LABORATOIRE

INFORMATIQUE, SIGNAUX ET SYSTÈMES
DE SOPHIA ANTIPOLIS
UMR 6070

COMBINING GEOMETRIC PRIOR AND STATISTICAL FEATURES FOR ACTIVE CONTOUR SEGMENTATION

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Projet CREATIVE

Rapport de recherche
ISRN I3S/RR–2003-10–FR

Mai 2003

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RÉSUMÉ :

Ce rapport traite de segmentation d'images et de vidéo par contours actifs. L'approche variationnelle proposée repose sur la minimisation d'un critère comprenant un terme géométrique d'a priori sur le contour, et un terme caractérisant les propriétés statistiques de la région à segmenter. L'a priori géométrique induit une déformation sans contrainte vers le contour de référence. Le critère est differencié terme à terme. Le bénéfice apporté par une telle déformation sans contrainte est démontré dans le cadre d'une segmentation d'image interactive. Le critère est enfin appliqué pour le suivi de regions homogènes.

MOTS CLÉS :

Segmentation, contours actifs, a priori géométrique, caractéristiques statistiques, suivi de visage

ABSTRACT:

This article deals with image and video segmentation using active contours. The proposed variational approachs is based on a criterion combining geometric prior and statistical features computed on the inside region of the contours. The geometric prior involves a free form deformation from a reference contour as opposed to a parametric transformation. Differentiation of this geometric prior criterion is provided. Introducing such a free form deformation has proven to be beneficial for interactive image segmentation. A tracking application where the geometric prior results from the segmentation of the previous frame is presented.

KEY WORDS :

Segmentation, active contours, geometric prior, statistical features, face tracking

Combining Geometric Prior And Statistical Features For Active Contour Segmentation

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***Abstract*— This article deals with image and video segmentation using active contours. The proposed variational approachs is based on a criterion combining geometric prior and statistical features computed on the inside region of the contours. The geometric prior involves a free form deformation from a reference contour as opposed to a parametric transformation. Differentiation of this geometric prior criterion is provided. Introducing such a free form deformation has proven to be beneficial for interactive image segmentation. A tracking application where the geometric prior results from the segmentation of the previous frame is presented.**

***Index Terms*— segmentation, active contour, geometric prior, statistical features, face tracking**

I. Introduction

The purpose of segmentation is to extract homogeneous image regions corresponding to semantic objects. The active contour method consists in applying a velocity to a closed curve such that the curve evolves towards the boundary of the object to be segmented. This velocity may result from the minimization of a criterion including region and boundary functionals. Boundary functionals were first proposed by Kass *et al.* [18] and by Caselles *et al.* [4], [5] for active contour segmentation using geodesic active contours. Region-based active contours were first introduced by Ronfard *et al.* [24] and Cohen *et al.* [10]. Zhu *et al.* [31], Chan *et al.* [7], Paragios *et al.* [22] introduced statistical descriptors. Jehan *et al.* [15]–[17] addressed the active contour segmentation problem where region features are involved in region functionals. Methods using prior shape information were proposed. First approaches were probabilistic method. Statistical models of shape variation were proposed by Wang and Staih [29], Cremers *et al.* [12], Rousson and Paragios [25]. Leventon *et al.* [19] defined a statistical shape model to guide the evolution process. Chen *et al.* [9] introduced a variational method with a shape prior assuming a parametric deformation between the model and the object segmented contour.

In this article, we propose a variational approach as opposed to previous statistical method. We defined a criterion combining geometric prior, statistical measures computed on image regions, and a boundary term. The deformation between the reference shape and the active contour is a free form deformation as opposed to a (constrained) parametric transformation [9].

This work was supported in part by France Telecom R&D grants nb 011BC22 and 021B406.

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This geometrical term was differentiated in order to compute the velocity of the active contour.

The proposed free form deformation algorithm was applied to interactive image segmentation based on a competition between a reference contour constraint and statistical, region-based features (or descriptor) of the region to be segmented. In Section 3, an application to region tracking on a real sequence is presented. In this application, the reference contour in a frame is deduced from the segmentation of the previous frame.

II. Setting A General Framework

The proposed method is based on the active contour technique. An initial contour is deformed towards the boundary of the object according to an evolution equation defined by the minimization of an energy criterion. The criterion combines a geometric prior characterizing the contour, a regularization term, and region-based descriptors. Differentiation of each term of the criterion leads to the evolution equation of the active contour. The main steps of the algorithm implementation are given in Section C.

A. A General Criterion

Let us assume that the current frame, I, of a video sequence is composed of two regions: Ω^{out}, the background, and Ω^{in}, the object. The common boundary of these domains is denoted by $\Gamma = \partial\Omega^{in}$. Let us define the tracking criterion by:

$$J(\Omega^{in}) = \int_{\Gamma} \varphi(d(\Gamma, \Gamma^{ref}))ds + \lambda \int_{\Omega^{in}} \psi^{in}(K^{in}(\Omega^{in}))\ dxdy + \epsilon \int_{\Gamma} k^b ds \quad (1)$$

where:

- Γ^{ref} is a reference contour. $d(\Gamma, \Gamma^{ref})$ is the geometric distance between contours Γ and Γ^{ref}, i.e., for each $x \in \Gamma$, $d(x, \Gamma^{ref}) = \min_{y_{ref} \in \Gamma^{ref}}(|x - y_{ref}|) = |x - y(x)|$ (see Fig. 1). φ is a differentiable function.
- $K^{in}(\Omega^{in})$ is the descriptor of the object of current frame I and ψ^{in} a differentiable function [6].
- k^b is a classical boundary descriptor [4].

The first term of criterion (1) is the geometric prior. It minimizes a "weighted" distance between the segmentation contour and a reference contour, Γ^{ref}. There are no specific conditions regarding the transformation between Γ and Γ^{ref} unlike similar shape priors [9]. The reference contour may be initialized from an atlas (for registration applications), it may be defined interactively by an operator, or it can be deduced

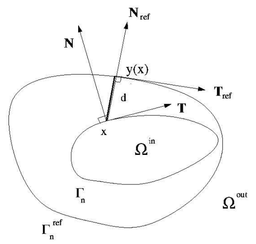


Fig. 1. Distance definition.

from the segmentation contour in the previous frame.
The second term of criterion (1) characterizes the object region, in terms of color texture or motion for instance. For homogeneous region segmentation, K^{in} may be a statistical descriptor such as the mean, the variance, the determinant of covariance matrix [16] or the region histogram [2].
The third term includes a smoothness constraint (regularization). For $k^b = 1$ it minimizes the length of the contour.

Let us introduce a dynamical auxiliary variable: Each region becomes dependent on an evolution parameter τ. $\Gamma(\tau)$ is the active contour evolving towards the object boundary.
Criterion $J(\Omega^{in})$ is rewritten as:

$$\begin{aligned} J(\tau) = J(\Omega^{in}(\tau)) &= \int_{\Gamma(\tau)} \varphi(d(\Gamma(\tau), \Gamma^{ref}))ds \\ &+ \lambda \int_{\Omega^{in}(\tau)} \psi^{in}(K^{in}(\Omega^{in}(\tau)))dxdy \\ &+ \epsilon \int_{\Gamma(\tau)} k^b ds \end{aligned} \tag{2}$$

B. Criterion Differentiation And Evolution Equation

The contour evolution equation is obtained by differentiating the criterion in an Eulerian framework. The differentiation calculus of the first term of the criterion, $J^1(\tau)$, is given in Appendix I.

$$J^1(\tau) = \int_{\Gamma(\tau)} \varphi(d(\Gamma(\tau), \Gamma^{ref}))ds \tag{3}$$

The derivative of J^1 with respect to τ is:

$$J^{1\prime}(\tau) = \int_0^1 \langle \frac{\partial x}{\partial \tau}, (\langle -\mathbf{N}_{ref}, \mathbf{N}\rangle \varphi'(d) - \varphi(d)\kappa)\mathbf{N}\rangle \left|\frac{\partial x}{\partial p}\right| dp \tag{4}$$

According to the Cauchy-Schwarz inequality, the fastest decrease of $J^1(\tau)$ is obtained with the following evolution equation:

$$\frac{\partial \Gamma^1(\tau)}{\partial \tau} = [\langle \mathbf{N}_{ref}, \mathbf{N}\rangle \varphi'(d) + \varphi(d)\kappa]\ \mathbf{N} \tag{5}$$

<u>Remark</u>: This equation is a combination of two terms: A hyperbolic term $\langle \mathbf{N}_{ref}, \mathbf{N}\rangle \varphi'(d)\mathbf{N}$ and a parabolic term $\varphi(d)\ \kappa\ \mathbf{N}$. To ensure stability of the numerical scheme, the discretization of the hyperbolic term must satisfy the Courant-Friedrich-Lewy (CFL) [11] condition:

$$|\langle \mathbf{N}_{ref}, \mathbf{N}\rangle \varphi'(d)\mathbf{N}| \frac{\Delta t}{\Delta x} \leq 1 \tag{6}$$

A sufficient condition for stability is to choose φ such that its derivative is bounded and Δt such that $\Delta t \leq \frac{\Delta x}{max(\varphi')}$.

The differentiation of the second term of criterion (2) is developed in [16]. The key steps are presented in Appendix II.

$$J^2(\tau) = \int_{\Omega^{in}(\tau)} \psi^{in}(K^{in}(\Omega^{in}(\tau)))dxdy \tag{7}$$

The derivative $J^2(\tau)$ with respect to τ is:

$$\begin{aligned} J^{2'}(\tau) = &\int_{\Gamma(\tau)} -\psi^{in}(\tau)\langle \mathbf{v}.\mathbf{N}\rangle ds \\ &- \sum_{j=1}^{m^{in}} A_j^{in}(\tau) \int_{\Gamma(\tau)} f_j^{in}(\tau)\langle \mathbf{v}.\mathbf{N}\rangle ds \\ &+ \sum_{j=1}^{m^{in}} A_j^{in}(\tau) \int_{\Omega^{in}(\tau)} \frac{\partial f_j^{in}(\tau)}{\partial \tau} dxdy \end{aligned} \tag{8}$$

According to the Cauchy-Schwarz inequality, the fastest decrease of $J^2(\tau)$ is obtained with the following evolution equation:

$$\frac{\partial \Gamma^2(\tau)}{\partial \tau} = [\psi^{in}(\tau) + \sum_{j=1}^{m^{in}} A_j^{in} f_j^{in}(\tau)]\ \mathbf{N} \tag{9}$$

The definitions of A and f are given in Appendix II.

The differentiation of the regularization term is classical and can be found in various articles [1], [4].

$$J^3(\tau) = \int_{\Gamma(\tau)} k^b ds \tag{10}$$

The derivative of $J^3(\tau)$ with respect to τ is:

$$J^{3'}(\tau) = \int_{\Gamma(\tau)} (k^b\kappa - \langle \nabla k^b, \mathbf{N}\rangle)\langle v, \mathbf{N}\rangle \tag{11}$$

The associated evolution equation is:

$$\frac{\partial \Gamma^3(\tau)}{\partial \tau} = [k^b\kappa - \langle \nabla k^b, \mathbf{N}\rangle]\mathbf{N} \tag{12}$$

The complete evolution equation is a weighted sum of evolution equations (5), (13), and (12):

$$\begin{aligned} \frac{\partial \Gamma(\tau)}{\partial \tau} = &[\langle \mathbf{N}_{ref}, \mathbf{N}\rangle \varphi'(d) + \varphi(d)\kappa + \lambda\psi^{in}(\tau) \\ &+ \lambda \sum_{j=1}^{m^{in}} A_j^{in} f_j^{in}(\tau) + \epsilon k^b\kappa - \epsilon\langle \nabla k^b, \mathbf{N}\rangle]\ \mathbf{N} \end{aligned} \tag{13}$$

This evolution equation can be implemented using the level set technique [21] or parametric tools such as B-Splines [23].

C. Algorithm

For an image segmentation application, the main steps of the algorithm are:

- *Reference contour*:
  The reference contour may be initialized interactively by an operator or it may be taken from an atlas.
- *Distance map*:
  A distance map from reference contour Γ^{ref} is computed using a fast algorithm [28]. The map is positive for pixels belonging to the object and negative for pixels in the background.
- *Initialization and propagation of the active contour*:
  Active contour Γ may be initially chosen equal to the reference contour or to a predefined form. Then it is deformed according to evolution equation (13) using the level set technique.

III. Application To Interactive Segmentation

A possible application of the proposed method is homogeneous region segmentation using a geometrical prior. After discussing the choice of descriptors adapted to this application, some experimental results are presented.

A. Criterion And Evolution Equation

Let us remind the general segmentation criterion:

$$J(\tau) = \int_{\Gamma(\tau)} \varphi(d(\Gamma(\tau), \Gamma^{ref}))ds + \lambda \int_{\Omega^{in}(\tau)} \psi^{in}(K^{in}(\Omega^{in}(\tau)))dxdy + \epsilon \int_{\Gamma(\tau)} k^b ds \quad (14)$$

• Using the geometric descriptor only, the criterion is defined by:

$$J^1(\tau) = \int_{\Gamma(\tau)} \varphi(d(\Gamma(\tau), \Gamma^{ref}))ds \quad (15)$$

The "weighting" function φ must be differentiable, even, and increasing on $\Re^+$. Moreover, the active contour velocity vector should be zero when $d = 0$. Thus φ must satisfy $\varphi(0) = 0$ and $\varphi'(0) = 0$.
We choose $\varphi(d) = \sqrt{1 + d^2} - 1$ [8].
The criterion becomes:

$$J^1(\tau) = \int_{\Gamma(\tau)} \sqrt{1 + d^2(\Gamma(\tau), \Gamma^{ref})} - 1 \quad ds \quad (16)$$

Then the evolution equation is:

$$\frac{\partial \Gamma^1(\tau)}{\partial \tau} = [\langle \mathbf{N}_{ref}, \mathbf{N} \rangle \frac{d}{\sqrt{1 + d^2}} + (\sqrt{1 + d^2} - 1)\ \kappa]\mathbf{N} \quad (17)$$

Since $\varphi'(d) = \frac{d}{\sqrt{1+d^2}}$, the contour expands or shrinks if it is inside the object ($d < 0$) or outside the object ($d > 0$), respectively.
Moreover $\phi'(d)$ is bounded by 1. Therefore the CFL condition is satisfied if $\Delta t \leq \frac{\Delta x}{max(\varphi')}$, ensuring stability of the numerical scheme.

• The statistical, region-based term is:

$$J^2(\tau) = \int_{\Omega^{in}(\tau)} \psi^{in}(K^{in}(\Omega^{in}(\tau))) \quad dxdy \quad (18)$$

Descriptor $K^{in}(\Omega^{in}(\tau))$ is chosen as the variance of the active contour inside region: $\sigma^2(\Omega^{in})$. It allows to segment homogeneous regions.

Criterion $J^2(\tau)$ can be extended to color images [16]:

$$J^2(\tau) = \int_{\Omega^{in}(\tau)} \psi^{in}(\sum_{i=1}^{3} \sigma_i^2(\Omega^{in}(\tau)))dxdy \quad (19)$$

where i is the color space component (1=Y, 2=U, 3=V), $\sigma_i^2(\Omega)$ is the variance of the i^{th} color component in domain Ω^{in}.
We chose $\psi^{in} = log(1 + r^2)$ ($\psi^{in\prime} = \frac{2r}{1+r^2}$).
For clarity the value $\sigma_1^2(\tau) + \sigma_2^2(\tau) + \sigma_3^2(\tau)$ is denoted by S.
The evolution equation is:

$$\frac{\partial \Gamma^2(\tau)}{\partial \tau} = [S + \psi'(S)(\sum_{i=1}^{3}(I_i - \mu_i)^2 - S)]\mathbf{N} \quad (20)$$

where I_i is the value of the i^{th} color component of frame I and μ_i is the mean value in domain Ω^{in}.

• The regularization term is:

$$J^3(\tau) = \int_{\Gamma(\tau)} k^b ds \quad (21)$$

We chose $k^b = 1$ in order to minimize the length of the contour. The evolution equation is:

$$\frac{\partial \Gamma^3(\tau)}{\partial \tau} = \kappa \mathbf{N} \quad (22)$$

The complete evolution equation is:

$$\frac{\partial \Gamma(\tau)}{\partial \tau} = [\langle \mathbf{N}_{ref}, \mathbf{N} \rangle \frac{d}{\sqrt{1 + d^2}} + (\sqrt{1 + d^2} - 1)\kappa + \lambda S + \lambda \psi'(S)(\sum_{i=1}^{3}(I_i - \mu_i)^2 - S) + \epsilon \kappa]\mathbf{N} \quad (23)$$

It was implemented using the level set technique [21].

B. Experimental Results

The purpose of this application was to evaluate the benefits of the geometrical prior compared to an unconstrained region-based segmentation.
The criterion of the unconstrained region-based segmentation method was the sum of the variances of the object and the background regions and it included a regularization term such as (21). It corresponded to a region competition technique between the object and the background regions. Without the background term, the contour shrinks and disapears.
The geometrical prior of the proposed method creates a competition between the object region and the geometrical constraint.

(a) Segmentation using the competition between regions (cf Section III-B.

(b) Reference contour.

(c) Segmentation using geometrical prior and object-based statistical features.

Fig. 2. Segmentation with and without geometric information.

Figure 2 shows the results of the unconstrained (Fig. 2.a) and geometrically constrained (Fig. 2.c) segmentation methods. Figure 2.b shows the reference contour as defined by an operator. Using the unconstrained segmentation, the contour drifts from the face towards the hand. This phenomenon is compensated for by the geometric prior based upon the distance between the active contour and the reference contour.

IV. Application To Tracking

This general framework is now applied for tracking purpose. First we highlight the additional steps of the algorithm required by the tracking application. Then we applied this algorithm to track a face through 50 frames.

A. Adaptation Of The Algorithm To Tracking

For a tracking application, reference contour Γ_0^{ref} is initialized (by an atlas or an operator..) only once for all the video sequence. For each frame I_n, reference contour Γ_n^{ref} is automatically deduced from previous frame final contour Γ_{n-1} compensated in motion.

The general algorithm is completed by the following steps:

- For each pixel X_{n-1} belonging to Γ_{n-1}, we estimate its position X_n in the frame I_n using a robust matching method.
- Affine model M best representing the whole displacement of contour Γ_{n-1} into the frame I_n is estimated. An affine model is a good trade-off between performance and computational cost. The parameters of the affine model are evaluated by minimizing:
$$\sum \phi(\|X_{n-1} - MX_n\|). \quad (24)$$
ϕ is Geman and Mc Clure penalization function [14] and is introduced to increase the robustness of the estimation. The minimization of criterion (24) is carried out thanks to Charbonnier *et al.* half quadratic theorem [8].
- Γ_{n-1} is compensated in motion into frame I_n according to estimated model M. The compensated contour is denoted Γ_n^{ref} and satisfies: $\Gamma_n^{ref} = M\Gamma_{n-1}$.

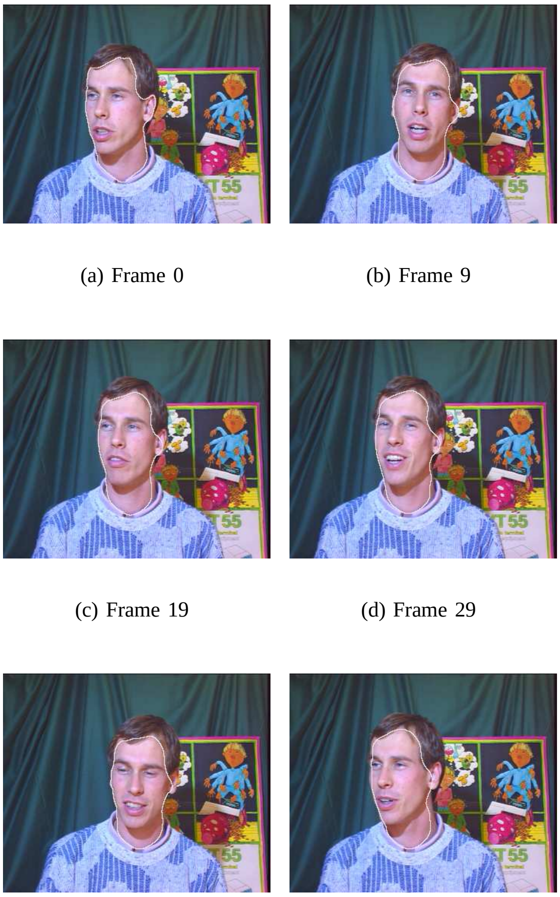
Once the reference contour is determined, the tracking algorithm follows the steps 2 and 3 of the segmentation algorithm (Section II.C).

B. Experimental Results

In this section, we present experimental results obtained by tracking a face through 50 frames of a sequence called *Erik*. We show results coming from our tracking algorithm using geometric, regularization, and statistical region-based terms. The evolution equation is the same as used in segmentation algorithm II.C. The main difference is the estimation of the reference contour. Image segmentation suffers from a lack of temporal coherency. The geometric prior increases the temporal coherency since the reference contour is computed from previous frame segmentation results.

V. Conclusion

This article proposed a segmentation algorithm in a variational active contour framework. A criterion combining geometrical prior term, statistical region-based term, and regularization term is defined. The geometrical term of the criterion minimizes the distance between the active contour and a reference contour. The reference contour may be defined interactively by an operator or initialized from an atlas. The statistical region-based term is chosen to provide the segmentation of homogeneous regions. Differentiation of each term of the criterion is provided and leads to the evolution equation of the active contour. This general framework is applied to homogeneous region segmentation, in order to evaluate the



(a) Frame 0 (b) Frame 9

(c) Frame 19 (d) Frame 29

(e) Frame 39 (f) Frame 49

Fig. 3. Tracking combining geometrical, statistical, region-based and regularization terms.

benefits of the geometrical constraint. Then the algorithm is adapted to track a face through 50 frames. In this case the reference contour comes from the previous final contour compensated in motion.

Applications of this approach to image and vision problems such as registration and reconstruction are straightforward. Registration of multi-modal images or 3D reconstruction should be highly improved by combining geometric a priori information of the shape and statistical measures features obtained from the data.

Appendix I
Differentiation Of The Geometrical Term Of The Criterion

We focus on the geometric term differentiation. From now on, let us denote by $J^1(\tau)$ the first term of criterion (2):

$$J^1(\tau) = \int_{\Gamma(\tau)} \varphi(d(\Gamma(\tau), \Gamma^{ref}))ds \tag{25}$$

If we suppose that $\Gamma(\tau)$ is parametrized by $p \in [0, 1]$, criterion $J^1(\tau)$ can be expressed in terms of $x(p, \tau)$ and $y(p, \tau)$:

$$J^1(\tau) = \int_0^1 \varphi(|x(p,\tau) - y(p,\tau)|) \left|\frac{\partial x}{\partial p}(p,\tau)\right| dp \tag{26}$$

By differentiating (26) with respect to τ, we obtain:

$$\begin{aligned} J^{1\prime}(\tau) &= \int_0^1 \left(\frac{\partial x}{\partial \tau} - \frac{\partial y}{\partial \tau}\right) \frac{x-y}{|x-y|}\ \varphi'(d) \left|\frac{\partial x}{\partial p}\right| dp \\ &+ \int_0^1 \varphi(d) \frac{\partial}{\partial \tau}\left(\left|\frac{\partial x}{\partial p}\right|\right) dp \end{aligned} \tag{27}$$

Let $\mathbf{N}$ and $\mathbf{T}$ be the outward normal and tangent of $\Gamma(\tau)$. After integration by part of the second term of (27), $J^{1\prime}(\tau)$ is of the form:

$$\begin{aligned} J^{1\prime}(\tau) = &-\int_0^1 \langle\langle \mathbf{T}, \frac{x-y}{|x-y|}\rangle\varphi'(d)\mathbf{T} - \varphi(d)\kappa\mathbf{N} \\ &+ \int_0^1 \langle \frac{\frac{\partial y}{\partial p}}{\left|\frac{\partial x}{\partial p}\right|}, \frac{x-y}{|x-y|}\rangle\varphi'(d)\mathbf{T}, \frac{\partial x}{\partial \tau}\rangle \left|\frac{\partial x}{\partial p}\right| dp \\ &+ \int_0^1 \left(\langle\frac{\partial x}{\partial \tau}, \frac{x-y}{|x-y|}\rangle - \langle\frac{\partial y}{\partial \tau}, \frac{x-y}{|x-y|}\rangle\right)\varphi'(d)\left|\frac{\partial x}{\partial p}\right| dp \end{aligned} \tag{28}$$

where κ is the mean curvature of $\Gamma(\tau)$.

Let us denote by $\mathbf{N}_{ref} = \nabla d = \frac{x-y}{|x-y|}$ and $\mathbf{T}_{ref}$ the outward normal and tangent of Γ^{ref}.
As $y(p, \tau) \in \Gamma^{ref}$, we deduce that:

$$d(y(p,\tau), \Gamma^{ref}) = 0 \tag{29}$$

By differentiating this expression with respect to τ and to p, we get:

$$\langle\frac{\partial y}{\partial \tau}.\nabla d\rangle = 0 \qquad \text{and} \qquad \langle\frac{\partial y}{\partial p}.\nabla d\rangle = 0 \tag{30}$$

i.e. $\frac{\partial y}{\partial \tau}$ and $\frac{\partial y}{\partial p}$ are collinear with $\mathbf{T}_{ref}$.

Therefore, we conclude that:

$$\langle\frac{\partial y}{\partial p}, \frac{x-y}{|x-y|}\rangle = 0 \text{ and } \langle\frac{\partial y}{\partial \tau}, \frac{x-y}{|x-y|}\rangle = 0 \tag{31}$$

Thus it follows:

$$\begin{aligned} J^{1\prime}(\tau) = \int_0^1 \langle\frac{\partial x}{\partial \tau}, &\frac{x-y}{|x-y|}\varphi'(d) - \langle\mathbf{T}, \frac{x-y}{|x-y|}\rangle\varphi'(d)\mathbf{T} \\ &- \varphi(d)\kappa\mathbf{N}\rangle \left|\frac{\partial x}{\partial p}\right| dp \end{aligned} \tag{32}$$

Then, writing $\frac{x-y}{|x-y|}$ as a combination of $\mathbf{N}$ and $\mathbf{T}$, $J^{1\prime}(\tau)$ is given by:

$$J^{1\prime}(\tau) = \int_0^1 \langle\frac{\partial x}{\partial \tau}, (\langle -\mathbf{N}_{ref}, \mathbf{N}\rangle\varphi'(d) - \varphi(d)\kappa)\mathbf{N}\rangle \left|\frac{\partial x}{\partial p}\right| dp \tag{33}$$

The derivative of $J^1(\tau)$ could also be achieved using the general derivation theorem of [13].
According to the Cauchy-Schwarz inequality, we deduce the PDE making the contour evolve:

$$\frac{\partial \Gamma_n(\tau)}{\partial \tau} = [\langle\mathbf{N}_{ref}, \mathbf{N}\rangle\varphi'(d) + \varphi(d)\ \kappa)]\mathbf{N} \tag{34}$$

APPENDIX II
DIFFERENTIATION OF THE REGION-BASED TERM OF THE CRITERION

We differentiate the object region-based term of the criterion following the method detailed in [16]:

$$J^{in}(\tau) = \int_{\Omega^{in}(\tau)} \psi^{in}(K^{in}(\Omega^{in}(\tau)))dxdy$$

Let us denote by $\psi^{in}(\tau)$ the function $\psi^{in}(K^{in}(\Omega^{in}(\tau)))$. Descriptor $K^{in}(\Omega^{in}(\tau))$ is modeled as a combination of m^{in} features $G_j^{in}(\tau)$ attached to the evolving object:

$$K^{in}(\Omega^{in}(\tau)) = g^{in}(G_1^{in}(\tau), \ldots, G_{m^{in}}^{in}(\tau)) \quad (35)$$

where $G_j^{in}(\tau)$ can be expressed as:

$$G_j^{in}(\tau) = \int_{\Omega^{in}(\tau)} f_j^{in}(x,y,\tau)dxdy \qquad \text{for} \quad j = 1, \ldots, m^{in} \quad (36)$$

The derivative of $J^{in}(\tau)$ according to τ can be written as:

$$\begin{aligned} J^{in'}(\tau) = & \int_{\Gamma(\tau)} -\psi^{in}(\tau)\langle \mathbf{v}.\mathbf{N}\rangle ds \\ & - \sum_{j=1}^{m^{in}} A_j^{in}(\tau) \int_{\Gamma(\tau)} f_j^{in}(\tau)\langle \mathbf{v}.\mathbf{N}\rangle ds \\ & + \sum_{j=1}^{m^{in}} A_j^{in}(\tau) \int_{\Omega^{in}(\tau)} \frac{\partial f_j^{in}(\tau)}{\partial \tau} dxdy \end{aligned} \quad (37)$$

where

$$A_j^{in}(\tau) = \int_{\Omega^{in}(\tau)} \frac{\partial \psi^{in}(\tau)}{\partial K^{in}(\tau)} \frac{\partial g^{in}(\tau)}{\partial G_j^{in}(\tau)} dxdy$$

and $\mathbf{v}$ is the velocity vector of the active contour.
We will only consider descriptors, such as mean or variance, for which the last integral is null.
The evolution equation is then given by:

$$\frac{\partial \Gamma^{in}(\tau)}{\partial \tau} = [\psi^{in}(\tau) + \sum_{j=1}^{m^{in}} A_j^{in} f_j^{in}(\tau)] \ \mathbf{N} \quad (38)$$

The contribution of the background region-based term is computed using the same approach.

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