t}])_3 - [(\mathbf{A_2}[\mathbf{R} \mid \mathbf{t}])_{-3}((\mathbf{A_1})_{-3})^{-1}(\mathbf{A_1})_3]\,\mathbf{n}\,[(\mathbf{A_1})_{-3} + (\mathbf{A_1})_3\mathbf{n}]^{-1}. \tag{7}$$

2.5. Homographic Relation in the Orthographic Case

The orthographic case is a particular case of paraperspective projection for which the gaze direction is the optical axis. Following a demonstration similar to the paraperspective case, we also obtain two constraints; the displacement constraint states that the rotation axis must be parallel to the optical axis, giving a homographic matrix

$$\mathbf{H}_{\infty_{\text{ortho}}} = (\mathbf{A_2}[\mathbf{R} \mid \mathbf{t}])_{-3}((\mathbf{A_1})_{-3})^{-1},$$

and the geometric constraint states that the 3D-points must belong to the same plane which does not contain the optical axis. The homographic matrix is

$$\mathbf{H}_{\text{ortho}} = \mathbf{H}_{\infty_{\text{ortho}}}(\mathbf{A_2}[\mathbf{R} \mid \mathbf{t}])_3\,\mathbf{n}^{\mathrm{T}}\,((\mathbf{A_1})_{-3})^{-1}.$$

All constraints on displacement and scene geometry for homographic relations are summarized in the following table:

Projection	Displacement constraint	Geometric constraint
Perspective	$t = 0$	Plane
Paraperspective	$\mathbf{r} \parallel \mathbf{CM_0}$	Plane $Z = f(X, Y)$
Orthographic	$\mathbf{r} \parallel \mathbf{0z}$	Plane $Z = f(X, Y)$

3. ALL PARTICULAR CASES DESCRIPTION

In order to do an exhaustive study of particular cases combinations, we first study every elementary particular case. We begin with particular camera parameter values, and then particular displacements of the camera.

3.1. Particular Cases of Projection and Intrinsic Parameters

In the previous Section 2.2, we studied particular cases of projection and their simplifications. Let **p1**, **p2**, and **p3** denote the different kinds of projection:

p1	$\lambda = 0$	and	$\mu = 0$	Orthographic
p2	$\lambda = 1$	and	$\mu = 0$	Paraperspective projection
p3	$\lambda = 1$	and	$\mu = 1$	Perspective projection

If no auto-focus and no zoom is used, for instance, it is possible to parameterize the model with fewer parameters than in the general case. This is one reason to study particular cases of intrinsic parameters.

Authors generally make several hypotheses regarding intrinsic parameters. For example, usually, in case of auto-calibration, common hypothesis states that the intrinsic parameters are constant. They may or may not be known. However, usually, some parameters are constant and some others are not.

We now detail all prior knowledge on parameters leading to particular cases.

3.1.1. The principal point. The principal point of coordinates (u_0, v_0) is not fixed at the image plane in the general case but can be fixed in some cases and its position can be known (for example, at the image center). We then change the reference frame, regarding the principal point position.

3.1.2. The γ parameter. This parameter is usually assumed to be zero or, at least, considered a constant value. Furthermore, the numerical precision of the model obtained by this parameter is not crucial for the paraperspective or the orthographic projection cases.

3.1.3. The α_u and α_v parameters. Enciso and Víeville [8] have experimentally proven that, for a large number of cameras, the $\frac{\alpha_u}{\alpha_v}$ ratio can be considered as a constant value even if other intrinsic parameters change. The constancy of this ratio can be expressed by the equality $f = \alpha_u = \alpha_v$, and the transformation

$$\begin{pmatrix} \alpha_u & \gamma & \lambda\beta_u + \mu u_0 & (1-\mu)u_0 \\ 0 & \alpha_v & \lambda\beta_v + \mu v_0 & (1-\mu)v_0 \\ 0 & 0 & \mu & (1-\mu) \end{pmatrix} = \begin{pmatrix} f & \gamma & \lambda\beta_u + \mu u_0 & (1-\mu)u_0 \\ 0 & f & \lambda\beta_v + \mu v_0 & (1-\mu)v_0 \\ 0 & 0 & \mu & (1-\mu) \end{pmatrix} \cdot \begin{pmatrix} \frac{\alpha_u}{\alpha_v} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

3.1.4. The β_u and β_v parameters. These parameters are zero except in the paraperspective projection case.

In the paraperspective case, β_u and β_v are related to the other intrinsic parameters by

$$\begin{cases} \beta_u = \alpha_u \frac{X_0}{Z_0} + \gamma \frac{Y_0}{Z_0} \\ \beta_v = \alpha_v \frac{Y_0}{Z_0}. \end{cases}$$

TABLE 1
Particular Cases of Intrinsic Parameters for 2 Frames

Label	Case	Description
g1	$\gamma = 0$	γ constant and zero
g2	$\gamma = \gamma_0$	γ constant
g3	$\gamma = \gamma(\tau)$	γ free
f1	$\alpha_v = 1$	α_v constant and known
f2	$\alpha_v = f_0$	α_v constant
f3	$\alpha_v = \alpha_v(\tau)$	α_v free
s1	$\alpha_u = \alpha_v(\tau)$	$\frac{\alpha_u}{\alpha_v}$ constant and known
s2	$\alpha_u = \alpha_u(\tau)$	α_u free
b1	$\beta_v = 0$	β_v constant and zero
b2	$\beta_v = \beta_0$	β_v constant
b3	$\beta_v = \beta_v(\tau)$	β_v free
B1	$\beta_u = \beta_v(\tau)$	β_u and β_v equal
B2	$\beta_u = \beta_v(\tau)$	$\frac{\beta_u}{\beta_v}$ constant
B3	$\beta_u = \beta_u(\tau)$	$\frac{\beta_u}{\beta_v}$ free
c1	$u_0 = v_0 = 0$	u_0 and v_0 constant and known
c2	$u_0 = u_{0_0}$ and $v_0 = v_{0_0}$	u_0 and v_0 constant
c3	$u_0 = u_0(\tau)$ and $v_0 = v_0(\tau)$	u_0 and v_0 free

Their ratio is

$$\frac{\beta_u}{\beta_v} = \frac{\alpha_u X_0 + \gamma Y_0}{\alpha_v Y_0}.$$

Thus, if we neglect γ with respect to $\alpha_u \frac{X_0}{Y_0}$, we obtain

$$\frac{\beta_u}{\beta_v} = \frac{\alpha_u}{\alpha_v} \frac{X_0}{Y_0},$$

which is also a constant ratio, known if the $\frac{X_0}{Y_0}$ value is known.

Table 1 summarizes, for each intrinsic parameter, the particular cases (constant values are indexed by 0).

Subsequently, we refer to each case by the label given in the first column.

3.2. Particular Cases of Displacement

A rigid displacement is parameterized by the rotation **R** and the translation **t** parameters.

3.2.1. Discrete motion–continuous motion. In an image sequence, if the displacement between two frames is small, we can approximate the rotation equations by their first-order expansion, using the notations $\mathbf{r} = \theta \mathbf{u}$:

$$\mathbf{R} = e^{\tilde{\mathbf{r}}} = \mathbf{I} + \tilde{\mathbf{r}} + o(\tilde{\mathbf{r}}) = \begin{pmatrix} 1 & -r_2 & r_1 \\ r_2 & 1 & -r_0 \\ -r & r_0 & 1 \end{pmatrix}.$$

Otherwise, if the motion is larger, we can also consider the second-order expansion

$$\mathbf{R} = \mathbf{I} + \tilde{\mathbf{r}} + \frac{\tilde{\mathbf{r}}^2}{2} + o(\tilde{\mathbf{r}}^2) = \begin{pmatrix} 1 - \left(r_1^2 + r_2^2\right) & r_1 r_0 - r_2 & r_2 r_0 + r_1 \\ r_1 r_0 + r_2 & 1 - \left(r_0^2 + r_2^2\right) & r_2 r_1 - r_0 \\ r_2 r_0 - r_1 & r_2 r_1 + r_0 & 1 - \left(r_0^2 + r_1^2\right) \end{pmatrix}.$$

3.2.2. About extrinsic parameters. The rotation parameters are related to the rotation axis and the rotation angle by $\mathbf{r} = 2\tan\frac{\theta}{2}\mathbf{u}$, in the general case and $\mathbf{r} = \theta\mathbf{u}$, in the first or second order of expansion. The vector $\mathbf{u}$ is an unary vector giving the direction of the rotation axis.

Some components of $\mathbf{u}$ can be known or assumed zero. Some values of θ may yield singularities; for example, $\theta = 0$ corresponds to a null rotation. Another particular case is the screw displacement for which $\theta = \frac{\pi}{4}$ and the rotation axis is parallel to the translation vector. The case $\theta = \pi$ is not considered in this paper but must be considered if the camera has an angle of view greater than 180°.

Some robotic systems give precise values of the robot displacements (angle, axis, translation). Some values may be known (we denote by $\_\theta_0$ a constant and known value of a parameter θ). Other information on parallelism or orthogonality to a known direction may be available.

For the translation vector, some components can also be known or assumed zero.

3.2.3. Relations between axis and direction. These relations in which we are interested are orthogonality and parallelism:

- The rotation axis is orthogonal to the translation plane (e.g., planar motion): $\mathbf{r} \perp \mathbf{t} \Leftrightarrow \mathbf{r} \cdot \mathbf{t} = 0 \Leftrightarrow r_0 t_0 + r_1 t_1 + r_2 t_2 = 0$,
- Screw displacement:

$$\mathbf{r} \parallel \mathbf{t} \Leftrightarrow \exists\kappa / \mathbf{r} = \kappa\mathbf{t} \Leftrightarrow \exists\kappa \bigg/ \begin{cases} r_0 = \kappa t_0 \\ r_1 = \kappa t_1 \\ r_2 = \kappa t_2, \end{cases}$$

- The rotation axis or the translation direction is parallel or orthogonal to a known direction denoted by $\_\mathbf{g}$ ($\_\mathbf{r}$ or $\_\mathbf{t}$).

3.2.4. All constraints on motion. All these constraints, also called "atomic particular cases," have simple expressions that can easily be combined. For this purpose, we use the fact that $\mathbf{u}$ is an unary vector and that, for monocular systems, the norm of translation cannot be recovered. To parameterize these vectors with only two parameters, we divide each component by a nonzero component. Then, the dot product and scalar product induce linear relations. For example, if $t_2 = 1$, $\mathbf{t} \perp \mathbf{r}$ is equivalent to $t_0 u_0 + t_1 u_1 + u_2 = 0 \Rightarrow u_2 = -t_0 u_0 - t_1 u_1$.

All cases are collected in Table 2.

3.3. Generation of All Cases

In this section, we combine all previous constraints in order to generate all possible cases. We then generate the simplified equations of our vision problem, i.e., the $\mathbf{F}$ or $\mathbf{H}$ matrix coefficients, depending of the cases.

TABLE 2
Particular Cases of Displacements

R1	$\mathbf{R} = \mathbf{I}$	Null rotation
R2	$\mathbf{R} = \mathbf{I} + \tilde{\mathbf{r}}$	First order
R3	$\mathbf{R} = \mathbf{I} + \tilde{\mathbf{r}} + \frac{1}{2}\tilde{\mathbf{r}}^2$	Second order
R4	$\mathbf{R} = \mathbf{I} + \dfrac{\tilde{\mathbf{r}} + \frac{1}{2}\tilde{\mathbf{r}}^2}{1 + \frac{\mathbf{r}^T\mathbf{r}}{4}}$	General case
r1	$\mathbf{r} = 2\tan\left(\dfrac{\theta}{2}\right)\dfrac{\mathbf{u}}{\Vert\mathbf{u}\Vert}$	General case
a1	$\theta = \dfrac{\pi}{2}$	Quarter turn
a2	θ	Free angle
T1	$\mathbf{t} = 0$	Null translation
T2	$\mathbf{t} = [t_0\, t_1\, t_2]^T$	Translation
t1	$t_1 = t_2 = 0, t_0 = 1$	Trans. $\Vert$ x axis
t2	$t_1 = 0, t_0 = 1$	Trans. $\perp$ y axis
t3	$t_2 = 0, t_0 = 1$	Trans. $\perp$ z axis
t4	$t_0 = 1$	General trans.
t5	$t_0 = t_2 = 0, t_1 = 1$	Trans. $\Vert$ y axis
t6	$t_0 = 0, t_1 = 1$	Trans. $\perp$ x axis
t7	$t_2 = 0, t_1 = 1$	Trans. $\perp$ z axis
t8	$t_1 = 1$	General trans.
t9	$t_0 = t_1 = 0, t_2 = 1$	Trans. $\Vert$ z axis
t10	$t_0 = 0, t_2 = 1$	Trans. $\perp$ x axis
t11	$t_1 = 0, t_2 = 1$	Trans. $\perp$ y axis
t12	$t_2 = 1$	General trans.
D1	$\mathbf{t} \cdot {\_}\mathbf{t} = 0$	Trans. $\perp$ known axis
D2	$\mathbf{t} \wedge {\_}\mathbf{t} = 0$	Trans. $\Vert$ known axis
D3		No relation
Z1	$\mathbf{t} \cdot \mathbf{u} = 0$	Trans. $\perp$ rotat. axis
Z2	$\mathbf{t} \wedge \mathbf{u} = 0$	Screw displacement
Z3		No relation

W1	$\mathbf{r}.{\_}\mathbf{r} = 0$	Axis $\perp$ known axis
W2	$\mathbf{r} \wedge {\_}\mathbf{r} = 0$	Axis $\Vert$ known axis
W3		General case
u1	$u_0 = u_2 = 0, u_1 = 1$	Axis $\Vert$ y axis
u2	$u_0 = 0, u_1 = 1$	Axis $\perp$ x axis
u3	$u_2 = 0, u_1 = 1$	Axis $\perp$ z axis
u4	$u_1 = 1$	General case
u5	$u_0 = u_2 = 0, u_1 = -1$	Axis $\Vert$ y axis
u6	$u_0 = 0, u_1 = -1$	Axis $\perp$ x axis
u7	$u_2 = 0, u_1 = -1$	Axis $\perp$ z axis
u8	$u_1 = -1$	General case
u9	$u_0 = u_1 = 0, u_2 = 1$	Axis $\Vert$ z axis
u10	$u_0 = 0, u_2 = 1$	Axis $\perp$ x axis
u11	$u_1 = 0, u_2 = 1$	Axis $\perp$ y axis
u12	$u_2 = 1$	General case
u13	$u_0 = u_1 = 0, u_2 = -1$	Axis $\Vert$ z axis
u14	$u_0 = 0, u_2 = -1$	Axis $\perp$ x axis
u15	$u_1 = 0, u_2 = -1$	Axis $\perp$ y axis
u16	$u_2 = -1$	General case
u17	$u_1 = u_2 = 0, u_0 = 1$	Axis $\Vert$ x axis
u18	$u_1 = 0, u_0 = 1$	Axis $\perp$ y axis
u19	$u_2 = 0, u_0 = 1$	Axis $\perp$ z axis
u20	$u_0 = 1$	General case
u21	$u_1 = u_2 = 0, u_0 = -1$	Axis $\Vert$ x axis
u22	$u_1 = 0, u_0 = -1$	Axis $\perp$ y axis
u23	$u_2 = 0, u_0 = -1$	Axis $\perp$ z axis
u24	$u_0 = -1$	General case

We call each case described above *atomic*. By combining *atomic cases*, we produce *molecular cases*, i.e., all possible particular cases. For each *molecular case*, constraints are solved by combining the *atomic cases* and solving the constraints by substitution[2] with some rules: one projection mode, one rotation mode, etc. This corresponds to choosing one case in each family, a family being named by a label. For example, in the `R` family, we must choose one of `R1`, `R2`, `R3`, and `R4`. We denote by `R[1–3]` the set {`R1`, `R2`, `R3`} and by `R[1; 3]` the set {`R1`, `R3`}. Thus, a molecular case is identified by the sequence

```
p[1-3]g[1-3]f[1-3]s[1-3]b[1-3]B[1-3]c[1-3]R[1-4]r1a[1-2]
  u[1-24]W[1-3]T[1-2]t[1-12]D[1-3]Z[1-3].
```

3.3.1. How many cases do we have? If we look at the expression of a particular case mentioned above, we obtain 3×10^8 particular cases. However, this is not the real number

[2] This work is done using Maple for symbolic computation.

of particular cases due to:

- Incompatibility of some atomic cases, for instance (the symbol $\otimes$ means "AND"),

$$(\mathbf{r} \parallel \mathbf{t}) \otimes (\mathbf{r} \perp \mathbf{t}) \otimes (\mathbf{r} \neq 0) \otimes (\mathbf{t} \neq 0).$$

- Redundancy of some constraints; two different set of atomic constraints can generate the same simplified model. For instance,

$$(r_0 = 0) \otimes (\mathbf{t} \perp \mathbf{r}) \text{ is the same case as } (t_1 = 0) \otimes (t_2 = 0) \otimes (\mathbf{t} \perp \mathbf{r}).$$

It is easy to eliminate incompatible constraints. To deal with redundant constraints requires comparing each set of combined constraint with the others in order to determine the similarity. The complexity of this process is $O(n^2)$, which makes this elimination intractable for large values of n.

Furthermore, redundant cases are not the main reason for the large amount of particular cases. Thus suppressing redundancies is not sufficient for reducing the huge number of cases to a computationally tractable amount.

We now propose an adapted strategy in order to deal with all cases. Previous works have tried to build a hierarchy of cases but they encounter problems in order to manage it. The idea of this paper is (i) to eliminate some of the redundant cases by some considerations on the atomic cases and (ii) to limit the number of cases by the study of the particular forms of the matrices. For this second step, we will separate cases into two subgroups: cases inducing homographies and cases inducing fundamental relations.

3.4. Reducing the Number of Cases

Some redundancies are obvious:

- in the case of a null rotation, (`R1`), we do not consider every case of axis and angle, one is sufficient;
- in the case of first and second-order rotation, (`R2`) and (`R3`), we do not consider the case (`a1`) where θ is equal to $\frac{\pi}{2}$;
- the case (`a1`) where θ is equal to $\frac{\pi}{2}$ is only considered if the rotation axis and the translation direction are parallel, (`Z2`);
- in the case of a null translation, we do not consider any relation of orthogonality or parallelism to other directions;
- in the case of nonparaperspective projection, (`p1`) and (`p3`), β_u and β_v are equal to 0.

We also consider the following experimental simplifications:

- when approximating a perspective projection, (`p1`) and (`p2`), we neglect the parameter γ with respect to other approximations;
- following previous studies [8, 27], we assume that the ratio $\frac{\alpha_u}{\alpha_v}$ is constant;
- these two previous items imply that the $\frac{\beta_u}{\beta_v}$ ratio is also constant.

Then there only remains, from the intrinsic parameters part, 117 cases and, from the extrinsic parameters part, 21,709 cases, leading to a total of 2,539,953 particular cases. This is approximately 100 times less than previously determined (see Appendix C for details).

3.5. Fundamental and Homographic Matrices

For each case, we have computed the set of reduced equations. Now, for each case, we compute the fundamental or homographic matrix expression.

As previously studied in Sections 2.3, 2.4, and 2.5, the displacements inducing homographic relations are;

- in the orthographic case (`p1`): $\mathbf{u} \parallel Oz$. The relations between $\mathbf{t}$ and $\mathbf{r}$ are equivalent to the nullity of some vector components. We will not consider (`Z1`) and (`Z2`). Previous studies on orthographic displacement have shown that the displacement is retinal (`t[1;3;5;7]`).
- in the paraperspective case (`p2`): $\mathbf{u} \parallel [X_0\ Y_0\ Z_0]$ (`D2`). Since the view axis has at least a component on the optical axis, we set $u_2 = \pm 1$. Moreover the view axis is not exactly the optical axis; thus we cannot have $u_0 = 0$ and $u_1 = 0$.
- in the perspective case (`p3`): $\mathbf{t} = 0$. Therefore we do not consider the parallelism and orthogonality constraints on $\mathbf{t}$.

We also note that, since we are dealing with only 2 views, relations between $\mathbf{r}$ or $\mathbf{t}$ with a known vector $\underline{\mathbf{g}}$ will not simplify the $\mathbf{H}$ matrix form, except in the paraperspective case, if $\underline{\mathbf{g}} = \mathbf{M_0}$.

The homographic relation cases lead to 351 cases of orthographic homographic relations, 18,360 cases of paraperspective homographic relations, and 2,619 cases of perspective homographic relations, leading to a total 21,330 cases of homographic relations (see explanations in Appendix C).

We will not study paraperspective and orthographic projection for fundamental matrices since the domain of validity of such projection approximations is included in conditions of existence of homographic relation. In the case of perspective projection, (`p3`): $\mathbf{t} \neq \mathbf{0}$ thus $u_0 = \pm 1$ or $u_1 = \pm 1$.

For perspective projection, there are 72,252 different cases as shown in Appendix C.

4. MATRIX FORMS

In the previous section, we have significantly reduced the number of cases in both fundamental matrices and homographic matrices sets. However, we still have to deal with a huge amount of cases that is numerically intractable. In this section, we introduce a new idea of splitting the two sets of matrices into a two-level tree.

Each set of matrices is first split into subsets of matrices, depending on their form. We determine a matrix form by a very simple parameterization. We consider (3×3) matrices to have 9 parameters (coefficients) and we use two simple rules:

- If a coefficient is equal to 0, then there is one less parameter.
- If a coefficient has the same expression or is opposite to another, then there is one less parameter again.

These operations are very simple and can be computed in each case in a reasonable time (approximately one day for the entire process to determine the matrix form).

This process, as illustrated in Fig. 4, reduces the 21,330 cases of homography matrices to only 108 subgroups and the 72,252 cases of fundamental matrices to only 188 subgroups.

The table in Appendix A shows all the particular forms of homography matrices and the table in Appendix B shows all the particular forms of fundamental matrices.

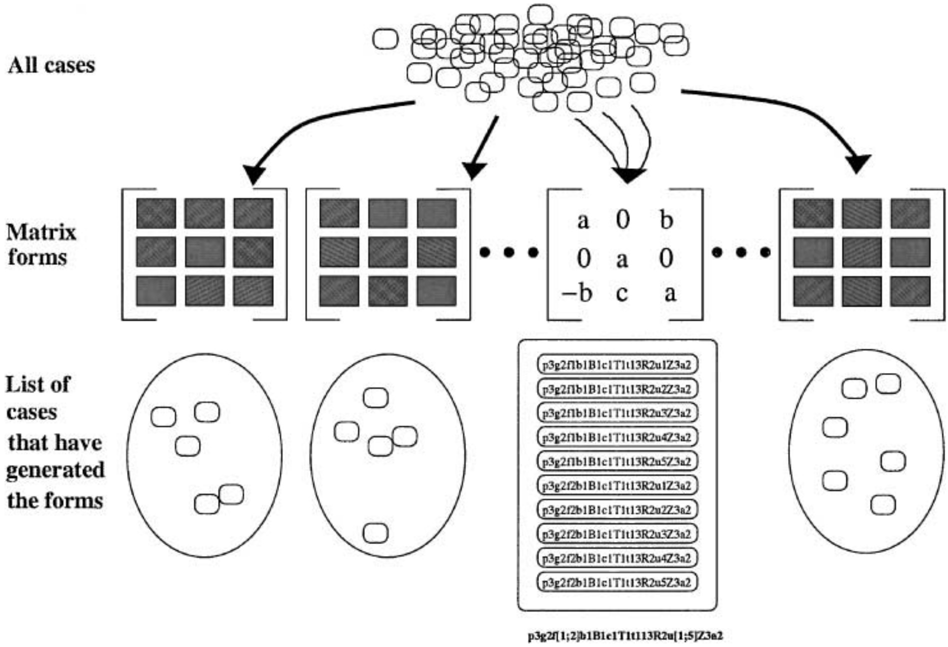

FIG. 4. Set of cases that generates the same matrix form. The central column shows an example of the homography matrix form (number 19 in the table form Appendix A).

Homography and fundamental matrices are defined up to a scale factor. This parameter has not been eliminated here. We take it into account in the numerical implementation. Fundamental matrices must also satisfy the constraint det $\mathbf{F} = 0$, We must check whether this constraint is satisfied in order to determine whether the number of degrees of freedom is reduced. This is important in order to properly use the Akaike [1] criterion at the numerical stage (see next section).

For each matrix form, we have collected all the cases that have generated them. Once the matrix form corresponding to an experiment is determined, it is possible to backtrack the source cases.

5. EXPERIMENTS

In a previous paper [26], we demonstrated, for several specific displacements, that the case which minimizes a error criterion is that corresponding to the motion performed by a robotic system. We present here two cases (one for homographies, another for fundamental matrices), performed by a precise robotic system. We then present an extension to real approximative displacements.

5.1. *Forms of Homography*

We have recorded several video sequences for which the camera displacement induces a homographic relation between image points $\mathbf{m_1}$ and $\mathbf{m_2}$. We have first extracted points of interest and determined matching points using the *image-matching* algorithm from Zhang *et al.* [29]. From each matrix form enumerated in Table A1, we have estimated the homography parameters with the robust least median of squares method [21] in order to minimize the distance between a 2D point $\mathbf{m_1}$ and its projected estimation $\mathbf{Hm_2}$,

$$\left\| \mathbf{m_2} - \frac{\mathbf{Hm_1}}{((\mathbf{h^2})^T\, \mathbf{m_1})} \right\|^2,$$

where $\mathbf{h^2}$ represents the last line of the $\mathbf{H}$ matrix and $\mathbf{m_1}$ and $\mathbf{m_2}$ are normalized.

It is not possible to have a symmetric criterion without inverting the $\mathbf{H}$ matrix (and we do not want to invert it).

To deal with cases with different degrees of freedom, we use an appropriate Akaike criterion [1].

For each video sequence, we have verified that the model with the smallest residual error indeed corresponds to the displacement performed by a robotic system. An example is proposed in Fig. 5. For each pair of consecutive images, the case with the least residual error is case 51 in the table in Appendix A, which corresponds to the matrix form

$$\mathbf{H}_{51} = \begin{pmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & x_6 \\ 0 & 0 & x_1 \end{pmatrix}.$$

FIG. 5. Frames 1, 2, and 8 of the video sequence. The robotic system performs a rotation around the optical axis.

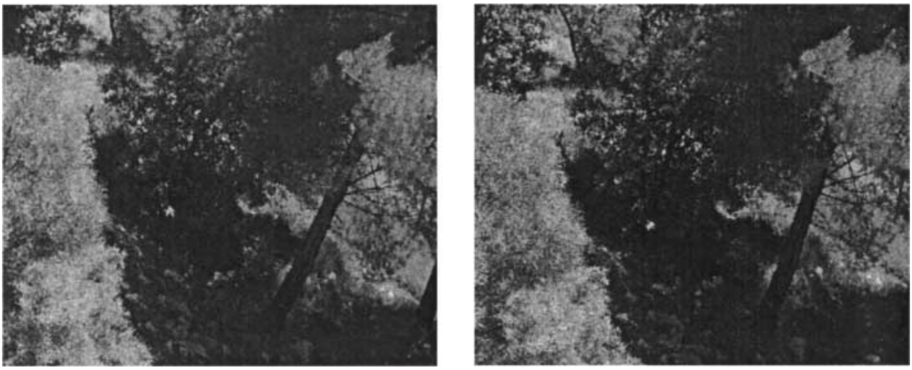

FIG. 6. Approximate rotation around the optical axis and translation.

We observe that this case corresponds to a first-order rotation (R2). If we consider only the first and the last frame, the rotation is general (R4).

We also performed several experiments without the help of a precise robotic system. A human manipulated a camera by hand and tried to realize different particular displacements. Figure 6 shows two frames of a video sequence. The camera motion was approximately a rotation around its optical axis followed by a translation. As the previous experiment with a robotic system, for each pair of consecutive images, the case with smallest residual error is case 51 in the table in Appendix A. This result demonstrates the robustness of the analysis of displacement by particular cases. Even an approximative displacement is best recovered by a close particular case than the general equation.

5.2. *Forms of Fundamental Matrices*

We have done the same experiment for a displacement that induces a fundamental relation. The criterion is using the distance between a 2D point $\mathbf{m_1}$ and its epipolar line $\mathbf{Fm_2}$ [17, 28]:

$$f_m(\mathbf{F}) = \frac{\left|\mathbf{m}_2^T \mathbf{F} \mathbf{m_1}\right|}{\sqrt{(\mathbf{F^T m_2})_1^2 + (\mathbf{F^T m_2})_2^2}}.$$

The camera has performed a translation parallel to the x axis, and a small pan rotation, and corrected focal length with auto-focus (Fig. 7). The case with less residual error corresponds to the fundamental matrix form (case 59 in the table in Appendix B)

$$\mathbf{F}_{59} = \begin{pmatrix} 0 & 0 & 0 \\ x_0 & x_1 & x_2 \\ 0 & -x_2 & x_3 \end{pmatrix}.$$

This particular form was obtained from cases where the rotation was approximated to its first and second order, the translation is parallel to the x axis, the rotation axis is orthogonal to the optical axis, and the intrinsic parameters are free.

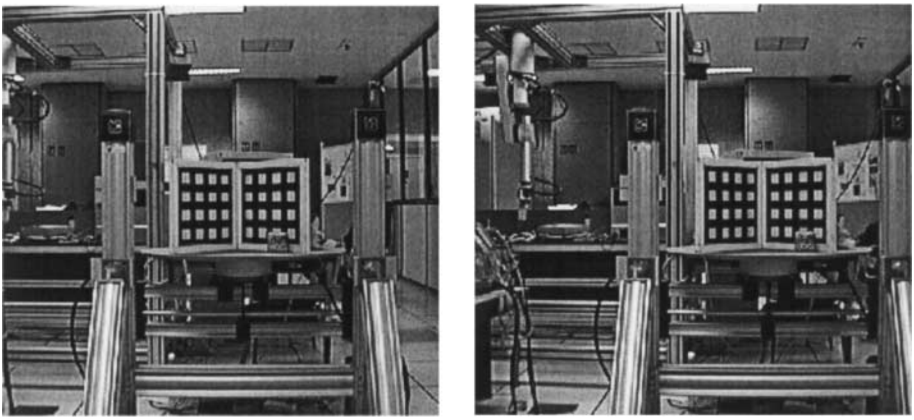

FIG. 7. x-axis translation and small pan rotation, with auto-focus.

5.3. Discussion

An interesting study should be considering a large number of pairs of frames. The idea is to extract from the $108 + 188$ cases presented in this paper a subset of cases that are selected at least once with respect to the corresponding criterion. We believe that this subset is really smaller than the whole set of the 296 cases.

We did preliminary work on fMRI (functionnal magnetic resonance images) sequences in order to register each image volume with respect to the first one. We used a strategy similar to that explained in this paper but using a different parameterization (there were only displacement parameters generating 120 different cases). We experimented on 323 pairs of image volumes in which only 63 cases were selected at least once. All details of this study have already been published in [15].

The next idea is to avoid cases that do not occur often and that obtain a criterion error similar to another case that is often selected.

6. HOW TO EXTEND THIS WORK TO VIDEO SEQUENCES?

In this paper, we have dealt with video sequences with pairs of frames. Two major extensions could be done: (i) an extension of this work to trilinearities (relations between 2D points from three frames) and (ii) an extension to video sequences of n frames.

In order to consider sequences of n images, we need to introduce several displacement cases. We have constraints on translation, axis and angle or rotation, and zoom factor. For each of these quantities, the questions are is the displacement between two frames constant? Is the acceleration constant? linear? following a known rule?

In our formalism, we need to introduce these constraints in the Maple code, which will generate other equations with more constraints but we will also have more data (n images) and more parameters. It is also easy to change the criterion used to measure the equation of the model to the data (one C function to be rewritten).

We must think about the fact that some objects may disappear along a video sequence and that their movements may be detected only in a few images of the video sequences. This is a problem that will be examined in another paper.

7. CONCLUSION

We have studied how to deal with video sequences and with particular cases of displacement and projection that often occur in real situations (man walking on a flat road, objects far from the retina etc). The general equations of the vision problem present singularities in some particular cases that are usually avoided.

In the present paper, we have proposed an alternative approach to this problem, using such singularities and other particular cases in order to obtain more information than in the general case instead of avoiding them.

Our major contributions are:

- We have determined the conditions of existence of homographic relations between projected 2D points for the orthographic, the paraperspective, and the perspective projections.
- We have used these conditions and other obvious redundancy properties to reduce the amount of homographic particular cases to study. Thus, we have determined all particular forms of matrices, and we have obtained, for each particular form, the list of cases that have generated this form. This result is a first fundamental step for further studies.

This study might be extended in two ways: (i) to be able, given a form, to analyze the molecular constraints, to determine which are redundant and which correspond to the case we are dealing with, and (ii) to do the same analysis with geometrical property of the 3D scene, meaning homography induced by planes. The structure of this analysis is as general as possible to extend this work to other kinds of cameras (conic mirror, etc.).

The applications are twofold:

- an incremental reconstruction of the scene using different cases: each case studied has fewer parameters than the next one, giving the ability to recover some parameters from others already determined. We have already studied the control of a robot on a particular case [16].
- the segmentation of objects moving with different displacements or with different geometric properties in video sequences: using a ν-trimmed square method instead of the least median square method, we can build sets of points with same matrix forms (and same numerical matrix forms).

APPENDIX A

Table of Particular Forms of Homographic Matrices

Table A1 shows the simplified forms obtained and, for each form, the cases that have generated them. We denote by # the form number, by **p** the number of parameters (we have not taken into account the fact that the homography matrix is defined up to a scale factor but we do it in our numerical implementation), and by **n** the number of molecular cases that have generated a form.

The interest of this table is for the reader who wants to implement the method presented in this paper and to interpret the results the method gives. An electronic form of this table and some Maple and C code can be sent upon simple request to the author.

TABLE A1

Particular Forms of Homography and Cases that Have Generated Them

#	p	Simplified form of homography	Generated by	n
1	1	$[x_1\ \ 0\ \ 0\ \ 0\ \ x_1\ \ 0\ \ 0\ \ 0\ \ x_1]$	p1g1f[1;2]b1B1c[1;2]T1t13R1u4Z3a2 ⊕ p3g1f[1;2]b1B1c[1;2]T1t13R1u4Z3a2 ⊕ p3g2f1b1B1c[1;2]T1t13R1u4Z3a2 ⊕ p3g2f2b1B1c1T1t13R1u4Z3a2	11
2	1	$[x_1\ \ 0\ \ 0\ \ 0\ \ x_1\ \ x_1\ \ 0\ \ 0\ \ x_1]$	p1g1f1b1B1c[1;2]T2t[5;7]R1u4Z3a2	4
3	1	$[x_1\ \ 0\ \ x_1\ \ 0\ \ x_1\ \ 0\ \ 0\ \ 0\ \ x_1]$	p1g1f[1;2]b1B1c[1;2]T2t1R1u4Z3a2	4
4	2	$[x_1\ \ 0\ \ 0\ \ 0\ \ x_1\ \ x_6\ \ 0\ \ -x_6\ \ x_1]$	p3g1f1b1B1c1T1t13R2u[17;21]23a2	2
5	2	$[x_1\ \ 0\ \ 0\ \ 0\ \ x_1\ \ x_6\ \ 0\ \ 0\ \ x_1]$	p1g1f2b1B1c[1;2]T2t[5;7]R1u4Z3a2	4
6	2	$[x_1\ \ 0\ \ 0\ \ 0\ \ x_5\ \ 0\ \ 0\ \ 0\ \ x_1]$	p1g1f3b1B1c1T1t13R1u4Z3a2 ⊕ p3g[1;2]f3b1B1c1T1t13R1u4Z3a2	3
7	2	$[x_1\ \ 0\ \ x_1\ \ 0\ \ x_1\ \ x_6\ \ 0\ \ 0\ \ x_1]$	p1g1f[1;2]b1B1c[1;2]T2t3R1u4Z3a2	4
8	2	$[x_1\ \ 0\ \ x_1\ \ 0\ \ x_5\ \ 0\ \ 0\ \ 0\ \ x_1]$	p1g1f3b1B1c1T2t1R1u4Z3a2	1
9	2	$[x_1\ \ 0\ \ x_3\ \ 0\ \ x_1\ \ 0\ \ -x_3\ \ 0\ \ x_1]$	p3g1f[1;2]b1B1c1T1t13R2u[1;5]Z3a2	4
10	2	$[x_1\ \ 0\ \ x_3\ \ 0\ \ x_1\ \ 0\ \ 0\ \ 0\ \ x_1]$	p3g2f2b1B1c2T1t13R1u4Z3a2	1
11	2	$[x_1\ \ x_2\ \ 0\ \ -x_2\ \ x_1\ \ 0\ \ 0\ \ 0\ \ x_1]$	p[1;3]g1f1b1B1c1T1t13R2u[13;9]Z3a2	4
12	2	$[x_1\ \ x_2\ \ 0\ \ -x_2\ \ x_1\ \ x_1\ \ 0\ \ 0\ \ x_1]$	p1g1f1b1B1c1T2t[5;7]R2u[13;9]Z3a2	4
13	2	$[x_1\ \ x_2\ \ 0\ \ 0\ \ x_1\ \ 0\ \ 0\ \ 0\ \ x_1]$	p3g3f1b1B1c1T1t13R1u4Z3a2	2
14	2	$[x_1\ \ x_2\ \ x_1\ \ -x_2\ \ x_1\ \ 0\ \ 0\ \ 0\ \ x_1]$	p1g1f[1;2]b1B1c1T2t1R2u13Z3a2	2
15	3	$[x_1\ \ 0\ \ 0\ \ 0\ \ x_1\ \ x_6\ \ 0\ \ x_8\ \ x_1]$	p3g1f2b1B1c1T1t13R2u[17;21]Z3a2	2
16	3	$[x_1\ \ 0\ \ 0\ \ 0\ \ x_5\ \ x_6\ \ 0\ \ -x_6\ \ x_5]$	p3g1f1b1B1c1T1t13R3u[17;21]Z3a2 ⊕ p3g1f1b1B1c1T1t13R4u[17;21]Z3a[1;2]	6
17	3	$[x_1\ \ 0\ \ 0\ \ 0\ \ x_5\ \ x_6\ \ 0\ \ 0\ \ x_1]$	p1g1f3b1B1c[1;2]T2t[5;7]R1u4Z3a2 ⊕ p1g1f3b1B1c2T1t13R1u4Z3a2 ⊕ p3g1f3b1B1c2T1t13R1u4Z3a2	6
18	3	$[x_1\ \ 0\ \ x_1\ \ 0\ \ x_5\ \ x_6\ \ 0\ \ 0\ \ x_1]$	p1g1f3b1B1c[1;2]T2t3R1u4Z3a2 ⊕ p1g1f3b1B1c2T2t1R1u4Z3a2	3
19	3	$[x_1\ \ 0\ \ x_3\ \ 0\ \ x_1\ \ 0\ \ -x_3\ \ x_8\ \ x_1]$	p3g2f[1;2]b1B1c1T1t13R2u[1;5]Z3a2	4
20	3	$[x_1\ \ 0\ \ x_3\ \ 0\ \ x_1\ \ x_6\ \ -x_3\ \ -x_6\ \ x_1]$	p3g1f1b1B1c1T1t13R2u[2;6;19;23]Z3a2	4
21	3	$[x_1\ \ 0\ \ x_3\ \ 0\ \ x_1\ \ x_6\ \ 0\ \ -x_6\ \ x_1]$	p3g2f1b1B1c1T1t13R2u[17;21]Z3a2	2
22	3	$[x_1\ \ 0\ \ x_3\ \ 0\ \ x_1\ \ x_6\ \ 0\ \ 0\ \ x_1]$	p1g1f[1;2]b1B1c3T1t13R1u4Z3a2 ⊕ p1g1f[1;2]b1B1c3T2t[1;3;5;7]R1u4Z3a2 ⊕ p3g[1;2]f[1;2]b1B1c3T1t13R1u4Z3a2	14
23	3	$[x_1\ \ 0\ \ x_3\ \ 0\ \ x_5\ \ 0\ \ -x_3\ \ 0\ \ x_1]$	p3g1f[1-3]b1B1c1T1t13R3u[1;5]Z3a2 ⊕ p3g1f3b1B1c1T1t13R2u[1;5]Z3a2 ⊕ p3g1f[1-3]b1B1c1T1t13R4u[1;5]Z3a[1;2]	20
24	3	$[x_1\ \ x_2\ \ 0\ \ -x_2\ \ x_1\ \ 0\ \ 0\ \ 0\ \ x_9]$	pp[1;3]g1f1b1B1c1T1t13R3u[9;13]Z3a2 ⊕ p[1;3]g1f1b1B1c1T1t13R4u[9;13]Z3a[1;2]	12
25	3	$[x_1\ \ x_2\ \ 0\ \ -x_2\ \ x_1\ \ x_6\ \ 0\ \ -x_6\ \ x_1]$	p3g1f1b1B1c1T1t13R2u[11;15;18;22]Z3a2	4
26	3	$[x_1\ \ x_2\ \ 0\ \ -x_2\ \ x_1\ \ x_6\ \ 0\ \ 0\ \ x_6]$	p1g1f1b1B1c1T2t[5;7]R[3;4]u[9;13]Z3a2	8
27	3	$[x_1\ \ x_2\ \ 0\ \ 0\ \ x_5\ \ 0\ \ 0\ \ 0\ \ 0x_1]$	p3g3f3b1B1c1T1t13R1u4Z3a2	1
28	3	$[x_1\ \ x_2\ \ 0\ \ x_4\ \ x_1\ \ 0\ \ 0\ \ 0\ \ x_1]$	p[1;3]g1f2b1B1c1T1t13R2u[9;13]Z3a2	4
29	3	$[x_1\ \ x_2\ \ x_1\ \ -x_2\ \ x_1\ \ x_6\ \ 0\ \ 0\ \ x_1]$	p1g1f1b1B1c1T2t3R2u[9;13]Z3a2	2
30	3	$[x_1\ \ x_2\ \ x_1\ \ x_4\ \ x_1\ \ 0\ \ 0\ \ 0\ \ x_1]$	p1g1f2b1B1c1T2t1R2u[9;13]Z3a2	2
31	3	$[x_1\ \ x_2\ \ x_3\ \ -x_2\ \ x_1\ \ 0\ \ -x_3\ \ 0\ \ x_1]$	p3g1f1b1B1c1T1t13R2u[3;7;10;14]Z3a2	4
32	3	$[x_1\ \ x_2\ \ x_3\ \ -x_2\ \ x_1\ \ 0\ \ 0\ \ 0\ \ x_3]$	p1g1f1b1B1c1T2t1R[3;4]u[9;13]Z3a2	4
33	3	$[x_1\ \ x_2\ \ x_3\ \ 0\ \ x_1\ \ 0\ \ 0\ \ 0\ \ x_1]$	p3g3f[1;2]b1B1c2T1t13R1u4Z3a2	2
34	4	$[x_1\ \ 0\ \ 0\ \ 0\ \ x_5\ \ x_6\ \ 0\ \ x_8\ \ x_1]$	p3g1f3b1B1c1T1t13R2u[17;21]Z3a2	2
35	4	$[x_1\ \ 0\ \ 0\ \ 0\ \ x_5\ \ x_6\ \ 0\ \ x_8\ \ x_5]$	p3g1f2b1B1c1T1t13R3u[17;21]Z3a2 ⊕ p3g1f2b1B1c1T1t13R4u[17;21]Z3a[1;2]	6
36	4	$[x_1\ \ 0\ \ x_3\ \ 0\ \ x_1\ \ x_6\ \ -x_3\ \ x_8\ \ x_1]$	p3g1f2b1B1c1T1t13R2u[2;6;19;23]Z3a2	4
37	4	$[x_1\ \ 0\ \ x_3\ \ 0\ \ x_1\ \ x_6\ \ 0\ \ x_8\ \ x_1]$	p3g2f2b1B1c1T1t13R2u[17;21]Z3a2	2
38	4	$[x_1\ \ 0\ \ x_3\ \ 0\ \ x_5\ \ 0\ \ -x_3\ \ x_8\ \ x_1]$	p3g2f3b1B1c1T1t13R2u[1;5]Z3a2	2
39	4	$[x_1\ \ 0\ \ x_3\ \ 0\ \ x_5\ \ x_6\ \ 0\ \ 0\ \ x_1]$	p[1;3]g1f3b1B1c3T1t13R1u4Z3a2 ⊕ p1g1f3b1B1c3T2t[1;3;5;7]R1u4Z3a2 ⊕ p3g2f3b1B1c[2;3]T1t13R1u4Z3a2	8
40	4	$[x_1\ \ x_2\ \ -x_2\ \ 0\ \ x_5\ \ x_6\ \ 0\ \ -x_6\ \ x_5]$	p3g2f1b1B1c1T1t13R4u21Z3a1	1
41	4	$[x_1\ \ x_2\ \ 0\ \ x_4\ \ x_1\ \ 0\ \ 0\ \ 0\ \ x_9]$	p[1;3]g1f2b1B1c1T1t[9;13]R3u13Z3a2 ⊕ p[1;3]g1f2b1B1c1T1t13R4u[9;13]Z3a[1;2]	12
42	4	$[x_1\ \ x_2\ \ 0\ \ x_4\ \ x_1\ \ x_6\ \ 0\ \ 0\ \ x_1]$	p1g1f2b1B1c1T2t[5;7]R2u[9;13]Z3a2	4
43	4	$[x_1\ \ x_2\ \ 0\ \ x_4\ \ x_5\ \ 0\ \ 0\ \ 0\ \ x_1]$	p[1;3]g1f3b1B1c1T1t13R2u[9;13]Z3a2 ⊕ p2g1f[1-3]b[2;3]B[1;2]c1T1t13R2u [11-15]Z3a2	28

TABLE A1—*Continued*

#	p	Simplified form of homography	Generated by	n
44	4	$[x_1\ \ x_2\ \ 0\ \ x_4\ \ x_5\ \ 0\ \ 0\ \ 0\ \ x_5]$	p2g1f[1;3]b[2;3]B[1;2]c1T1t13R2u [10;14]Z3a2	16
45	4	$[x_1\ \ x_2\ \ 0\ \ x_4\ \ x_5\ \ x_1\ \ 0\ \ 0\ \ x_1]$	p2g1f1b[2;3]B[1;2]c1T2t5R2u[11;15]Z3a2	8
46	4	$[x_1\ \ x_2\ \ 0\ \ x_4\ \ x_5\ \ x_5\ \ 0\ \ 0\ \ x_5]$	p2g1f1b2B1c1T2t5R2u[10;14]Z3a2	8
47	4	$[x_1\ \ x_2\ \ x_1\ \ x_4\ \ x_1\ \ x_6\ \ 0\ \ 0\ \ x_1]$	p1g1f2b1B1c1T2t3R2u[9;13]Z3a2	2
48	4	$[x_1\ \ x_2\ \ x_1\ \ x_4\ \ x_5\ \ 0\ \ 0\ \ 0\ \ x_1]$	p1g1f3b1B1c1T2t1R2u[9;13]Z3a2 ⊕ p2g1f[1-3]b[2;3]B[1;2]c1T2t1R2u [11;15]Z3a2	26
49	4	$[x_1\ \ x_2\ \ x_2\ \ 0\ \ x_5\ \ x_6\ \ 0\ \ -x_6\ \ x_5]$	p3g2f1b1B1c1T1t13R4u17Z3a1	1
50	4	$[x_1\ \ x_2\ \ x_3\ \ -x_2\ \ x_1\ \ x_6\ \ -x_3\ \ -x_6\ \ x_1]$	p3g1f1b1B1c1T1t13R2u[4;8;12;16;20;24]Z3a2	6
51	4	$[x_1\ \ x_2\ \ x_3\ \ -x_2\ \ x_1\ \ x_6\ \ 0\ \ 0\ \ x_1]$	p[1;3]g1f1b1B1c[2;3]T1t13R2u[9-13]Z3a2 ⊕ p1g1f1b1B1c[2;3]T2t[1;3;5;7]R2u [9;13]Z3a2 ⊕ p2g1f1b[2;3]B[1;2]c[1-3]T2t9R2u [10-12;14-16]Z1a2 ⊕ p2g1f1b[2;3]B[1;2]c[1-3]T2t10R2u [11;15]Z1a2 ⊕ p2g1f1b[2;3]B[1;2]c[1-3]T2t11R2u [10;14]Z1a2	144
52	4	$[x_1\ \ x_2\ \ x_3\ \ -x_2\ \ x_1\ \ x_6\ \ 0\ \ 0\ \ x_3]$	p1g1fb1B1c1T2t3R[3;4]u[9;13]Z3a2	4
53	4	$[x_1\ \ x_2\ \ x_3\ \ 0\ \ x_1\ \ 0\ \ -x_3\ \ x_8\ \ x_1]$	p3g3f[1;2]b1B1c1T1t13R2u[1;5]Z3a2	4
54	4	$[x_1\ \ x_2\ \ x_3\ \ 0\ \ x_1\ \ x_6\ \ 0\ \ -x_6\ \ x_1]$	p3g3f1b1B1c1T1t13R2u[17;21]Z3a2	2
55	4	$[x_1\ \ x_2\ \ x_3\ \ 0\ \ x_1\ \ x_6\ \ 0\ \ 0\ \ x_1]$	p3g3f[1;2]b1B1c3T1t13R1u4Z3a2	2
56	4	$[x_1\ \ x_2\ \ x_3\ \ 0\ \ x_5\ \ 0\ \ -x_3\ \ -x_2\ \ x_1]$	p3g2f[1-3]b1B1c1T1t13R4u5Z3a1	3
57	4	$[x_1\ \ x_2\ \ x_3\ \ 0\ \ x_5\ \ 0\ \ -x_3\ \ x_2\ \ x_1]$	p3g2f[1-3]b1B1c1T1t13R4u1Z3a1	3
58	4	$[x_1\ \ x_2\ \ x_3\ \ x_4\ \ x_1\ \ 0\ \ -x_3\ \ 0\ \ x_1]$	p3g1f2b1B1c1T1t13R2u[3;7;10;14]Z3a2	4
59	4	$[x_1\ \ x_2\ \ x_3\ \ x_4\ \ x_1\ \ 0\ \ 0\ \ 0\ \ x_3]$	p1g1f2b1B1c1T2t1R[3;4]u[9;13]Z3a2 ⊕ p2g1f[1;2]b[2;3]B[1;2]c1T2t1R2u [10;14]Z3a2	20
60	5	$[x_1\ \ 0\ \ 0\ \ 0\ \ x_5\ \ x_6\ \ 0\ \ x_3\ \ x_9]$	p3g1f3b1B1c1T1t13R3u[17;21]Z3a2 ⊕ p3g1f3b1B1c1T1t13R4u[17;21]Z3a[1;2]	6
61	5	$[x_1\ \ 0\ \ x_3\ \ 0\ \ x_1\ \ x_6\ \ x_7\ \ x_8\ \ x_1]$	p3g2f[1;2]b1B1c1T1t13R2u [2;6;19;23]Z3a2	8
62	5	$[x_1\ \ 0\ \ x_3\ \ 0\ \ x_5\ \ x_6\ \ -x_3\ \ x_8\ \ x_1]$	p3g1f3b1B1c1T1t13R2u[2;6;19;23]Z3a2	4
63	5	$[x_1\ \ 0\ \ x_3\ \ 0\ \ x_5\ \ x_6\ \ 0\ \ x_8\ \ x_1]$	p3g2f3b1B1c1T1t13R2u[17;21]Z3a2	2
64	5	$[x_1\ \ x_2\ \ 0\ \ x_4\ \ x_1\ \ x_6\ \ 0\ \ 0\ \ x_9]$	p1g1f2b1B1c1T2t[5;7]R[3;4]u[9;13]Z3a2	8
65	5	$[x_1\ \ x_2\ \ 0\ \ x_4\ \ x_1\ \ x_6\ \ 0\ \ x_8\ \ x_1]$	p3g1f2b1B1c1T1t13R2u[11;15;18;22]Z3a2	4
66	5	$[x_1\ \ x_2\ \ 0\ \ x_4\ \ x_5\ \ 0\ \ 0\ \ 0\ \ x_9]$	p1g1f3b1B1c1T1t13R[3;4]u[9;13]Z3a2 ⊕ p1g1f3b1B1c1T1t13R4u[9;13]Z3a1 ⊕ p2g1f[1-3]b[2;3]B[1;2]c1T1t13R[2-4]u [12;16]Z3a2 ⊕ p2g1f3b[2;3]B[1;2]c1T1t13R2u [10;14]Z3a2 ⊕ p2g1f[1-3]b[2;3]B[1;2]c1T1t13R[3;4]u [10;11;14;15]Z3a2 ⊕ p2g1f[1-3]b[2;3]B[1;2]c1T1t13R4u [10-12;14-16]Z3a1 ⊕ p3g1f3b1B1c1T1t13R3u[9;13]Z3a2 ⊕ p3g1f3b1B1c1T1t13R4u[9;13]Z3a[1;2] ⊕ p3g[2;3]f[1-3]b1B1c1T1t13R[2-4]u [9;13]Z3a2 ⊕ p3g[2;3]f[1-3]b1B1c1T1t13R4u[9-13]Z3a1	308
67	5	$[x_1\ \ x_2\ \ 0\ \ x_4\ \ x_5\ \ x_6\ \ 0\ \ 0\ \ x_1]$	p1g1f3b1B1c1T2t[5;7]R2u[9;13]Z3a2 ⊕ p2g1f[2;3]b[2;3]B[1;2]c1T2t5R2u [11;15]Z3a2	20
68	5	$[x_1\ \ x_2\ \ 0\ \ x_4\ \ x_5\ \ x_6\ \ 0\ \ 0\ \ x_5]$	p2g1f2b[2;3]B[1;2]c1T2t5R2u[10;14]Z3a2	8
69	5	$[x_1\ \ x_2\ \ 0\ \ x_4\ \ x_5\ \ x_6\ \ 0\ \ 0\ \ x_6]$	p2g1f1b[2;3]B[1;2]c1T2t5R[2-4]u [12;16]Z3a2 ⊕ p2g1f1b[2;3]B[1;2]c1T2t5R[3;4]u [10;11;14;15]Z3a2	56
70	5	$[x_1\ \ x_2\ \ x_1\ \ x_4\ \ x_5\ \ x_6\ \ 0\ \ 0\ \ x_1]$	p1g1f3b1B1c1T2t3R2u[9;13]Z3a2 ⊕ p2g1f[1-3]b[2;3]B[1;2]c1T2t3R2u [11;15]Z3a2	26

TABLE A1—*Continued*

#	p	Simplified form of homography	Generated by	n
71	5	$[x_1 \; x_2 \; x_3 \; -x_2 \; x_1 \; x_6 \; 0 \; 0 \; x_9]$	p1g1f1b1B1c[1-3]T2t12R4u[9;13]Z3a1 ⊕ p1g1f1b1B1c[2;3]T1t13R[3;4]u [9;13]Z3a2 ⊕ p1g1f1b1B1c[2;3]T2t13R4u[9;13]Z3a1 ⊕ p1g1f1b1B1c[2;3]T2t[1;3;5;7]R[3;4]u [9-13]Z3a2 ⊕ p3g1f1b1B1c[2;3]T1t13R3u[9;13]Z3a2 ⊕ p3g1f1b1B1c[2;3]T1t13R4u[9;13]Z3a[1;2]	62
72	5	$[x_1 \; x_2 \; x_3 \; 0 \; x_1 \; x_6 \; 0 \; x_8 \; x_1]$	p3g3f2b1B1c1T1t13R2u[17;21]Z3a2	2
73	5	$[x_1 \; x_2 \; x_3 \; 0 \; x_5 \; 0 \; -x_3 \; x_8 \; x_1]$	p3g[2;3]f[1-3]b1B1c1T1t13R[3;4]u [;5]Z3a2 ⊕ p3g3f[1-3]b1B1c1T1t13R4u[1;5]Z3a1 ⊕ p3g3f3b1B1c1T1t13R2u[1;5]Z3a2	32
74	5	$[x_1 \; x_2 \; x_3 \; 0 \; x_5 \; x_6 \; 0 \; -x_6 \; x_5]$	p3g[2;3]f1b1B1c1T1t13R[3;4] u[17;21]Z3a2 ⊕ p3g3f1b1B1c1T1t13R4u[17;21]Z3a1	10
75	5	$[x_1 \; x_2 \; x_3 \; 0 \; x_5 \; x_6 \; 0 \; 0 \; x_1]$	p3g3f3b1B1c[2;3]T1t13R1u4Z3a2	2
76	5	$[x_1 \; x_2 \; x_3 \; x_4 \; x_1 \; x_6 \; 0 \; 0 \; x_1]$	p1g1f2b1B1c[2;3]T2t[1;3;5;7]R2u [9;13]Z3a2 ⊕ p[1;3]g1f2b1B1c[2;3]T1t13R2 u[9;13]Z3a2 ⊕ p2g1f2b[2;3]B[1;2]c[1-3]T2t10R2 u[11;16]Z1a2 ⊕ p2g1f2b[2;3]B[1;2]c[1-3]T2t11R2 u[10;14]Z1a2 ⊕ p2g1f2b[2;3]B[1;2]c[1-3]T2t9R2u [10-12;14-16]Z1a2	144
77	5	$[x_1 \; x_2 \; x_3 \; x_4 \; x_1 \; x_6 \; 0 \; 0 \; x_3]$	p1g1f2b1B1c1T2t3R[3;4]u[9;13]Z3a2 ⊕ p2g1f[1;2]b[2;3]B[1;2]c1T2t3R2u [10;14]Z3a2	20
78	5	$[x_1 \; x_2 \; x_3 \; x_4 \; x_5 \; 0 \; -x_3 \; 0 \; x_1]$	p3g1f3b1B1c1T1t13R2u[3;7;10;14]Z3a2	4
79	5	$[x_1 \; x_2 \; x_3 \; x_4 \; x_5 \; 0 \; 0 \; 0 \; x_3]$	p1g1f3b1B1c1T2t1R[3;4]u[9;13]Z3a2 ⊕ p2g1f[1-3]b[2;3]B[1;2]c1T2t1R[2-4] u[12;16]Z3a2 ⊕ p2g1f[1-3]b[2;3]B[1;2]c1T2t1R[3;4] u[10;11;14;15]Z3a2	180
80	5	$[x_1 \; x_2 \; x_3 \; x_4 \; x_5 \; x_1 \; 0 \; 0 \; x_1]$	p2g1f1b[2;3]B[1;2]c1T2t6R2u[11;15]Z3a2	8
81	5	$[x_1 \; x_2 \; x_3 \; x_4 \; x_5 \; x_5 \; 0 \; 0 \; x_5]$	p2g1f1b[2;3]B[1;2]c1T2t6R2u[10;14]Z3a2	8
82	6	$[x_1 \; 0 \; x_3 \; 0 \; x_5 \; x_6 \; x_7 \; x_8 \; x_1]$	p3g2f3b1B1c1T1t13R2u[2;6;19;23]Z3a2	4
83	6	$[x_1 \; x_2 \; 0 \; x_4 \; x_5 \; x_6 \; 0 \; 0 \; x_9]$	p1g1f3s1b1B1c1T2t[5-7]r1U1R[3-4]u[9;13] Z3W3D3a2 ⊕ p2g1f[2;3]s1b[2-3]B[1;2]c1T2t5r1U1R [3-4]u[10-12;14-16]Z3W3D3a2 ⊕ p2g1f2s1b[2-3]B[1;2]c1T2t5r1U1R2u [12;16]Z3W3D3a2 ⊕ p2g1f3s1b[2-3]B[1;2]c1T2t5r1U1R2u [10;12;14;16]Z3W3D3a2	128
84	6	$[x_1 \; x_2 \; 0 \; x_4 \; x_5 \; x_6 \; 0 \; x_8 \; x_1]$	p3g1f3b1B1c1T1t13R2u[11;15;18;22]Z3a2	4
85	6	$[x_1 \; x_2 \; x_3 \; -x_2 \; x_5 \; x_6 \; -x_3 \; x_6 \; x_9]$	p3g1f1b1B1c1T1t13R3u[3;7;10;14]Z3a2 ⊕ p3g1f1b1B1c1T1t13R4u[3;7;10;14]Z3a[1;2]	12
86	6	$[x_1 \; x_2 \; x_3 \; -x_2 \; x_5 \; x_6 \; x_3 \; -x_6 \; x_9]$	p3g1f1b1B1c1T1t13R3u[11;15;18;22]Z3a2 ⊕ p3g1f1b1B1c1T1t13R4u[11;15;18;22]Z3a [1;2]	12
87	6	$[x_1 \; x_2 \; x_3 \; 0 \; x_1 \; x_6 \; x_7 \; x_8 \; x_1]$	p3g3f[1;2]b1B1c1T1t13R2u[2;6;19;23]Z3a2	8
88	6	$[x_1 \; x_2 \; x_3 \; 0 \; x_5 \; x_6 \; 0 \; x_8 \; x_1]$	p3g3f3b1B1c1T1t13R2u[17;21]Z3a2	2

TABLE A1—*Continued*

#	p	Simplified form of homography	Generated by	n
89	6	$[x_1\ \ x_2\ \ x_3\ \ 0\ \ x_5\ \ x_6\ \ 0\ \ x_8\ \ x_5]$	p3g[2;3]f2b1B1c1T1t13R3u[17;21]Z3a2 ⊕ p3g[2;3]f2b1B1c1T1t13R4u[17;21]Z3a[1;2]	12
90	6	$[x_1\ \ x_2\ \ x_3\ \ x_2\ \ x_5\ \ x_6\ \ -x_3\ \ -x_6\ \ x_9]$	p3g1f1b1B1c1T1t13R3u[2;6;19;23]Z3a2 ⊕ p3g1f1b1B1c1T1t13R4u[2;6;19;23]Z3a[1;2]	12
91	6	$[x_1\ \ x_2\ \ x_3\ \ x_4\ \ x_1\ \ x_6\ \ -x_3\ \ x_8\ \ x_1]$	p3g1f2b1B1c1T1t13R2u[4;8;12;16;20;24]Z3a2	6
92	6	$[x_1\ \ x_2\ \ x_3\ \ x_4\ \ x_1\ \ x_6\ \ 0\ \ 0\ \ x_9]$	p1g1f2b1B1c[1-3]T2t12R4u[9;13]Z3a1 ⊕ p1g1f2b1B1c2T1t13R[3;4]u[9;13]Z3a2 ⊕ p1g1f2b1B1c[2;3]T1t13R4u[9;13]Z3a1 ⊕ p1g1f2b1B1c[2;3]T2t[1;3;5;7]R[3;4]u[9;13]Z3a2 ⊕ p3g1f2b1B1c[2;3]T1t13R[3;4]u[9;13]Z3a2 ⊕ p3g1f2b1B1c[2;3]T1t13R4u[9;13]Z3a1 ⊕ p2g1f2b[2;3]B[1;2]c[1-3]T2t10R[3-4]u[11;15]Z1a2 ⊕ p2g1f2b[2;3]B[1;2]c[1-3]T2t11R[3;4]u[10;14]Z1a2 ⊕ p2g1f2b[2;3]B[1;2]c[1-3]T2t9R[3;4]u[10-12;14-16]Z1a2	302
93	6	$[x_1\ \ x_2\ \ x_3\ \ x_4\ \ x_5\ \ x_6\ \ 0\ \ 0\ \ x_1]$	p1g1f3b1B1c[2;3]T1t13R2u[9;13]Z3a2 ⊕ p1g1f3b1B1c[2;3]T2t[1;3;5;7]R2u[9;13]Z3a2 ⊕ p2g1f[1-3]b[2;3]B[1;2]c[1-3]T2t[2;4;7;8;9;10-12]R2u[11;15]Z3a2 ⊕ p2g1f[2;3]b[2;3]B[1;2]c1T2t6R2u[11;15]Z3a2 ⊕ p2g1f[1-3]b[2;3]B[1;2]c[2;3]T2t[1;3;5;6]R2u[11;15]Z3a2 ⊕ p2g1f[1-3]b[2;3]B[1;2]c[2;3]T1t13R2u[11;15]Z3a2 ⊕ p3g1f3b1B1c[2;3]T1t13R2u[9;13]Z3a2 ⊕ p2g1f[1;2]b[2;3]B[1;2]c[1-3]T2t[10-12]R2u[11;15]Z2a2 ⊕ p2g1f[1;2]b[2;3]B[1;2]c[1-3]T2t[2;4;11;12]R2u[11;15]Z1a2 ⊕ p2g1f[1;2]b[2;3]B[1;2]c[1-3]T2t[2;11]R2u[12;16]Z1a2 ⊕ p2g1f3b2B1c1T2t[2;4;9;10-12]R2u[11;15]Z1a2 ⊕ p2g1f3b2B1c1T2t[9;11]R2u[10;14]Z1a2 ⊕ p2g1f3b2B1c1T2t[2;9;11]R2u[12;16]Z1a2 ⊕ p2g1f3b2B1c1T2t[10-12]R2u[11;15]Z2a2	1624
94	6	$[x_1\ \ x_2\ \ x_3\ \ x_4\ \ x_5\ \ x_6\ \ 0\ \ 0\ \ x_3]$	p1g1f3b1B1c1T2t3R[3;4]u[9;13]Z3a2 ⊕ p2g[1-3]f1b[2;3]B[1;2]c1T2t3R[2-4]u[12;16]Z3a2 ⊕ p2g1f[1-3]b[2;3]B[1;2]c1T2t3R[3;4]u[10;11;14;15]Z3a2	180
95	6	$[x_1\ \ x_2\ \ x_3\ \ x_4\ \ x_5\ \ x_6\ \ 0\ \ 0\ \ x_5]$	p2g1f[1;2]b[2;3]B[1;2]c[1-3]T2t[2;4;7-12]R2u[10;14]Z3a2 ⊕ p2g1f[1;2]b[2;3]B[1;2]c[2;3]T1t13R2u[10;14]Z3a2 ⊕ p2g1f2b[2;3]B[1;2]c1T2t6R2u[10;14]Z3a2 ⊕ p2g1f[1;2]b[2;3]B[1;2]c[2;3]T2t[1;3;5;6]R2u[10;14]Z3a2 ⊕ p2g1f[1;2]b[2;3]B[1;2]c[1-3]T2t[10-12]R2u[10;14]Z2a2 ⊕ p2g1f[1;2]b[2;3]B[1;2]c[1-3]T2t[7;8;10;12]R2u[10;14]Z1a2 ⊕	984

TABLE A1—*Continued*

#	p	Simplified form of homography	Generated by	n
			`p2g1f[1;2]b[2;3]B[1;2]c[1-3]T2t [7;10]R2u[12;16]Z1a2`	
96	6	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ 0\ 0\ x_6]$	`p2g1f1b[2;3]B[1;2]c1T2t6R[2-4] u[12;16]Z3a2` ⊕ `p2g1f1b[2;3]B[1;2]c1T2t6R[3;4] u[10;11;14;15]Z3a2`	56
97	7	$[x_1\ 0\ x_3\ x_4\ x_5\ x_6\ x_7\ 0\ x_9]$	`p3g1f[1-3]b1B1c[2;3]T1t13R[2;3] u[1;5]Z3a2` ⊕ `p3g1f[1-3]b1B1c[2;3]T1t13R4u [1;5]Z3a[1;2]`	48
98	7	$[x_1\ x_2\ x_3\ 0\ x_5\ x_6\ 0\ x_8\ x_9]$	`p3g[1-3]f[1-3]b1B1c[2;3]T1t13R[2-4] u[17;21]Z3a2` ⊕ `p3g[1-3]f[1-3]b1B1c[2;3]T1t13R4u [17;21]Z3a1` ⊕ `p3g[2;3]f3b1B1c1T1t13R[3;4] u[17;21]Z3a2` ⊕ `p3g[2;3]f3b1B1c1T1t13R4u[17;21]Z3a1`	156
99	7	$[x_1\ x_2\ x_3\ 0\ x_5\ x_6\ x_7\ x_8\ x_1]$	`p3g3f3b1B1c1T1t13R2u[2;6;19;23]Z3a2`	4
100	7	$[x_1\ x_2\ x_3\ x_4\ x_5\ 0\ -x_3\ x_8\ x_9]$	`p3g[2;3]f[1-3]b1B1c1T1t13R2u [3;7;10;14]Z3a2`	24
101	7	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ -x_3\ x_8\ x_1]$	`p3g1f3b1B1c1T1t13R2u [4;8;12;16;20;24]Z3a2`	6
102	7	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ 0\ -x_6\ x_9]$	`p3g[2;3]f1b1B1c1T1t13R2u [11;15;18;22]Z3a2`	8
103	7	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ 0\ 0\ x_9]$	`p3g1f3b1B1c[2;3]T1t13R[3;4] u[9;13Z3a2` ⊕ `p3g1f3b1B1c[2;3]T1t13R4u[9;13]Z3a1` ⊕ `p3g[2;3]f[1-3]b1B1c[2;3]T1t13R[2-4] u[9;13]Z3a2` ⊕ `p3g[2;3]f[1-3]b1B1c[2;3]T1t13R4u [9;13]Z3a1` `and all other para and ortho`	10318
104	8	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ -x_3\ x_8\ x_9]$	`p3g1f2b1B1c1T1t13R3u10Z3a2`	48
105	8	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ 0\ x_8\ x_9]$	`p3g[1-3]f[1-3]s1b1B1c[2;3]T1t13r1U1R2u [11;15;18;22]Z3W3D3a2` ⊕ `p3g[2;3]f[2;3]s1b1B1c1T1t13r1U1R2u [11;15;18;22]Z3W3D3a2`	88
106	8	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_3\ x_8\ x_9]$	`p3g1f[2;3]b1B1c1T1t13R3u [11;15;18;22]Z3a2` ⊕ `p3g1f[2;3]b1B1c1T1t13R4u [11;15;18;22]Z3a[1;2]`	24
107	8	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ 0\ x_9]$	`p3g1f[1-3]b1B1c[2;3]T1t13R2u [3;7;10;14]Z3a2`	24
108	9	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9]$	`all the other perspective cases`	3630

APPENDIX B

Table of Particular Forms of Fundamental Matrices

Please note that in Table B1, as it is for Table A1, the number of parameters **p** does not take into account that the fundamental matrix is defined up to a scale factor and that its determinant is zero. This is done is the numerical implementation.

TABLE B1
Particular Forms of Fundamental Matrices

#	p	Simplified form of fundamental matrices	For example generated by:	n
1	1	$[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ x_6\ \ 0\ \ -x_6\ \ 0]$	g1f1s1c1t1R1u24Z3a2	24
2	1	$[0\ \ 0\ \ x_3\ \ 0\ \ 0\ \ 0\ \ -x_3\ \ 0\ \ 0]$	g1f1s1c1t5R1u24Z3a2	4
3	1	$[0\ \ x_2\ \ 0\ \ -x_2\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]$	g1f1s1c1t9R1u24Z3a2	5
4	2	$[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ x_6\ \ 0\ \ -x_6\ \ x_9]$	g1f1s1c3t1R1u24Z3a2	12
5	2	$[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ x_6\ \ 0\ \ x_8\ \ 0]$	g1f3s1c1t1R1u24Z3a2	6
6	2	$[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ x_6\ \ x_7\ \ -x_6\ \ 0]$	g1f1s1c1t1R2u13Z2a2	16
7	2	$[0\ \ 0\ \ 0\ \ 0\ \ x_5\ \ x_6\ \ 0\ \ -x_6\ \ x_5]$	g1f1s1c1t1R2u17Z1a2	396
8	2	$[0\ \ 0\ \ 0\ \ x_4\ \ 0\ \ x_6\ \ 0\ \ -x_6\ \ 0]$	g1f1s1c1t1R2u1Z2a2	16
9	2	$[0\ \ 0\ \ x_3\ \ 0\ \ 0\ \ 0\ \ -x_3\ \ 0\ \ x_9]$	g1f1s1c3t5R1u24Z3a2	2
10	2	$[0\ \ 0\ \ x_3\ \ 0\ \ 0\ \ 0\ \ -x_3\ \ x_8\ \ 0]$	g1f1s1c1t5R2u13Z2a2	8
11	2	$[0\ \ 0\ \ x_3\ \ 0\ \ 0\ \ 0\ \ x_7\ \ 0\ \ 0]$	g1f1s2c1t5R1u24Z3a2	4
12	2	$[0\ \ 0\ \ x_3\ \ 0\ \ 0\ \ x_6\ \ -x_3\ \ -x_6\ \ 0]$	g1f1s1c1t3R1u24Z3a2	17
13	2	$[0\ \ x_2\ \ 0\ \ -x_2\ \ 0\ \ x_6\ \ 0\ \ -x_6\ \ 0]$	g1f1s1c1t1R1u24Z3a2	8
14	2	$[0\ \ x_2\ \ 0\ \ -x_2\ \ 0\ \ x_6\ \ 0\ \ 0\ \ 0]$	g1f1s1c1t9R2u1Z2a2	24
15	2	$[0\ \ x_2\ \ 0\ \ -x_2\ \ x_5\ \ 0\ \ 0\ \ 0\ \ 0]$	g2f3s1c1t9R1u24Z3a2	4
16	2	$[0\ \ x_2\ \ 0\ \ x_4\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]$	g1f1s2c1t9R1u24Z3a2	3
17	2	$[0\ \ x_2\ \ x_3\ \ -x_2\ \ 0\ \ 0\ \ -x_3\ \ 0\ \ 0]$	g1f1s1c1t10R1u24Z3a2	4
18	2	$[0\ \ x_2\ \ x_3\ \ -x_2\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]$	g1f1s1c1t9R2u17Z2a2	12
19	2	$[0\ \ x_2\ \ x_3\ \ 0\ \ 0\ \ 0\ \ -x_3\ \ 0\ \ 0]$	g1f1s1c1t5R2u17Z2a2	8
20	2	$[x_1\ \ 0\ \ x_3\ \ 0\ \ 0\ \ 0\ \ -x_3\ \ 0\ \ x_1]$	g1f1s1c1t5R2u1Z1a2	66
21	2	$[x_1\ \ x_2\ \ 0\ \ -x_2\ \ x_1\ \ 0\ \ 0\ \ 0\ \ 0]$	g1f1s1c1t10R2u11Z1a2	198
22	3	$[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ x_6\ \ 0\ \ x_8\ \ x_9]$	g1f3s1c2t1R1u24Z3a2	12
23	3	$[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ x_6\ \ x_7\ \ -x_6\ \ x_9]$	g1f1s1c2t1R2u13Z2a2	32
24	3	$[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ x_6\ \ x_7\ \ x_8\ \ 0]$	g1f1s1c1t1R3u13Z2a2	200
25	3	$[0\ \ 0\ \ 0\ \ 0\ \ x_5\ \ x_6\ \ 0\ \ -x_6\ \ x_9]$	g1f2s1c1t1R2u17Z1a2	396
26	3	$[0\ \ 0\ \ 0\ \ 0\ \ x_5\ \ x_6\ \ x_7\ \ -x_6\ \ x_5]$	g1f1s1c1t1R2u11Z2a2	16
27	3	$[0\ \ 0\ \ 0\ \ x_4\ \ 0\ \ x_6\ \ 0\ \ x_8\ \ 0]$	g1f1s1c1t1R3u1Z2a2	56
28	3	$[0\ \ 0\ \ 0\ \ x_4\ \ 0\ \ x_6\ \ x_7\ \ -x_6\ \ 0]$	g1f1s1c1t1R2u10Z2a2	32
29	3	$[0\ \ 0\ \ 0\ \ x_4\ \ x_5\ \ x_6\ \ 0\ \ -x_6\ \ 0]$	g2f1s1c1t1R2u1Z2a2	32
30	3	$[0\ \ 0\ \ 0\ \ x_4\ \ x_5\ \ x_6\ \ 0\ \ -x_6\ \ x_5]$	g1f1s1c1t1R2u19Z2a2	16
31	3	$[0\ \ 0\ \ x_3\ \ 0\ \ 0\ \ 0\ \ -x_3\ \ x_8\ \ x_9]$	g1f1s1c2t5R2u13Z2a2	16
32	3	$[0\ \ 0\ \ x_3\ \ 0\ \ 0\ \ 0\ \ x_7\ \ 0\ \ x_9]$	g1f1s2c2t5R1u24Z3a2	8
33	3	$[0\ \ 0\ \ x_3\ \ 0\ \ 0\ \ 0\ \ x_7\ \ x_8\ \ 0]$	g1f1s1c1t5R3u13Z2a2	64
34	3	$[0\ \ 0\ \ x_3\ \ 0\ \ 0\ \ x_6\ \ -x_3\ \ -x_6\ \ x_9]$	g1f1s1c3t3R1u24Z3a2	13
35	3	$[0\ \ 0\ \ x_3\ \ 0\ \ 0\ \ x_6\ \ -x_3\ \ x_8\ \ 0]$	g2f1s1c1t5R2u13Z2a2	22
36	3	$[0\ \ 0\ \ x_3\ \ 0\ \ 0\ \ x_6\ \ x_7\ \ -x_6\ \ 0]$	g1f1s2c1t3R1u24Z3a2	4
37	3	$[0\ \ x_2\ \ 0\ \ -x_2\ \ 0\ \ x_6\ \ 0\ \ x_8\ \ 0]$	g1f3s1c1t11R1u24Z3a2	2
38	3	$[0\ \ x_2\ \ 0\ \ -x_2\ \ x_5\ \ x_6\ \ 0\ \ -x_6\ \ 0]$	g3f1s1c1t11R1u24Z3a2	4
39	3	$[0\ \ x_2\ \ 0\ \ -x_2\ \ x_5\ \ x_6\ \ 0\ \ 0\ \ 0]$	g2f3s1c1t9R2u1Z2a2	12
40	3	$[0\ \ x_2\ \ 0\ \ x_4\ \ 0\ \ x_6\ \ 0\ \ -x_6\ \ 0]$	g1f1s2c1t11R1u24Z3a2	4
41	3	$[0\ \ x_2\ \ 0\ \ x_4\ \ 0\ \ x_6\ \ 0\ \ 0\ \ 0]$	g1f1s1c1t9R3u1Z2a2	60
42	3	$[0\ \ x_2\ \ 0\ \ x_4\ \ x_5\ \ 0\ \ 0\ \ 0\ \ 0]$	g2f1s2c1t9R1u24Z3a2	6
43	3	$[0\ \ x_2\ \ x_3\ \ -x_2\ \ 0\ \ 0\ \ x_7\ \ 0\ \ 0]$	g1f3s1c1t10R1u24Z3a2	2
44	3	$[0\ \ x_2\ \ x_3\ \ -x_2\ \ 0\ \ x_6\ \ -x_3\ \ -x_6\ \ 0]$	g1f1s1c1t12R1u24Z3a2	40
45	3	$[0\ \ x_2\ \ x_3\ \ -x_2\ \ 0\ \ x_6\ \ 0\ \ 0\ \ 0]$	g1f1s1c1t9R2u19Z2a2	60
46	3	$[0\ \ x_2\ \ x_3\ \ 0\ \ 0\ \ 0\ \ -x_3\ \ x_8\ \ 0]$	g1f1s1c1t5R2u11Z2a2	16
47	3	$[0\ \ x_2\ \ x_3\ \ 0\ \ 0\ \ 0\ \ x_7\ \ 0\ \ 0]$	g1f1s1c1t5R3u17Z2a2	64
48	3	$[0\ \ x_2\ \ x_3\ \ x_4\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]$	g1f1s1c1t9R3u17Z2a2	60
49	3	$[x_1\ \ 0\ \ x_3\ \ 0\ \ 0\ \ 0\ \ -x_3\ \ 0\ \ x_9]$	g1f2s1c1t5R2u1Z1a2	66
50	3	$[x_1\ \ 0\ \ x_3\ \ 0\ \ 0\ \ 0\ \ -x_3\ \ x_8\ \ x_1]$	g1f1s1c1t5R2u10Z2a2	8

TABLE B1—*Continued*

#	p	Simplified form of fundamental matrices	For example generated by:	n
51	3	$[x_1 \quad x_2 \quad 0 \quad -x_2 \quad x_1 \quad x_6 \quad 0 \quad 0 \quad 0]$	g1f1s1c1t9R2u10Z2a2	24
52	3	$[x_1 \quad x_2 \quad x_3 \quad -x_2 \quad x_1 \quad 0 \quad 0 \quad 0 \quad 0]$	g1f1s1c1t9R2u11Z2a2	24
53	3	$[x_1 \quad x_2 \quad x_3 \quad 0 \quad 0 \quad 0 \quad -x_3 \quad 0 \quad x_1]$	g1f1s1c1t5R2u19Z2a2	8
54	4	$[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad x_6 \quad x_7 \quad x_8 \quad x_9]$	g1f1s1c2t1R3u13Z2a2	400
55	4	$[0 \quad 0 \quad 0 \quad 0 \quad x_5 \quad x_6 \quad 0 \quad x_8 \quad x_9]$	g1f1s1c2t1R2u17Z1a2	2772
56	4	$[0 \quad 0 \quad 0 \quad 0 \quad x_5 \quad x_6 \quad x_7 \quad -x_6 \quad x_9]$	g1f2s1c1t1R2u11Z2a2	16
57	4	$[0 \quad 0 \quad 0 \quad 0 \quad x_5 \quad x_6 \quad x_7 \quad x_8 \quad x_5]$	g2f1s1c1t1R2u11Z2a2	32
58	4	$[0 \quad 0 \quad 0 \quad x_4 \quad 0 \quad x_6 \quad x_7 \quad x_8 \quad 0]$	g1f3s1c1t1R2u10Z2a2	16
59	4	$[0 \quad 0 \quad 0 \quad x_4 \quad x_5 \quad x_6 \quad 0 \quad -x_6 \quad x_9]$	g1f2s1c1t1R2u19Z2a2	80
60	4	$[0 \quad 0 \quad 0 \quad x_4 \quad x_5 \quad x_6 \quad 0 \quad x_8 \quad 0]$	g2f1s1c1t1R3u1Z2a2	112
61	4	$[0 \quad 0 \quad 0 \quad x_4 \quad x_5 \quad x_6 \quad x_7 \quad -x_6 \quad x_5]$	g1f1s1c1t1R2u12Z2a2	24
62	4	$[0 \quad 0 \quad x_3 \quad 0 \quad 0 \quad 0 \quad x_7 \quad x_8 \quad x_9]$	g1f1s1c2t5R3u13Z2a2	128
63	4	$[0 \quad 0 \quad x_3 \quad 0 \quad 0 \quad x_6 \quad -x_3 \quad x_8 \quad x_9]$	g2f1s1c2t5R2u13Z2a2	44
64	4	$[0 \quad 0 \quad x_3 \quad 0 \quad 0 \quad x_6 \quad x_7 \quad -x_6 \quad x_9]$	g1f1s2c2t3R1u24Z3a2	8
65	4	$[0 \quad 0 \quad x_3 \quad 0 \quad 0 \quad x_6 \quad x_7 \quad x_8 \quad 0]$	g1f1s1c1t3R2u13Z2a2	588
66	4	$[0 \quad x_2 \quad 0 \quad -x_2 \quad x_5 \quad x_6 \quad 0 \quad x_8 \quad 0]$	g2f3s1c1t11R1u24Z3a2	4
67	4	$[0 \quad x_2 \quad 0 \quad x_4 \quad 0 \quad x_6 \quad 0 \quad x_8 \quad 0]$	g1f1s1c1t11R2u1Z2a2	146
68	4	$[0 \quad x_2 \quad 0 \quad x_4 \quad x_5 \quad x_6 \quad 0 \quad -x_6 \quad 0]$	g2f1s2c1t11R1u24Z3a2	8
69	4	$[0 \quad x_2 \quad 0 \quad x_4 \quad x_5 \quad x_6 \quad 0 \quad 0 \quad 0]$	g2f1s1c1t9R3u1Z2a2	120
70	4	$[0 \quad x_2 \quad x_3 \quad -x_2 \quad 0 \quad x_6 \quad -x_3 \quad -x_6 \quad x_9]$	g1f3s1c2t9R1u24Z3a2	9
71	4	$[0 \quad x_2 \quad x_3 \quad -x_2 \quad x_5 \quad x_6 \quad 0 \quad -x_6 \quad x_5]$	g1f1s1c1t11R2u17Z2a2	8
72	4	$[0 \quad x_2 \quad x_3 \quad -x_2 \quad x_5 \quad x_6 \quad 0 \quad 0 \quad 0]$	g2f3s1c1t9R2u17Z2a2	36
73	4	$[0 \quad x_2 \quad x_3 \quad 0 \quad 0 \quad 0 \quad x_7 \quad x_8 \quad 0]$	g1f1s2c1t5R2u11Z2a2	32
74	4	$[0 \quad x_2 \quad x_3 \quad 0 \quad x_5 \quad x_6 \quad -x_3 \quad -x_6 \quad 0]$	g2f1s1c1t5R2u17Z2a2	12
75	4	$[0 \quad x_2 \quad x_3 \quad 0 \quad x_5 \quad x_6 \quad -x_3 \quad -x_6 \quad x_5]$	g1f1s1c1t3R2u17Z2a2	8
76	4	$[0 \quad x_2 \quad x_3 \quad x_4 \quad 0 \quad 0 \quad x_7 \quad 0 \quad 0]$	g1f1s1c1t10R2u17Z2a2	150
77	4	$[0 \quad x_2 \quad x_3 \quad x_4 \quad 0 \quad x_6 \quad 0 \quad 0 \quad 0]$	g1f1s2c1t9R2u19Z2a2	24
78	4	$[x_1 \quad 0 \quad x_3 \quad 0 \quad 0 \quad 0 \quad -x_3 \quad x_8 \quad x_9]$	g1f2s1c1t5R2u10Z2a2	8
79	4	$[x_1 \quad 0 \quad x_3 \quad 0 \quad 0 \quad 0 \quad x_7 \quad 0 \quad x_9]$	g1f1s1c2t5R2u1Z1a2	1056
80	4	$[x_1 \quad 0 \quad x_3 \quad x_4 \quad 0 \quad x_6 \quad -x_3 \quad -x_6 \quad x_1]$	g1f1s1c1t3R2u1Z2a2	8
81	4	$[x_1 \quad x_2 \quad 0 \quad -x_2 \quad x_1 \quad x_6 \quad x_7 \quad -x_6 \quad 0]$	g1f1s1c1t11R2u13Z2a2	16
82	4	$[x_1 \quad x_2 \quad 0 \quad x_4 \quad x_5 \quad 0 \quad 0 \quad 0 \quad 0]$	g1f1s2c1t10R2u11Z1a2	990
83	4	$[x_1 \quad x_2 \quad x_3 \quad -x_2 \quad 0 \quad x_6 \quad -x_3 \quad 0 \quad x_1]$	g1f1s1c1t10R2u1Z2a2	8
84	4	$[x_1 \quad x_2 \quad x_3 \quad -x_2 \quad x_1 \quad 0 \quad -x_3 \quad x_8 \quad 0]$	g1f1s1c1t10R2u13Z2a2	16
85	4	$[x_1 \quad x_2 \quad x_3 \quad -x_2 \quad x_1 \quad x_6 \quad 0 \quad 0 \quad 0]$	g1f1s1c1t9R2u12Z2a2	36
86	4	$[x_1 \quad x_2 \quad x_3 \quad 0 \quad 0 \quad 0 \quad -x_3 \quad 0 \quad x_9]$	g1f2s1c1t5R2u19Z2a2	8
87	4	$[x_1 \quad x_2 \quad x_3 \quad 0 \quad 0 \quad 0 \quad -x_3 \quad x_8 \quad x_1]$	g1f1s1c1t5R2u12Z2a2	12
88	5	$[0 \quad 0 \quad 0 \quad 0 \quad x_5 \quad x_6 \quad x_7 \quad x_8 \quad x_9]$	g1f1s1c2t1R2u11Z2a2	368
89	5	$[0 \quad 0 \quad 0 \quad x_4 \quad 0 \quad x_6 \quad x_7 \quad x_8 \quad x_9]$	g1f1s1c2t1R2u10Z2a2	240
90	5	$[0 \quad 0 \quad 0 \quad x_4 \quad x_5 \quad x_6 \quad 0 \quad x_8 \quad x_9]$	g1f3s1c1t1R2u19Z2a2	48
91	5	$[0 \quad 0 \quad 0 \quad x_4 \quad x_5 \quad x_6 \quad x_7 \quad -x_6 \quad x_9]$	g1f2s1c1t1R2u12Z2a2	24
92	5	$[0 \quad 0 \quad 0 \quad x_4 \quad x_5 \quad x_6 \quad x_7 \quad x_8 \quad -x_5]$	g1f1s1c1t1R3u10Z2a2	32
93	5	$[0 \quad 0 \quad 0 \quad x_4 \quad x_5 \quad x_6 \quad x_7 \quad x_8 \quad 0]$	g2f1s1c1t1R2u10Z2a2	96
94	5	$[0 \quad 0 \quad 0 \quad x_4 \quad x_5 \quad x_6 \quad x_7 \quad x_8 \quad x_5]$	g1f1s1c1t1R3u11Z2a2	64
95	5	$[0 \quad 0 \quad x_3 \quad 0 \quad 0 \quad x_6 \quad x_7 \quad x_8 \quad x_9]$	g1f1s1c2t3R2u13Z2a2	1176
96	5	$[0 \quad x_2 \quad 0 \quad x_4 \quad x_5 \quad x_6 \quad 0 \quad x_8 \quad 0]$	g2f1s1c1t11R2u1Z2a2	292
97	5	$[0 \quad x_2 \quad x_3 \quad -x_2 \quad 0 \quad x_6 \quad -x_3 \quad x_8 \quad x_9]$	g1f1s1c2t9R2u1Z2a2	26
98	5	$[0 \quad x_2 \quad x_3 \quad -x_2 \quad 0 \quad x_6 \quad x_7 \quad -x_6 \quad x_9]$	g1f1s1c2t9R2u17Z2a2	14
99	5	$[0 \quad x_2 \quad x_3 \quad -x_2 \quad 0 \quad x_6 \quad x_7 \quad x_8 \quad 0]$	g1f3s1c1t12R1u24Z3a2	3
100	5	$[0 \quad x_2 \quad x_3 \quad -x_2 \quad x_5 \quad x_6 \quad -x_3 \quad x_8 \quad 0]$	g3f1s1c1t10R1u24Z3a2	10
101	5	$[0 \quad x_2 \quad x_3 \quad -x_2 \quad x_5 \quad x_6 \quad 0 \quad -x_6 \quad x_9]$	g1f2s1c1t11R2u17Z2a2	8

TABLE B1—*Continued*

#	p	Simplified form of fundamental matrices	For example generated by:	n
102	5	$[0 \;\; x_2 \;\; x_3 \;\; -x_2 \;\; x_5 \;\; x_6 \;\; 0 \;\; x_8 \;\; x_5]$	g2f1s1c1t11R2u17Z2a2	12
103	5	$[0 \;\; x_2 \;\; x_3 \;\; 0 \;\; 0 \;\; 0 \;\; x_7 \;\; x_8 \;\; x_9]$	g1f1s1c2t5R2u11Z2a2	240
104	5	$[0 \;\; x_2 \;\; x_3 \;\; 0 \;\; x_5 \;\; x_6 \;\; -x_3 \;\; -x_6 \;\; x_9]$	g1f2s1c1t3R2u17Z2a2	32
105	5	$[0 \;\; x_2 \;\; x_3 \;\; 0 \;\; x_5 \;\; x_6 \;\; -x_3 \;\; x_8 \;\; 0]$	g2f1s1c1t5R2u11Z2a2	36
106	5	$[0 \;\; x_2 \;\; x_3 \;\; 0 \;\; x_5 \;\; x_6 \;\; x_7 \;\; -x_6 \;\; x_5]$	g1f1s1c1t3R3u17Z2a2	40
107	5	$[0 \;\; x_2 \;\; x_3 \;\; x_4 \;\; 0 \;\; x_6 \;\; x_7 \;\; -x_6 \;\; 0]$	g1f1s2c1t12R1u24Z3a2	6
108	5	$[0 \;\; x_2 \;\; x_3 \;\; x_4 \;\; x_5 \;\; x_6 \;\; 0 \;\; -x_6 \;\; x_5]$	g1f1s1c1t11R3u17Z2a2	40
109	5	$[0 \;\; x_2 \;\; x_3 \;\; x_4 \;\; x_5 \;\; x_6 \;\; 0 \;\; 0 \;\; 0]$	g2f1s1c1t9R3u17Z2a2	168
110	5	$[x_1 \;\; 0 \;\; x_3 \;\; 0 \;\; 0 \;\; 0 \;\; x_7 \;\; x_8 \;\; x_9]$	g1f1s1c2t5R2u10Z2a2	128
111	5	$[x_1 \;\; 0 \;\; x_3 \;\; x_4 \;\; 0 \;\; x_6 \;\; -x_3 \;\; -x_6 \;\; x_9]$	g1f2s1c1t3R2u1Z2a2	8
112	5	$[x_1 \;\; 0 \;\; x_3 \;\; x_4 \;\; 0 \;\; x_6 \;\; -x_3 \;\; x_8 \;\; x_1]$	g1f1s1c1t3R3u1Z2a2	16
113	5	$[x_1 \;\; x_2 \;\; 0 \;\; -x_2 \;\; x_1 \;\; x_6 \;\; x_7 \;\; x_8 \;\; 0]$	g1f1s1c1t11R3u13Z2a2	56
114	5	$[x_1 \;\; x_2 \;\; 0 \;\; x_4 \;\; x_5 \;\; x_6 \;\; 0 \;\; 0 \;\; 0]$	g1f1s2c1t9R2u10Z2a2	120
115	5	$[x_1 \;\; x_2 \;\; x_3 \;\; -x_2 \;\; 0 \;\; x_6 \;\; -x_3 \;\; 0 \;\; x_9]$	g1f2s1c1t10R2u1Z2a2	8
116	5	$[x_1 \;\; x_2 \;\; x_3 \;\; -x_2 \;\; x_1 \;\; 0 \;\; x_7 \;\; x_8 \;\; 0]$	g1f1s1c1t10R3u13Z2a2	56
117	5	$[x_1 \;\; x_2 \;\; x_3 \;\; 0 \;\; 0 \;\; 0 \;\; -x_3 \;\; x_8 \;\; x_9]$	g1f2s1c1t5R2u12Z2a2	12
118	5	$[x_1 \;\; x_2 \;\; x_3 \;\; 0 \;\; 0 \;\; 0 \;\; x_7 \;\; 0 \;\; x_9]$	g1f1s2c1t5R2u19Z2a2	32
119	5	$[x_1 \;\; x_2 \;\; x_3 \;\; 0 \;\; 0 \;\; 0 \;\; x_7 \;\; x_8 \;\; -x_1]$	g1f1s1c1t5R3u11Z2a2	16
120	5	$[x_1 \;\; x_2 \;\; x_3 \;\; 0 \;\; 0 \;\; 0 \;\; x_7 \;\; x_8 \;\; x_1]$	g1f1s1c1t5R3u10Z2a2	32
121	5	$[x_1 \;\; x_2 \;\; x_3 \;\; x_2 \;\; x_5 \;\; x_6 \;\; -x_3 \;\; -x_6 \;\; x_1]$	g2f1s1c1t5R2u1Z1a2	70
122	5	$[x_1 \;\; x_2 \;\; x_3 \;\; x_4 \;\; -x_1 \;\; x_6 \;\; 0 \;\; 0 \;\; 0]$	g1f1s1c1t9R3u19Z2a2	48
123	5	$[x_1 \;\; x_2 \;\; x_3 \;\; x_4 \;\; 0 \;\; x_6 \;\; -x_3 \;\; 0 \;\; x_1]$	g1f1s1c1t10R3u1Z2a2	16
124	5	$[x_1 \;\; x_2 \;\; x_3 \;\; x_4 \;\; x_1 \;\; x_6 \;\; 0 \;\; 0 \;\; 0]$	g1f1s1c1t9R3u10Z2a2	96
125	5	$[x_1 \;\; x_2 \;\; x_3 \;\; x_4 \;\; x_5 \;\; 0 \;\; 0 \;\; 0 \;\; 0]$	g1f1s2c1t9R2u11Z2a2	24
126	6	$[0 \;\; 0 \;\; 0 \;\; x_4 \;\; x_5 \;\; x_6 \;\; x_7 \;\; x_8 \;\; x_9]$	g1f1s1c1t1R3u12Z2a2	5160
127	6	$[0 \;\; x_2 \;\; x_3 \;\; -x_2 \;\; 0 \;\; x_6 \;\; x_7 \;\; x_8 \;\; x_9]$	g1f1s1c2t9R2u19Z2a2	199
128	6	$[0 \;\; x_2 \;\; x_3 \;\; -x_2 \;\; x_5 \;\; x_6 \;\; -x_3 \;\; x_8 \;\; x_9]$	g2f3s1c2t11R1u24Z3a2	34
129	6	$[0 \;\; x_2 \;\; x_3 \;\; -x_2 \;\; x_5 \;\; x_6 \;\; 0 \;\; x_8 \;\; x_9]$	g1f3s1c1t11R2u17Z2a2	44
130	6	$[0 \;\; x_2 \;\; x_3 \;\; -x_2 \;\; x_5 \;\; x_6 \;\; x_7 \;\; x_8 \;\; 0]$	g2f3s1c1t10R1u24Z3a2	10
131	6	$[0 \;\; x_2 \;\; x_3 \;\; 0 \;\; x_5 \;\; x_6 \;\; -x_3 \;\; x_8 \;\; x_9]$	g3f1s1c1t3R2u17Z2a2	8
132	6	$[0 \;\; x_2 \;\; x_3 \;\; 0 \;\; x_5 \;\; x_6 \;\; x_7 \;\; -x_6 \;\; x_9]$	g1f2s1c1t3R3u17Z2a2	40
133	6	$[0 \;\; x_2 \;\; x_3 \;\; 0 \;\; x_5 \;\; x_6 \;\; x_7 \;\; x_8 \;\; 0]$	g2f1s1c1t5R3u17Z2a2	192
134	6	$[0 \;\; x_2 \;\; x_3 \;\; 0 \;\; x_5 \;\; x_6 \;\; x_7 \;\; x_8 \;\; x_5]$	g1f1s1c1t3R2u11Z2a2	32
135	6	$[0 \;\; x_2 \;\; x_3 \;\; x_4 \;\; 0 \;\; x_6 \;\; x_7 \;\; x_8 \;\; 0]$	g1f3s2c1t12R1u24Z3a2	3
136	6	$[0 \;\; x_2 \;\; x_3 \;\; x_4 \;\; x_5 \;\; x_6 \;\; 0 \;\; -x_6 \;\; x_9]$	g1f2s1c1t11R3u17Z2a2	40
137	6	$[0 \;\; x_2 \;\; x_3 \;\; x_4 \;\; x_5 \;\; x_6 \;\; 0 \;\; x_8 \;\; x_5]$	g1f1s1c1t11R2u19Z2a2	32
138	6	$[0 \;\; x_2 \;\; x_3 \;\; x_4 \;\; x_5 \;\; x_6 \;\; x_7 \;\; -x_6 \;\; x_5]$	g1f1s1c1t12R2u17Z2a2	84
139	6	$[x_1 \;\; 0 \;\; x_3 \;\; x_4 \;\; 0 \;\; x_6 \;\; -x_3 \;\; x_8 \;\; x_9]$	g1f2s1c1t3R3u1Z2a2	16
140	6	$[x_1 \;\; 0 \;\; x_3 \;\; x_4 \;\; 0 \;\; x_6 \;\; x_7 \;\; -x_6 \;\; x_9]$	g1f1s2c1t3R2u1Z2a2	16
141	6	$[x_1 \;\; 0 \;\; x_3 \;\; x_4 \;\; 0 \;\; x_6 \;\; x_7 \;\; -x_6 \;\; x_9]$	g1f2s2c1t5R2u5Z3a2	16
142	6	$[x_1 \;\; x_2 \;\; 0 \;\; x_4 \;\; x_1 \;\; x_6 \;\; x_7 \;\; x_8 \;\; 0]$	g1f1s1c1t11R2u10Z2a2	48
143	6	$[x_1 \;\; x_2 \;\; 0 \;\; x_4 \;\; x_5 \;\; x_6 \;\; x_7 \;\; -x_6 \;\; 0]$	g1f1s2c1t11R2u13Z2a2	16
144	6	$[x_1 \;\; x_2 \;\; x_3 \;\; -x_2 \;\; 0 \;\; x_6 \;\; x_7 \;\; 0 \;\; x_9]$	g1f3s1c1t10R2u1Z2a2	8
145	6	$[x_1 \;\; x_2 \;\; x_3 \;\; -x_2 \;\; x_1 \;\; x_6 \;\; x_7 \;\; x_8 \;\; 0]$	g1f1s1c1t12R2u13Z2a2	126
146	6	$[x_1 \;\; x_2 \;\; x_3 \;\; -x_2 \;\; x_5 \;\; x_6 \;\; -x_3 \;\; x_6 \;\; x_9]$	g1f1s1c1t10R2u10Z1a2	144
147	6	$[x_1 \;\; x_2 \;\; x_3 \;\; -x_2 \;\; x_5 \;\; x_6 \;\; x_3 \;\; -x_6 \;\; x_9]$	g1f1s1c1t11R2u11Z1a2	144
148	6	$[x_1 \;\; x_2 \;\; x_3 \;\; 0 \;\; 0 \;\; 0 \;\; x_7 \;\; x_8 \;\; x_9]$	g1f1s1c1t5R3u12Z2a2	1536
149	6	$[x_1 \;\; x_2 \;\; x_3 \;\; x_2 \;\; x_5 \;\; x_6 \;\; -x_3 \;\; -x_6 \;\; x_9]$	g1f1s1c1t3R2u19Z1a2	358
150	6	$[x_1 \;\; x_2 \;\; x_3 \;\; x_2 \;\; x_5 \;\; x_6 \;\; -x_3 \;\; x_8 \;\; x_1]$	g2f1s1c1t5R2u10Z2a2	12
151	6	$[x_1 \;\; x_2 \;\; x_3 \;\; x_4 \;\; 0 \;\; x_6 \;\; -x_3 \;\; 0 \;\; x_9]$	g1f2s1c1t10R3u1Z2a2	16
152	6	$[x_1 \;\; x_2 \;\; x_3 \;\; x_4 \;\; 0 \;\; x_6 \;\; -x_3 \;\; x_8 \;\; x_1]$	g1f1s1c1t12R2u1Z2a2	42

TABLE B1—*Continued*

#	p	Simplified form of fundamental matrices	For example generated by:	n
153	6	$[x_1\ x_2\ x_3\ x_4\ 0\ x_6\ x_7\ 0\ x_1]$	g1f1s1c1t10R2u19Z2a2	16
154	6	$[x_1\ x_2\ x_3\ x_4\ x_1\ 0\ x_7\ x_8\ 0]$	g1f1s1c1t10R2u11Z2a2	48
155	6	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ -x_3\ -x_6\ x_1]$	g2f1s1c1t3R2u1Z2a2	24
156	6	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ 0\ 0\ 0]$	g1f1s1c1t9R3u12Z2a2	1428
157	7	$[0\ x_2\ x_3\ -x_2\ x_5\ x_6\ x_7\ x_8\ x_9]$	g1f1s1c2t11R2u17Z2a2	270
158	7	$[0\ x_2\ x_3\ 0\ x_5\ x_6\ x_7\ x_8\ x_9]$	g1f1s1c2t3R2u11Z2a2	2480
159	7	$[0\ x_2\ x_3\ x_4\ 0\ x_6\ x_7\ x_8\ x_9]$	g1f1s1c2t10R2u17Z2a2	912
160	7	$[0\ x_2\ x_3\ x_4\ x_5\ x_6\ 0\ x_8\ x_9]$	g1f2s1c1t11R2u19Z2a2	536
161	7	$[0\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ -x_6\ x_9]$	g1f2s1c1t12R2u17Z2a2	84
162	7	$[0\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ 0]$	g2f1s1c1t10R2u17Z2a2	318
163	7	$[x_1\ 0\ x_3\ x_4\ 0\ x_6\ x_7\ x_8\ x_9]$	g1f1s1c2t3R2u10Z2a2	640
164	7	$[x_1\ x_2\ 0\ x_4\ x_5\ x_6\ x_7\ x_8\ 0]$	g1f1s2c1t11R2u10Z2a2	584
165	7	$[x_1\ x_2\ x_3\ -x_2\ 0\ x_6\ x_7\ x_8\ x_9]$	g1f1s1c2t10R2u1Z2a2	48
166	7	$[x_1\ x_2\ x_3\ -x_2\ x_1\ x_6\ x_7\ x_8\ x_9]$	g1f1s1c2t10R2u11Z1a2	1104
167	7	$[x_1\ x_2\ x_3\ -x_2\ x_5\ x_6\ -x_3\ x_8\ x_9]$	g1f1s1c1t10R2u10Z2a2	32
168	7	$[x_1\ x_2\ x_3\ -x_2\ x_5\ x_6\ x_7\ -x_6\ x_9]$	g1f1s1c1t11R2u11Z2a2	32
169	7	$[x_1\ x_2\ x_3\ x_2\ x_5\ x_6\ -x_3\ x_8\ x_9]$	g2f2s1c1t5R2u10Z2a2	12
170	7	$[x_1\ x_2\ x_3\ x_4\ 0\ x_6\ -x_3\ x_8\ x_9]$	g1f2s1c1t12R2u1Z2a2	42
171	7	$[x_1\ x_2\ x_3\ x_4\ 0\ x_6\ x_7\ 0\ x_9]$	g1f1s2c1t10R2u19Z2a2	168
172	7	$[x_1\ x_2\ x_3\ x_4\ x_5\ 0\ x_7\ x_8\ 0]$	g1f1s2c1t10R2u11Z2a2	120
173	7	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ -x_3\ -x_6\ x_9]$	g1f1s1c1t3R2u19Z2a2	104
174	7	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ -x_3\ x_8\ 0]$	g2f1s1c1t10R2u13Z2a2	32
175	7	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ -x_3\ x_8\ x_1]$	g2f1s1c1t10R2u1Z2a2	262
176	8	$[0\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9]$	g1f1s1c2t11R2u19Z2a2	5220
177	8	$[x_1\ x_2\ x_3\ -x_2\ x_5\ x_6\ x_7\ x_8\ x_9]$	g1f1s1c2t10R2u10Z1a2	1232
178	8	$[x_1\ x_2\ x_3\ x_2\ x_5\ x_6\ x_7\ x_8\ x_9]$	g1f1s1c2t3R2u19Z1a2	1564
179	8	$[x_1\ x_2\ x_3\ x_4\ -x_1\ x_6\ x_7\ x_8\ x_9]$	g1f1s1c2t9R3u19Z2a2	96
180	8	$[x_1\ x_2\ x_3\ x_4\ 0\ x_6\ x_7\ x_8\ x_9]$	g1f1s1c2t10R2u19Z2a2	1104
181	8	$[x_1\ x_2\ x_3\ x_4\ x_1\ x_6\ x_7\ x_8\ x_9]$	g1f1s1c2t10R2u11Z2a2	384
182	8	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ -x_3\ x_8\ x_9]$	g2f1s1c1t10R2u10Z1a2	774
183	8	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_3\ x_8\ x_9]$	g2f1s1c1t11R2u11Z1a2	288
184	8	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ -x_6\ x_9]$	g1f1s2c1t11R2u11Z1a2	352
185	8	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_6\ x_9]$	g1f1s2c1t10R2u10Z1a2	144
186	8	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ -x_1]$	g2f1s1c1t5R3u11Z2a2	32
187	8	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ 0]$	g1f1s2c1t12R2u13Z2a2	1078
188	8	$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_1]$	g2f1s1c1t10R2u19Z2a2	128

APPENDIX C

Details on the Computations of Sections 3.4 and 3.5

Here we denote by $\oplus$ the "AND" symbol and by brackets [ ] an interval (unix-like notation). For example, p1g[1–2] $\oplus$ p2g3 represents the set of the three cases: p1g1, p1g2 and p2g3.

Considering the simplification rules given in Section 3.4, there only remains, from the intrinsic part,

p1g1f[1-3]s2b1B1c[1-3] $\oplus$ p2g1f[1-3]s2b[1-3]B[1-3]c[1-3] $\oplus$ p3g[1-3]
f[1-3]s2b1B1c[1-3]

which is 117 cases, and from the extrinsic parameters part,

R1r1a2u1W3T1t1D3Z3 $\oplus$ R1r1a2u1W3T2t[1-12]D[1-3]Z3 $\oplus$ R[2-3]r1a2u[1-24]
W[1-3]T1t1D3Z3 $\oplus$ R[2-3]r1a2u[1-24]W[1-3]T2t[1-12]D[1-3]Z[1-3]
$\oplus$ R4r1a[1-2]u[1-24]W[1-3]T1t1D3Z3 $\oplus$ R4r1a1u[1-24]W[1-3]T2t[4;8;12]
D2Z[1-3] $\oplus$ R4r1a2u[1-24]W[1-3]T2t[1-12]D[1;3]Z[1-3]

which is 21,709 cases, leading to a total of 2,539,953 particular cases. This is approximately 100 times less than previously determined.

Continuing in Section 3.5, the homographic relation cases are

p1g1f[1-3]s1b1B1c[1-3].MVTortho
p2g1f[1-3]s1b[2-3]B[1-2]c[1-3].MVTpara
p3g[1-3]f[1-3]s1b1B1c[1-3].MVTpersp

where

MVTpersp = R1r1a2u1W3T1t1D3Z3
R[2-3]r1a2u[1-24]W3T1t1D3Z3
R4r1a[1-2]u[1-24]W3T1t1D3Z3

MVTpara = R[2-3]r1a2u[10-12;14-16]W2T1t1D2Z3
R[2-3]r1a2u[10-12;14-16]W2T2t[10-12]D2Z2
R[2-3]r1a2u[10-12;14-16]W2T2t[1-12]D2Z[1;3]
R4r1a[1-2]u[10-12;14-16]W2T1t1D2Z3
R4r1a1u[10-12;14-16]W2T2t[10-12]D2Z2
R4r1a2u[10-12;14-16]W2T2t[1-12]D2Z[1;3]

MVTortho = R1r1a2u1W3T1t1D3Z3
R1r1a2u1W3T2t[1;3;5;7]D3Z3
R[2-3]r1a2u[9;13]W3T1t1D3Z3
R[2-3]r1a2u[9;13]W3T2t[1;3;5;7]D3Z3
R4r1a[1-2]u[9;13]W3T1t1D3Z3
R4r1a1u[9;13]W3T2t12D3Z3
R4r1a2u[9;13]W3T2t[1;3;5;7]D3Z3

which is 351 cases of orthographic homographic relations, 18,360 cases of paraperspective homographic relations, and 2,619 cases of perspective homographic relations, leading to a total 21,330 cases of homographic relations.

We will not study paraperspective and orthographic projection for fundamental matrices since the domain of validity of such projection approximations is included in conditions of existence of homographic relation. In the case of perspective projection, (p3): $\mathbf{t} \neq \mathbf{0}$ thus $u_0 = \pm 1$ or $u_1 = \pm 1$.

As previously determined,

$$\begin{aligned}\texttt{MVTpersp} = \ &\texttt{R1r1a2u1W3T2t[1-12]D3Z3}\\ &\texttt{R[2-3]r1a2u[1-24]W3T2t[1-12]D3Z[1-3]}\\ &\texttt{R4r1a1u[1-24]W3T2t[4;8;12]D3Z2}\\ &\texttt{R4r1a2u[1-24]W3T2t[1-12]D3Z[1-3]}\end{aligned}$$

inducing 72,252 cases of fundamental relations.

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