LABORATOIRE

INFORMATIQUE, SIGNAUX ET SYSTÈMES
DE SOPHIA ANTIPOLIS
UMR 6070

ANALYSIS METHODS AND MODELS FOR THE RESPIRATORY AND CARDIAC SYSTEMS COUPLING IN GRADED EXERCISE USING A TIME-VARYING MODEL

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Projet BIOMED

Rapport de recherche
ISRN I3S/RR–2004-19–FR

Juillet 2004

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RÉSUMÉ :

Analyser la période cardiaque est une étape difficile lors de tests à l'effort graduels. Le signal appelé "période cardiaque" contient un nombre important d'information sur le système nerveux autonome mais nous nous intéresserons uniquement au couplage entre la respiration et le système cardiaque. Le suivi de l'évolution des fréquences en fonction du temps sera réalisé à l'aide d'algorithmes de pistage, ce qui permettra l'estimation de leur amplitude. Le rythme sinusal étant modulé par la respiration nous établirons les relations non linéaires existante entre ces fréquences et amplitudes issues de la période cardiaque et celles de la modulation. Cette analyse est conduite sous l'hypothèse de stationnarité mais également lorsque la période moyenne, l'amplitude et la fréquence de la respiration varient en fonction du temps.

MOTS CLÉS :

variabilitéde la fréquence cardiaque, respiration, modulation de fréquence, modèle variant dans le temps, exercice

ABSTRACT:

The analysis of heart period series is a difficult task especially under graded exercise conditions. Among all the information present in these series, we are more interested in the coupling between respiratory and cardiac systems, known as respiratory sinus arrythmia. We show in this work that from the heart period series precise patterns concerning the respiratory frequency can be extracted. Evolutive model is introduced in order to achieve the tracking of the main frequencies and their time-varying amplitude. The respiration being a modulating signal of the sinus rhythm, we relate these frequencies and amplitude to this modulation by analyzing in detail its non linear transformation giving giving the heart period signal. This analysis is performed assuming stationary conditions but also in the realistic case when the mean heart period, the amplitude and frequency of the respiration are time-varying.

KEY WORDS :

heart rate variability, tidal volume, frequency modulation, time-varying model, exercise

Analysis methods and Models for the Respiratory and Cardiac System Coupling in Graded Exercise using a Time-Varying Model

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Abstract

The analysis of heart period series is a difficult task especially under graded exercise conditions. Of all the information present in these series, we are most interested in the coupling between respiratory and cardiac systems, known as respiratory sinus arrythmia. In this work we show that precise patterns concerning the respiratory frequency can be extracted from the heart period series. An evolutive model is introduced in order to achieve tracking of the main respiratory-related frequencies and their time-varying amplitudes. Since respiration acts to modulate the sinus rhythm, we relate the frequencies and amplitudes to this modulation by analyzing in detail its non linear transformation giving giving the heart period signal. This analysis is performed assuming stationary conditions but also in the realistic case where the mean heart period, the amplitude, and the frequency of the respiration are time-varying.

I. Introduction

PLEASE NOTE THAT AN EXTENSION OF THIS REPORT IS BEING SUBMITTED TO THE JOURNAL IEEE TRANS. BIOMEDICAL ENGINEERING.

Under resting conditions, power spectral parameters of heart rate variability (HRV) represent mainly the autonomic nervous system influence on the sinus node. Similarly, traditional spectral analysis of the HRV during steady state maximal exercise demonstrates similarity between the high frequency band (HF) and the respiratory frequency (RF) of the studied subject. However, vaso-vagal syncope, tilt manoeuvre, acute ischemic period or graded exercise are not commonly studied due to the non-stationary behavior of the information extracted from the HRV. In the field of exercise physiology, a validated method of HRV analysis in non-stationary conditions would be of interest to study the coupling between respiratory and cardiac systems.

The Prony model, usually encountered in stationary cases when damped or pure sinusoids are assumed to be recorded, makes use of a linear model for the frequency estimation and a Maximum Likelihood estimation for the amplitudes of the sinusoids. In the presence of time-varying frequencies, the classical way to follow these variations is tracking, where a pole representation is used. Some results have been given in [1] in the case of abrupt change of the model (or the frequencies) using the RLS algorithm in order to update the values of the autoregressive (AR) coefficients. In order to reduce the large variance of the AR coefficients estimated from the RLS algorithm we propose to use an offline estimation of the AR coefficients by introducing an evolutive Prony based model. This approach allows continuous variation of the AR coefficients. This lead to an effective frequency tracker based on the relation between the frequencies and the corresponding pole location. Since this first stage of the process gives only the time-varying frequencies, the remaining difficulty is to estimate the time-varying amplitude of the frequencies in order to define a criteria for the selection of the principal frequencies.

The second part of the paper is devoted to relating the information extracted using the proposed analysis method to the continuous signal, which is the respiration. It is commonly accepted that the IPFM model is a possible model for the generation of the R wave time occurrences t_k. The use of the outputs t_k for the frequency analysis of the heart rate variability has been studied in detail in several papers. The latest [2], [3] gives some recommendation in how to use these outputs in order to reduce or cancel the distortion of the spectrum. The problem of choosing the model input is not usually addressed, and is often simplified by using a single (or a sum) of cosines, but not a real physiological signal, such as the respiration. Since Respiratory Sinus Arrythmia (RSA) is one of the most important components of heart rate variability, it is reasonable to propose using the respiration as an input of the model. We have proposed in [4] a model for the generation of the P or R waves by introducing an external modulation which could represent respiration. In this paper, we will show a time occurrences generator, called the PFM model, which is analogous to the IPFM model. A condition for the correct use of the heart period signal defined as $hp(k) = t_{k+1} - t_k$ will also be given. The model analysis presented here novel since it addresses the complex situation where the parameters of the model are no longer constant or stationary, but can be time-varying. The PFM model is assumed in performing this analysis, but some parts of the presented approach are still valid for the IPFM model. This time-varying analysis is a step forward, because it does not require the physiological experiment to occur under near constant conditions but opens the gate to dynamic condition analysis such as those encountered during intense physical exercise. We will show that this approach, which includes a Prony based evolutive model, pole tracking, and time-varying filtering leads to clear results in both realistic simulations, and in experiments where healthy subjects were administered a pyramidal exercise.

II. Analysis Methods

Prior to presenting the methodology for the processing of the HRV, we must choose the signal assumed to contain the relevant information concerning the respiratory frequency (RF). First of all we will state that the heart period (HP) signal $hp(k)$ is defined as:

$$hp(k) = t_k - t_{k-1} \tag{1}$$

where t_k is the occurence time of the kth beat. This signal is assumed to reveal the modulation signal $r(t)$ [5] containing the expected information. The first processing applied to $hp(k)$ is to remove the trend due to the varying conditions of the experiment using a polynomial fitting $po(k)$ (order equal to 20). The second preprocessing step is a 100th order high-pass FIR filtering (the cut-off frequency is 0.03 with 0.5 corresponding to the half of the normalized sample rate) applied to $hp(k)$ where the trend $po(k)$ has been removed. The resulting signal will be referenced as $m(k)$

A. Frequencies estimation

As mentioned in [6], when a signal contains spectral lines it can be modeled, according to the Prony method, as the output of a system with null input, under stochastic initial conditions.

$$s(k) = \sum_{i=1}^{p} a_i s(k-i), \quad p+1 \leq k \leq N \tag{2}$$

where the relation between the coefficients a_i, called AR coefficients by extension, and the frequencies f_i of the spectral lines is given by:

$$1 - \sum_{i=1}^{p} a_i z^{-i} = \prod_{i=1}^{p} (1 - z_i z^{-1}), where \;\; z_i = e^{j2\pi f_i} \tag{3}$$

Then, the frequencies f_i can be obtained by computing the roots of the characteristic polynomial given in (3). When white measurement noise is added to $s(k)$, the noisy observation $m(k) = s(k) + \nu(k)$ is not exactly the output of an AR model, but is that of an ARMA approximated by an AR using an order p sufficiently high. When the signal is non-stationary, however, the model (2) no longer applies, and is extended to give the time-varying AR model:

$$m(k) = \sum_{i=1}^{p} a_i(k) m(k-i) + \nu(k), \quad p+1 \leq k \leq N \tag{4}$$

in which AR parameters are also time-varying. Instead of updating the AR coefficients using algorithms such as RLS [1], the evolutive approach constrains the $a_i(k)$ to be a linear combination of some known basis functions [7]:

$$a_i(k) = \sum_{l=0}^{q} a_{il} u_l(k) \tag{5}$$

In this work we will use the Akaike criteria for the optimal determination of the order q. The order p will be arbitrarily set equal to 12. The basis functions will be choosen as the orthogonal Fourier functions on the interval $[p+1, N]$.

$$\begin{cases} u_0(k) = 1 \\ u_l(k) = cos(k\frac{l}{2}\pi/(N-p)) \quad for\ l\ even \\ u_l(k) = sin(k\frac{(l+1)}{2}\pi/(N-p)) \quad for\ l\ odd \end{cases} \tag{6}$$

Using this linear combination, (4) can be replaced by:

$$m(k) = \mathbf{M}(k-1)\mathbf{a} + \nu(k) \tag{7}$$

where

$$\mathbf{M}(k-1) = [u_0(k-1)m(k-1)\ \ u_1(k-1)m(k-1)\cdots$$
$$u_{q-1}(k-1)m(k-p)\ \ u_q(k-1)m(k-p)]$$

and

$$\mathbf{a} = [a_{10}\ a_{11}\ a_{12}\cdots a_{p(q-2)}\ a_{p(q-1)}\ a_{pq}]^T$$

Then the parameter vector $\mathbf{a}$ is estimated using the well known least square estimator:

$$\hat{\mathbf{a}} = [\mathbf{M}^T\mathbf{M}]^{-1}\mathbf{M}^T\mathbf{m}\ \ where\ \ \mathbf{M} = \begin{bmatrix} \mathbf{M}(p) \\ \vdots \\ \mathbf{M}(N-1) \end{bmatrix} \tag{8}$$
$$and\ \ \mathbf{m} = [m(p+1)\ldots m(N)]^T$$

hence **a** is estimated using (8), the $a_i(k)$ are computed using (5). Since in this approach we are interested in following the variations of a limited set of frequencies, the next difficulty will be to track these frequencies from the estimated time-varying $\hat{a}_i(k)$.

B. Frequency tracking

Tracking varying frequencies is a difficult task where solutions correspond to given assumptions. For example, in the presence of pure sinusoids, efficient algorithms exist [8] but they are unfortunately not suitable for our problem since, as we will see in the application, poles deduced from (4) are not necessarily located on the unit circle. In [1] and [9], frequency trackers are developed using the RLS algorithm for solving the expression (4). However, unlike [1] and [9] the evolutive model of (4) gives a continuous-time description of the a_i allowing a more robust tracker as presented in the following.

The basic idea is to find a relation linking $\frac{\partial a_i(k)}{\partial k}$ (noted $\dot{a}_i(k)$) and the corresponding $z_i(k)$ which are the solution of (3). This relation is obtained by expanding (3) and is given by:

$$\dot{z}_i(k) = \sum_{n=1}^{p} \frac{\delta z_i}{\delta a_n}(k)\dot{a}_n(k) \tag{9}$$

where

$$\frac{\delta z_i}{\delta a_n}(k) = \frac{z_i^{p-n}(k)}{\prod_{l=1|l\neq i}^{p}(z_i(k) - z_l(k))} \tag{10}$$

Since an analytic formulation of $\hat{a}_i(k)$ (5) is available, the calculation of $\dot{a}_n(k)$ is straightforward. Assuming that the value obtained by (9) is equivalent to an increment, a predicted value $\tilde{z}_i(k+1)$ of $z_i(k+1)$ is given by:

$$\tilde{z}_i(k+1) = z_i(k) + \dot{z}_i(k) \tag{11}$$

The pole $z_x(k+1)$, obtained from computing the roots of (3) with the set of a_i at index $k+1$, which minimize the distance with $\tilde{z}_i(k+1)$ will be chosen as a member of the pole set defining a track. Since the previous prediction can be made for each z_i, the last calculation will be to deduce the frequency tracks from the angle of the poles tracks. Obviously several problems arise due to pole variations, especially when two poles are equal or when a complex pair becomes two real poles. In order to avoid these problems due to direct tracking on the poles, we propose to factorize (3) using order two polynomials since the order of the AR model is even, giving :

$$\prod_{i=1}^{p}(1 - z_i z^{-1}) = \prod_{i=1}^{p/2}(1 - b_{i1}z^{-1} - b_{i2}z^{-2}) \tag{12}$$

Then the tracking will be achieved on the coefficients (b_{i1}, b_{i2}) instead of z_i. This approach has been proposed in [4] with success but a better solution is to adopt the order two factorization and the algorithm proposed in [10]. In the following the time index k will be omitted since the algorithm will be applied to any $a_i(k)$ noted a_i. The starting point is the relation linking the a_i and the corresponding (b_{i1}, b_{i2}), which is:

$$1 - \sum_{i=1}^{p} a_i z^{-i} = \prod_{i=1}^{p/2}(1 - b_{i1}z^{-1} - b_{i2}z^{-2}) \tag{13}$$

We will assume that the rearrangment of the (b_{i1}, b_{i2}) will produce only real elements. The key of the algorithm is the estimation of the (b_{i1}, b_{i2}) using a minimization of the cost function:

$$V(\mathbf{b}) = \boldsymbol{\epsilon}^T(\mathbf{b})\boldsymbol{\epsilon}(\mathbf{b}) \tag{14}$$

with the error defined as:

$$\boldsymbol{\epsilon} = \mathbf{a} - \hat{\mathbf{a}}(\mathbf{b}) \tag{15}$$

where $\hat{\mathbf{a}}(\mathbf{b})$ is the vector of coefficients derived from the current estimate of the $\mathbf{b}$ and with

$$\mathbf{a}^T = [a_1 \dots a_p]^T \tag{16}$$

$$\mathbf{b}^T = [\mathbf{b}_1^T \dots \mathbf{b}_{p/2}^T]^T \;\; using \;\; \mathbf{b}_i^T = [b_{i1} \;\; b_{i2}]^T \tag{17}$$

An approximated newton algorithm for the minimization of (14) is given by the recursion for the $(r+1)$ iteration:

$$\hat{\mathbf{b}}^{(r+1)} = \hat{\mathbf{b}}^{(r)} - \left[\left[\frac{\partial \boldsymbol{\epsilon}(\mathbf{b})}{\partial \mathbf{b}^T} \right]^{-1} \boldsymbol{\epsilon}(\mathbf{b}) \right]_{\mathbf{b}=\hat{\mathbf{b}}^{(r)}} \tag{18}$$

From (15) we get that

$$\frac{\partial \boldsymbol{\epsilon}(\mathbf{b})}{\partial \mathbf{b}^T} = -\frac{\partial \hat{\mathbf{a}}(\mathbf{b})}{\partial \mathbf{b}^T} \tag{19}$$

It is shown in Appendix A that this differentiation can be directly computed using a matrix formed by multiple convolution and called $\mathbf{B}(\mathbf{b})$. Then the recursion is:

$$\hat{\mathbf{b}}^{(r+1)} = \hat{\mathbf{b}}^{(r)} - \left[\mathbf{B}(\mathbf{b})^{-1} \boldsymbol{\epsilon}(\mathbf{b}) \right]_{\mathbf{b}=\hat{\mathbf{b}}^{(r)}} \tag{20}$$

Note that the convergence determination is not tedious since this algorithm converges sufficiently fast to arbitrarily freeze the iteration maximum number equal to 5. When this algorithm is applied to the time varying case, the first initial value of $\hat{\mathbf{b}}^{(0)}$ is chosen as the ordered $\mathbf{b}$ such that the elements are all real valued and obtained using a classical roots calculation from the $a_i(1)$ coefficients. For the $a_i(k)$ with $k > 1$, the initial value $\hat{\mathbf{b}}^{(0)}$ is chosen as the final $\hat{\mathbf{b}}^{(5)}$ obtained after convergence on the previous set of coefficients $a_i(k-1)$. Tracking the evolution of the (b_{i1}, b_{i2}) function of time index k being achieved, the corresponding z_i tracks are directly computed using classical root calculation from order two polynomial. Recalling that the goal of the tracking stage is to follow continuously the change of the frequencies characterizing the spectral lines, the phase of the roots z_i will provide us the normalized instantaneous time-varying frequencies $f_i(k)$.

C. Time-varying amplitude estimation

Several procedures are available depending on the accuracy of the previous frequency estimation. The more robust characterization is probably the calculation of the instantaneous power of the signal in frequency bands whose centers are defined by the frequency tracks [4]. An alternative is to use time-varying filters defined by the frequency tracks in order to get the phase of the signal on the contrary to only working on the power.

Defining the discrete Short Time Fourier Transform of $m(k)$ as $HP(k,f)$:

$$HP(k,f) = \sum_u m(u)h(u-k)e^{j2\pi\frac{\ell}{K}u} \text{ with } -K/2 \leq \ell \leq K/2-1 \text{ integer and } f = \ell/K \quad (21)$$

where $h(u)$ is a weighting function such as the Hanning windows, and K is an even number of frequencies. For each frequency $f_i(k)$ define us a binary template or filter $G_i(k,f)$ in the time-frequency plane such that:

$$G_i(k,f) = \begin{cases} 1 \;\; for \;\; |f| \in [f_i(k)-\delta; f_i(k)+\delta] \\ \\ 0 \;\; for \;\; |f| \notin [f_i(k)-\delta; f_i(k)+\delta] \end{cases} \quad (22)$$

The selectivity of the time-varying filter $G_i(k,f)$ will depend on the value δ. The filtered signal $m_i(k)$ is then obtained using the Inverse Short Time Fourier Transform applied on the modified $hp(k,f)$ such that:

$$m_i(k) = \frac{1}{K}\sum_u \sum_{\ell=-K/2}^{K/2-1} G_i(u,\ell/K)HP(u,\ell/K)h(k-u)e^{j2\pi\frac{\ell}{K}k} \quad (23)$$

Note that care should be taken for designing the time-varying filter $G_i(k,f)$ since the negative frequency part must taken into account. This point is obvious assuming that the relation (3) is in the time-varying case:

$$1 - \sum_{i=1}^{p} a_i(k)z^{-i} = \prod_{i=1}^{p/2}(1-z_i(k)z^{-1})(1-z_i^*(k)z^{-1}), where \;\; z_i(k) = e^{j2\pi f_i(k)} \quad (24)$$

The filtered signal $m_i(k)$ being a narrow band signal, we will use the Hilbert transform in order to extract the envelope $A_i(k)$ of the signal, which is defined as the modulus of the analytical signal:

$$\tilde{m}_i(k) = m_i(k) + j\mathcal{H}[m_i(k)] = A_i(k)e^{j\varphi_i(k)} \quad (25)$$

where $\mathcal{H}[.]$ stands for the Hilbert transform. The signal $A_i(k)$, called $A_{RSA}(k)$, showing the highest energy was assumed to be the observed modulation caused by breathing, and the corresponding $f_i(k)$, called $f_{RSA}(k)$, was referred as the normalized instantaneous frequency of the RSA. This normalization is caused by the use of the index k as beat number instead of the running time t_k in the definition of $hp(k)$ (1). In [4], we have proposed to multiply $f_{RSA}(k)$ by $1/po(k)$ in the constant case, with $po(k)$ the trend of $hp(k)$, to get Hertz. As we will see in the following, this procedure is an approximation and depends on the underlying model driving the Heart Period but also that it could be valid in the time-varying case.

III. Models definitions and properties

In [11], we have proposed a model for the generation of the P or R waves:

$$ecg(t) = (cos(c_1 t + c_2 cos(c_3 t + c_4) + c_5) + 1)^{200} \; with \; c_5 \in [0; \pi] \tag{26}$$

Note that the exponent in (26) will only influence the width of the wave and has not other significance meaning. The function $c_2 cos(c_3 t + c_4)$ can be interpreted as an external modulation such as tidal volume. Assuming that the maxima of $ecg(t)$ correspond to the time occurrences of interest t_k, it is clear that the t_ks are the solution of the equation:

$$c_1 t_k + c_2 cos(c_3 t_k + c_4) + c_5 = 2\pi k \tag{27}$$

with $k = 0 \ldots K$. The proposed solution of this equation is:

$$t_k = \frac{2\pi}{c_1} k - \frac{c_2}{c_1} cos(\frac{c_3}{c_1} 2\pi k + c_4 - \frac{c_3 c_5}{c_1}) - \frac{c_5}{c_1} \tag{28}$$

This is an approximated solution in the simple case that the modulating function is a single cosine. It can be shown that it is an acceptable solution when the condition:

$$\frac{c_2 c_3}{c_1} \ll \pi \tag{29}$$

is satisfied.From (28) the heart period signal can be calculated by $hp(k) = t_{k+1} - t_k$ and is finally:

$$hp(k) = \frac{2\pi}{c_1} + 2sin(\frac{c_3}{c_1}\pi)\frac{c_2}{c_1}sin(\frac{c_3}{c_1}2\pi k + \frac{c_3}{c_1}\pi + c_4 - \frac{c_3 c_5}{c_1}) \tag{30}$$

We can state that if the model proposed in (26) synthesizes a real ECG (R-waves), the deduced $hp(k)$ is a cosine function uniformly sampled whose modulation period is $\frac{c_3}{c_1}2\pi$ with a magnitude equal to $2sin(\frac{c_3}{c_1}\pi)\frac{c_2}{c_1}$ added to an offset corresponding to the mean heart period T. This permits us to establish the relation $c_1 = 2\pi/T$. When the quantity T is expressed in seconds, it plays the role of the sampling period giving the relation $c_3 = 2\pi f_v$ with f_v being the frequency of the variability expressed in Hertz. In summary, introducing T as the mean heart period, f_v the frequency of the variability, and A as the magnitude of the variations, we obtain the equalities:

$$\begin{cases} c_1 = \frac{2\pi}{T} \\ c_2 = \frac{A\pi}{T\sin(Tf_v\pi)} \\ c_3 = 2\pi f_v \end{cases} \tag{31}$$

The validity condition (29) is then

$$\frac{Af_v}{\sin(\pi T f_v)} \ll 1 \tag{32}$$

From (31), we conclude that the magnitude A of the $hp(k)$ variability is a function of the magnitude c_2 of the modulation, the mean heart period T and the frequency of the variability f_v, and is given by:

$$A = \frac{Tc_2}{\pi}\sin(f_v T\pi) \tag{33}$$

In fact, the modulation function is $r_1(t) = c_2\cos(c_3 t + c_4)$ and appears in (27) as:

$$kT = t_k + \frac{T}{2\pi}r_1(t_k) + \frac{T}{2\pi}c_5 \tag{34}$$

This equation is similar to that from the IPFM model [2] which is:

$$kT = t_k + \int_0^{t_k} r_2(t)dt \tag{35}$$

Since, unlike (35), the modulation function is not integrated in (34) we will call the model leading to Equation (34) a PFM (Pulse Frequency Modulation).

In general, when choosing the modulation function in (35) the expression:

$$r_2(t) = a_2 \sin(a_3 t + a_4) \tag{36}$$

is often used to model respiration. The HP function deduced from the solution of (35) called hp_i is given by:

$$hp_i(k) = \frac{2\pi}{a_1} - 2\frac{a_2}{a_3}\sin(\frac{a_3\pi}{a_1})\sin(\frac{a_3}{a_1}2\pi + a_4 - a_2 cos(a_4)) \tag{37}$$

As for the PFM model the following relations can be derived:

$$\begin{cases} a_1 = \frac{2\pi}{T} \\ a_2 = -\frac{A\pi f_v}{sin(f_v T\pi} \\ a_3 = 2\pi f_v \end{cases} \tag{38}$$

We can deduce from above the expected magnitude of the variability, under the IPFM model assumption:

$$A = -\frac{a_2}{\pi f_v}\sin(f_v T\pi) \tag{39}$$

This result can be compared to (33) where the PFM model is used. It is clear that these quantities differ significantly and will help us to understand the results from both simulation. This difference is certainly important under dynamic conditions where both the mean heart period T and the frequency of the modulation c_3 can be considered as time varying. This more realistic condition can be addressed using the PFM model, by replacing c_1 with $c_1(t)$ and c_3 with $c_3(t)$ in (26) to give the expression:

$$t_k = \frac{2\pi}{c_1(t_k)}k - \frac{c_2}{c_1(t_k)}cos(\frac{c_3(t_k)}{c_1(t_k)}2\pi k + c_4 - \frac{c_3(t_k)c_5}{c_1(t_k)}) - \frac{c_5}{c_1(t_k)} \tag{40}$$

The calculation of $hp(k)$ from (40) is tedious and can be achieved using the instantaneous period $\beta_1(t_k)$ defined by:

$$\beta_1(t_k) = \frac{d}{dt}(tc_1(t))\Big|_{t=t_k} \tag{41}$$

which is approximated by:

$$\beta_1(t_k) \approx \frac{t_{k+1}c_1(t_{k+1}) - t_k c_1(t_k)}{t_{k+1} - t_k} \tag{42}$$

Since the definition of $hp(k)$ is $hp(k) = t_{k+1} - t_k$, we deduce from (42):

$$hp(k) = \frac{1}{\beta_1(t_k)}[t_{k+1}c_1(t_{k+1}) - t_k c_1(t_k)] \tag{43}$$

After some realistic approximations we finally obtain for the heart period $hp(k)$ (see Appendix B) as:

$$hp(k) \approx \frac{2\pi}{\beta_1(t_k)} + 2\frac{c_2}{\beta_1(t_k)} sin\left(\frac{\varphi^{(1)}(k)}{2}\right) sin\left(\varphi(k)\right) \tag{44}$$

where

$$\varphi(k) = \frac{c_3(t_k)}{c_1(t_k)}(2\pi k - c_5 + \pi) + c_4 \tag{45}$$

We can deduce from the expression of $hp(k)$ a definition of the "instantaneous" mean heart period $T(k)$:

$$T(k) = \frac{2\pi}{\beta_1(t_k)} \tag{46}$$

and also the magnitude $A(k)$ of the variability which is given by:

$$A(k) = 2\frac{c_2}{\beta_1(t_k)} sin\left(\frac{\varphi^{(1)}(k)}{2}\right) \tag{47}$$

The value $\varphi^{(1)}(k)$ is the instantaneous period of the observed $hp(k)$ which is not simply $\frac{d}{dt}(tc_3(t))\Big|_{t=t_k}$ as could be expected. The value $T(k)$ in (46) can be directly calculated from $hp(k)$ since it is the trend $po(k)$. From $T(k)$ and (41) the quantity $c_1(t_k)$ can be computed using a discrete integrator. The frequency $f_i(k)$ corresponding to the RSA observed in $hp(k)$, as described in Part C is in fact $\varphi^{(1)}(k)/2\pi$, so that the instantaneous phase $\varphi(k)$ can be computed also using a discrete integrator. Finally, approximating $\varphi(k)$ as:

$$\varphi(k) \approx \frac{c_3(t_k)}{c_1(t_k)} 2\pi k \tag{48}$$

when k increases, allows us to define $\tilde{c}_3(t_k)$ as an approximation of $c_3(t_k)$. The respiratory frequency is usually observed as a instantaneous frequency in the time domain, but not as a function of k. Assuming that the modulating function $r_1(t)$ in (34) is the respiration, this instantaneous frequency is $F_{RSA}(t) = \frac{1}{2\pi}\frac{d}{dt}(tc_3(t))$ and can be retrieved from the $hp(k)$ signal using the previous computation of $\tilde{c}_3(t_k)$. In practice, this solution exhibits small errors due to the integration computations. We will prefer the solution given in Appendix C where the relation:

$$\varphi^{(1)}(k)\frac{\beta_1(t_k)}{2\pi} \approx \frac{d}{dt}\left(c_3(t)t\right)\Big|_{t=t_k} \tag{49}$$

is shown. Then, since the function $\frac{\beta_1(t_k)}{2\pi}$ is the inverse of $po(k)$, i.e, the trend of $hp(k)$, and $\varphi^{(1)}(k)/2\pi$ is the instantaneous frequency $f_{RSA}(k)$ observed in $hp(k)$, the instantaneous frequency $F_{RSA}(t)$ can be directly computed.

It should be remarked that the procedures to retrieve the magnitude (47) of the $hp(k)$ variability, called now A_{RSA}, and the instantaneous frequency $F_{RSA}(t)$ are the same for the constant and time-varying case when $hp(k)$ is written as:

$$hp(k) = po(k) + A_{RSA} sin(\varphi(k)) \tag{50}$$

These procedures are:

$$\begin{cases} A_{RSA}(k) \approx c_2 \frac{po(k)}{\pi} sin\left(\pi f_{RSA}(k)\right) \\ \\ F_{RSA}(t_k) \approx \frac{f_{RSA}(k)}{po(k)} \end{cases} \tag{51}$$

The respiration or modulating function $r_1(t)$ can then be fully described by extracting from (51) the coefficient c_2 called now c_{RSA} and $F_{RSA}(t)$ using an interpolation.

IV. Simulations

In this section we will present two simulations whose aim is to show that the global procedure is able to reveal the time-varying frequency content of the heart period signal. However, instead of directly providing $hp(k)$, these simulations will first produce a noisy ($\sigma_\nu = 0.05$) synthesized ECG $ecg(n)$ (Fig. 1) from which the times of occurrence t_k's can be estimated using a fixed threshold technique to provide provide the $hp(k)$ (Fig. 2). In both simulations, the frequency $F_{RSA}(t)$ (Fig. 5) and the instantaneous mean heart period $T(k)$ are time varying. In the first simulation, the amplitude c_{RSA} (Fig. 7) is also time-varying, unlike the second simulation where c_{RSA} is held constant (Fig. 9). Note that in the section "Model definitions and properties" the case of time-varying c_{RSA} has not been explicitly addressed. However, by assuming that c_{RSA} is slowly varying, then simple mainupulation will show that it will appear as a function of time in the previous expressions. From $hp(k)$, the trend $po(k)$ (Fig. 3) and the variability $m(k)$ (Fig. 4) have been extracted, as previously described. In fig. 5, the estimated $F_{RSA}(t_k)$ from (51) and the real one are compared. We can see the good agreement between the two curves even in the presence of a spike of noise around the epoch $k = 850$. The estimated amplitude c_{RSA} of the modulation is also compared to the real one in fig. 7 where both the effect of the spike is visible and the border effect of the time-varying filter described previously. It is important to compare this figure to fig. 6 where A_{RSA} is plotted since it shows the significance of the correction expression in (51) relating A_{RSA} to c_2, called c_{RSA}. In order to clarify this fact, the comparison of fig. 8 and fig. 9 shows that instead of being constant, as is the case of the real c_{RSA}, the amplitude A_{RSA} of the frequency observed in $m(k)$ is not at all constant but is varying because of the variation of both $T(k)$ and $F_{RSA}(t)$.

These simulations whose aim is to validate the global approach presented in this paper have been extended to real data as shown in [4]. The simulations highlight the capacity of the methods to robustly extract information from the heart period signal but do not validate the proposed PFM model. In [11], some validations are shown introducing the arch signal [12] but also using real respiration records as the tidal volume.

V. Conclusion

We have presented a simple model for generating the time series called the PFM model of heart beats. This model proposed in (26) is not intended to reflect the inherent complexity of HRV. However, since Although investigation of HRV under exercise stress conditions tends to reduce such complexity, the proposed model can prove quite robust and accurate. Indeed, both simulated data and real data show that respiratory frequency can be accurately predicted from the HP series analysis, which may have utility in the field of cardio-respiratory physiology. Even if a larger set of observations is needed to confirm the utility of our method in the understanding of phenomenon contributing to HRV [13], it does demonstrate clearly that care should be taken in interpreting the information directly extracted from the heart period signal in relation to the respiration. We have shown that this relation is not linear and can be addressed in the case of time variations of the variables describing the respiration.

VI. Appendix A

In the following, we will give the form of the derivative of order p polynomial coefficients over the order 2 polynomials coefficients.

From the relation:

$$1-\sum_{i=1}^{p} a_i z^{-i} = \prod_{i=1}^{p/2}(1-b_{i1}z^{-1}-b_{i2}z^{-2}) \tag{52}$$

we define the vector $\mathbf{a}_p$ and $\mathbf{b}_i$ as:

$$\mathbf{a}_p^T = [a_1 \ldots a_p]^T \tag{53}$$

$$\mathbf{b}_i^T = [b_{i1} \;\; b_{i2}]^T. \tag{54}$$

It is well known that the coefficients of the product of two polynomial are equal to the convolution of the coefficients from the two polynomials. Then, the coefficients of the polynoms given in(52) are deduced from the following relation :

$$\begin{bmatrix} 1 \\ -\mathbf{a}_p \end{bmatrix} = \bigodot_{i=1}^{p/2} \begin{bmatrix} 1 \\ -\mathbf{b}_i \end{bmatrix} \tag{55}$$

using the multiple discrete convolution product introduced as :

$$\bigodot_{l=1}^{L} \mathbf{x}_l = \mathbf{x}_1 * \mathbf{x}_2 * \ldots * \mathbf{x}_L \tag{56}$$

where the discrete convolution product $\mathbf{x}_1 * \mathbf{x}_2$ gives the vector $\mathbf{w}$, such that the k'th component is:

$$w(k) = \sum_j x_1(j) x_2(k-j) \tag{57}$$

Now we can calculate the derivative $\partial \mathbf{a}_p / \partial \mathbf{b}$ with $\mathbf{b}^T = [\mathbf{b}_1^T \ldots \mathbf{b}_q^T]^T$ using :

$$\frac{\partial \mathbf{a}_p}{\partial \mathbf{b}^T} = \left[\frac{\partial \mathbf{a}_p}{\partial \mathbf{b}_1^T} \cdots \frac{\partial \mathbf{a}_p}{\partial \mathbf{b}_q^T} \right] \tag{58}$$

Since the convolution product commutes, the derivative of (53) with respect to $\mathbf{b}_i$ is :

$$\frac{\partial}{\partial \mathbf{b}_i^T}\begin{bmatrix} 1 \\ -\mathbf{a}_p \end{bmatrix} = \frac{\partial}{\partial \mathbf{b}_i^T}\begin{bmatrix} 1 \\ -\mathbf{b}_i \end{bmatrix} * \bigodot_{l=1|l\neq i}^{p/2}\begin{bmatrix} 1 \\ -\mathbf{b}_l \end{bmatrix} \tag{59}$$

leading to:

$$\begin{bmatrix} 0 \quad 0 \\ -\frac{\partial \mathbf{a}_p}{\partial \mathbf{b}_i^T} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} * \bigodot_{l=1|l\neq i}^{p/2}\begin{bmatrix} 1 \\ -\mathbf{b}_l \end{bmatrix} \tag{60}$$

which finally gives :

$$\frac{\partial \mathbf{a}_p}{\partial \mathbf{b}_i^T} = \mathbf{I} * \bigodot_{l=1|l\neq i}^{p/2}\begin{bmatrix} 1 \\ -\mathbf{b}_l \end{bmatrix} \tag{61}$$

where $\mathbf{I}$ is the 2x2 dimensional identity matrix.

Then (59) is :

$$\frac{\partial \mathbf{a}_p}{\partial \mathbf{b}^T} = \begin{bmatrix} \mathbf{c}_1 & \cdots & \mathbf{c}_{p/2} \end{bmatrix} \tag{62}$$

with

$$\mathbf{c}_i = \mathbf{I} * \bigodot_{l=1|l\neq i}^{p/2}\begin{bmatrix} 1 \\ -\mathbf{b}_l \end{bmatrix} \tag{63}$$

VII. Appendix B

From the expression of t_k (40) we directly obtain

$$t_{k+1} = \frac{2\pi}{c_1(t_{k+1})}(k+1) - \frac{c_2}{c_1(t_{k+1})}cos\left(\frac{c_3(t_{k+1})}{c_1(t_{k+1})}2\pi(k+1) + c_4 - \frac{c_3(t_{k+1})c_5}{c_1(t_{k+1})}\right) - \frac{c_5}{c_1(t_{k+1})} \tag{64}$$

Using (40) and (64), the heart period $hp(k)$ calculated by (44) is then:

$$\begin{aligned} hp(k) = \frac{1}{\beta_1(t_k)}\Bigg[2\pi - c_2 cos\left(\frac{c_3(t_{k+1})}{c_1(t_{k+1})}2\pi(k+1) + c_4 - \frac{c_3(t_{k+1})c_5}{c_1(t_{k+1})}\right) \\ + c_2 cos\left(\frac{c_3(t_k)}{c_1(t_k)}2\pi(k) + c_4 - \frac{c_3(t_k)c_5}{c_1(t_k)}\right)\Bigg] \end{aligned} \tag{65}$$

which is simplified as:

$$hp(k) = \frac{1}{\beta_1(t_k)}\left[2\pi - 2c_2 sin(A) sin(B)\right] \tag{66}$$

In the domain of application, which is heart rate variability, the term A can be simplified by assuming first that the time variation of both quantities $c_1(t_k)$ and $c_3(t_k)$ is slow, and second, that these quantities vary in almost the same way. This second assumption can be illustrated by relating $c_1(t_k)$ to the mean heart frequency and $c_3(t_k)$ to the respiration frequency when the heart period is recorded under increasing exercise conditions. In this case the approximation

$$\frac{c_3(t_k)}{c_1(t_k)} \approx \frac{c_3(t_{k+1})}{c_1(t_{k+1})} \tag{67}$$

is valid. Then A is approximated by

$$A \approx \frac{c_3(t_k)}{c_1(t_k)}(2\pi k - c_5 + \pi) + c_4 \tag{68}$$

The simplification of B is obtained using the approximation of the derivative with regards to k of any function $F(t_k)k$ such that

$$(F(t_k)k)^{(1)} \approx F(t_{k+1})(k+1) - F(t_k)k \tag{69}$$

where $(.)^{(1)}$ stands for the derivation. Then B is given by:

$$B = \frac{1}{2}\left[-\left(2\pi k\frac{c_3(t_k)}{c_1(t_k)}\right)^{(1)} + \left(\frac{c_3(t_k)}{c_1(t_k)}\right)^{(1)} c_5\right] \tag{70}$$

Renaming the quantity A as $\varphi(k)$, we can point out the relation between B and $\varphi(k)$ such that:

$$B = \frac{1}{2}\varphi^{(1)}(k) - \frac{\pi}{2}\left(\frac{c_3(t_k)}{c_1(t_k)}\right)^{(1)} \tag{71}$$

Using the previous assumption related to $c_1(t_k)$ and $c_3(t_k)$, the second term in (71) can be neglected.

Using (66), (68) and (71), the heart period $hp(k)$ can then be expressed as:

$$hp(k) \approx \frac{2\pi}{\beta_1(t_k)} + 2\frac{c_2}{\beta_1(t_k)} sin\left(\frac{\varphi^{(1)}(k)}{2}\right) sin\left(\varphi(k)\right) \tag{72}$$

VIII. Appendix C

Using (45), the derivative of the instantaneous phase $\varphi(k)$ with respect to the index k is given by:

$$\varphi^{(1)}(k) = t_k^{(1)}(2\pi k + \pi - c_5)\, \frac{d}{dt}\left(\frac{c_3(t)}{c_1(t)}\right)\Bigg|_{t=t_k} + \frac{c_3(t_k)}{c_1(t_k)}2\pi \tag{73}$$

It is common to approximate [3] the t_k as a trend plus a small variation, which could be neglected. Then (40) can be approximated by:

$$t_k \approx \frac{2\pi}{c_1(t_k)}k \tag{74}$$

then the derivative $t_k^{(1)}$ is:

$$t_k^{(1)} \approx t_{k+1} - t_k \approx \frac{2\pi}{\beta_1(t_k)} \tag{75}$$

since substituting (74) in (42) gives $\beta_1(t_k) = 2\pi/(t_{k+1} - t_k)$. The second derivative in (73) is :

$$\frac{d}{dt}\left(\frac{c_3(t)}{c_1(t)}\right)\Bigg|_{t=t_k} = \frac{\frac{dc_3(t)}{dt}c_1(t) - \frac{dc_1(t)}{dt}c_3(t)}{c_1^2(t_k)}\Bigg|_{t=t_k} \tag{76}$$

In (73), the terms π and c_5 can be neglected when k increases, then using (76), (74) and (75) we get the product:

$$\varphi^{(1)}(k)\frac{\beta_1(t_k)}{2\pi} \approx \left(\frac{dc_3(t)}{dt} - \frac{c_3(t)}{c_1(t)}\frac{dc_1(t)}{dt}\right)\Bigg|_{t=t_k} + \beta_1(t_k)\frac{c_3(t_k)}{c_1(t_k)} \tag{77}$$

Finally, using a second definition of $\beta_1(t_k)$ as:

$$\beta_1(t_k) = \frac{d}{dt}\left(c_1(t)t\right)\Bigg|_{t=t_k} = \left(\frac{dc_1(t)}{dt}t - c_1(t)\right)\Bigg|_{t=t_k} \tag{78}$$

the expression (77) becomes:

$$\varphi^{(1)}(k)\frac{\beta_1(t_k)}{2\pi} \approx \left(\frac{dc_3(t)}{dt}t - c_3(t)\right)\Bigg|_{t=t_k} = \frac{d}{dt}\left(c_3(t)t\right)\Bigg|_{t=t_k} \tag{79}$$

which is the instantaneous period of the modulating function at each time of occurence t_k.

References

[1] L Mainardi, A Bianchi, G Baselli, and S Cerutti, "Pole-tracking algorithms for the extraction of time-variant heart rate variability spectral parameters," *IEEE Trans. Biomed. Eng.*, vol. 42, no. 3, pp. 250–259, 1995.

[2] Javier Mateo and Pablo Laguna, "Improved heart rate variability signal analysis from the beat occurence times according to the ipfm model," *IEEE Trans. Biomed. Eng.*, vol. 47, no. 8, pp. 985–996, 2000.

[3] Michael Brennan, Marimuthu Malaniswami, and Peter Kamen, "Distortion properties of the interval spectrum of ipfm generated heartbeats for the heart rate variability analysis," *IEEE Trans. Biomed. Eng.*, vol. 48, no. 11, pp. 1251–1264, 2001.

[4] Olivier Meste, G. Blain, and Stéphane Bermon, "Analysis of the respiratory and cardiac systems coupling in pyramidal exercise using a time-varying model," in *29th annual conference on Computers in Cardiology*, Memphis, USA, Sept. 2002.

[5] J Mateo and P Laguna, "Improved heart rate variability signal analysis from the beat occurence times according to the ipfm model," *IEEE Trans. Biomed. Eng.*, vol. 47, no. 8, pp. 985–996, 2000.

[6] Kay S M, *Modern Spectral estimation: theory and application*, Englewood Cliffs: Prentice-Hall, 1988.

[7] Grenier Y, "Time-dependent arma modeling of nonstationary signals," *IEEE Trans. Signal Processing*, vol. 31, no. 4, pp. 899–911, 1983.

[8] P Strobach, "The recursive companion matrix root tracker," *IEEE Trans. Signal Processing*, vol. 45, no. 8, pp. 1931–1942, 1997.

[9] F A Lee and A Nehorai, "Adaptive power spectrum estimation algorithm for heart variability analysis," in *Computers in Cardiology*, Sept. 1991, pp. 273–276.

[10] D Starer and A Nehorai, "Adaptive polynomial factorization by coefficient matching," *IEEE Trans. Signal Processing*, vol. 39, no. 2, pp. 527–530, 1991.

[11] Olivier Meste, Grégory Blain, and Stéphane Bermon, "Validation of the IPFM model for the heart rate variability analysis," in *30th annual conference on Computers in Cardiology*, Thessalonique, Grece, Sept. 2003.

[12] Katrin Suder, Friedhelm R. Drepper, Michael Schiek, and Hans-Henning Abel, "One-dimensional, nonlinear determinism characterizes heart rate pattern during paced respiration," *Am. J. Physiol.*, pp. H1092–H1102, 1998.

[13] H Cammann and J Michel, "How to avoid misinterpretation of heart rate variability power spectra ?," *Computer Methods and Programs in Biomedicine*, vol. 68, pp. 15–23, 2002.

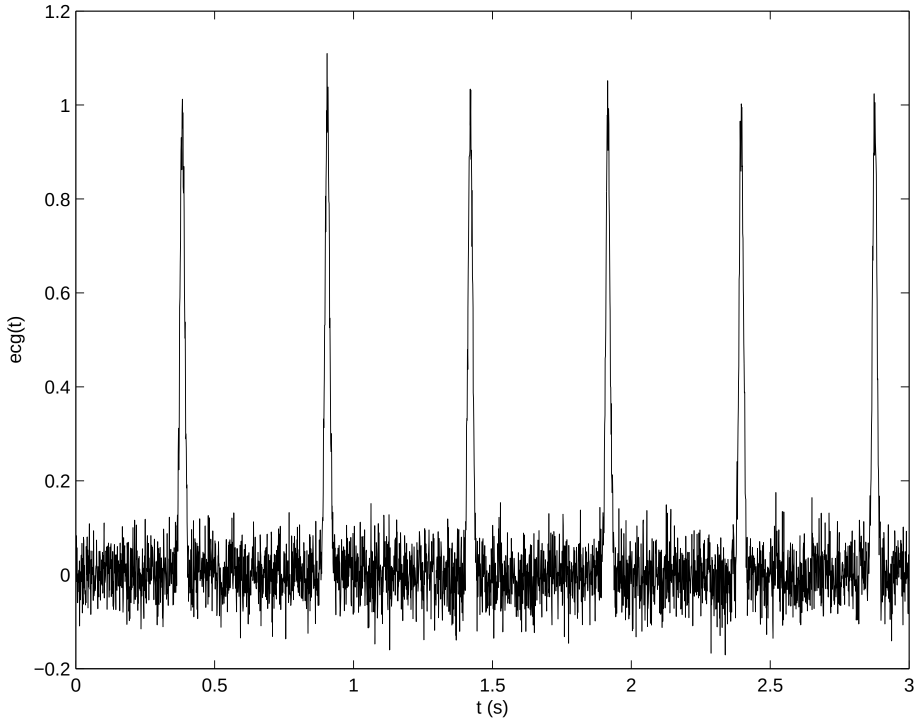

Fig. 1. Magnified section of $ecg(t)$ generated by (26)

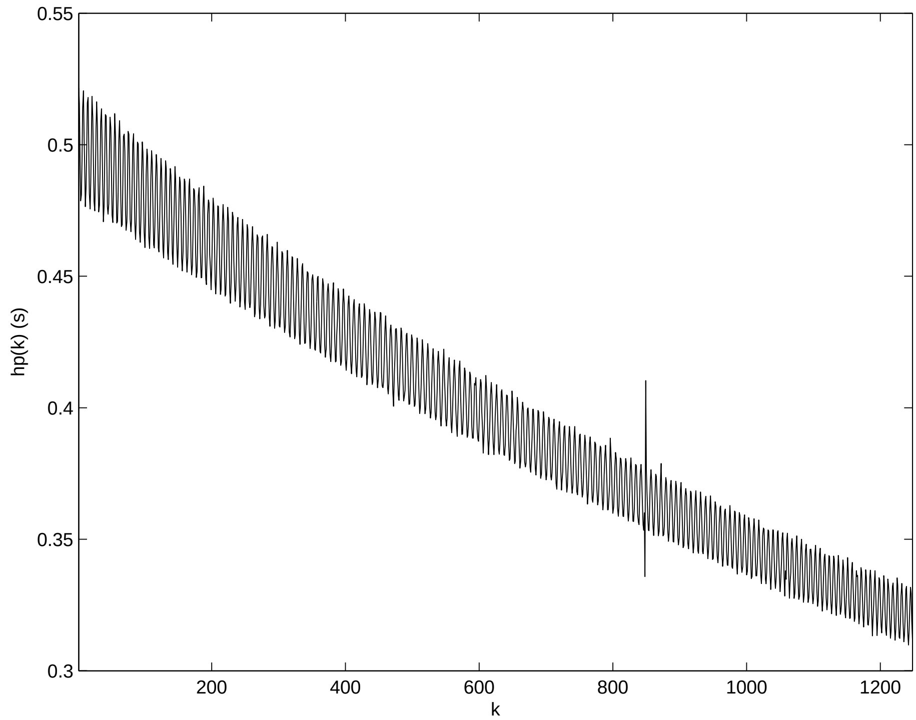

Fig. 2. The heart period signal $hp(k)$. Note the spike of noise at epoch $k = 850$

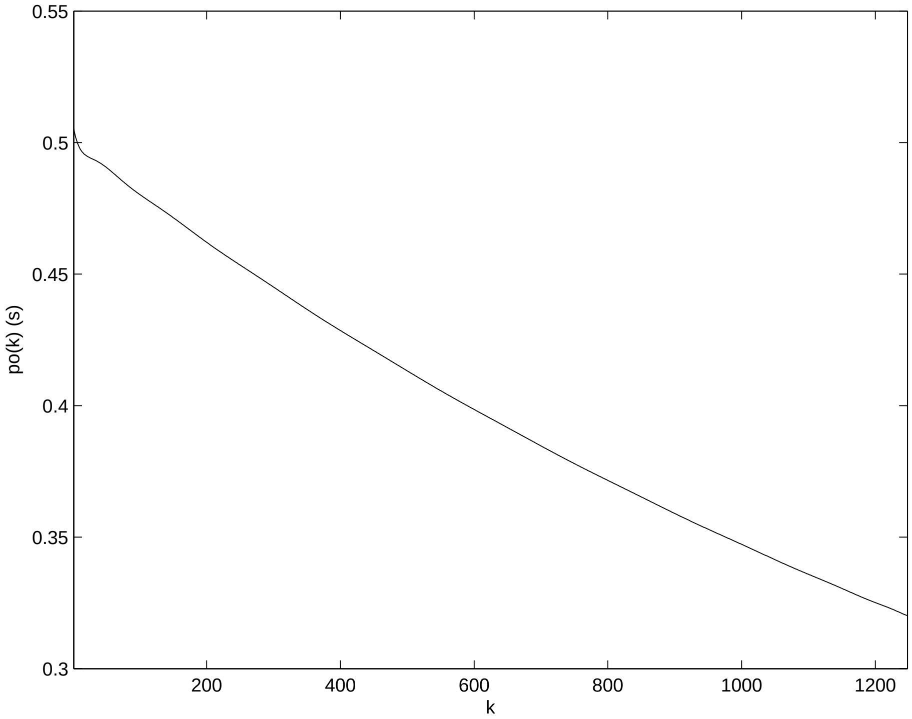

Fig. 3. The trend $po(k)$ or $T(k)$ of the heart period

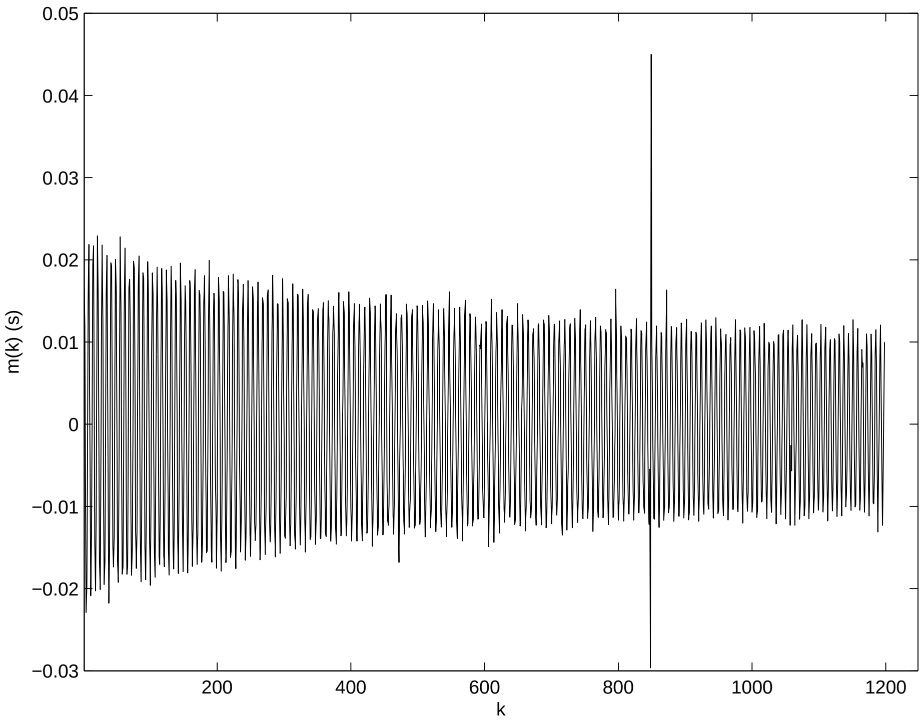

Fig. 4. The variability $m(k)$ of the heart period after the preprocessing stage

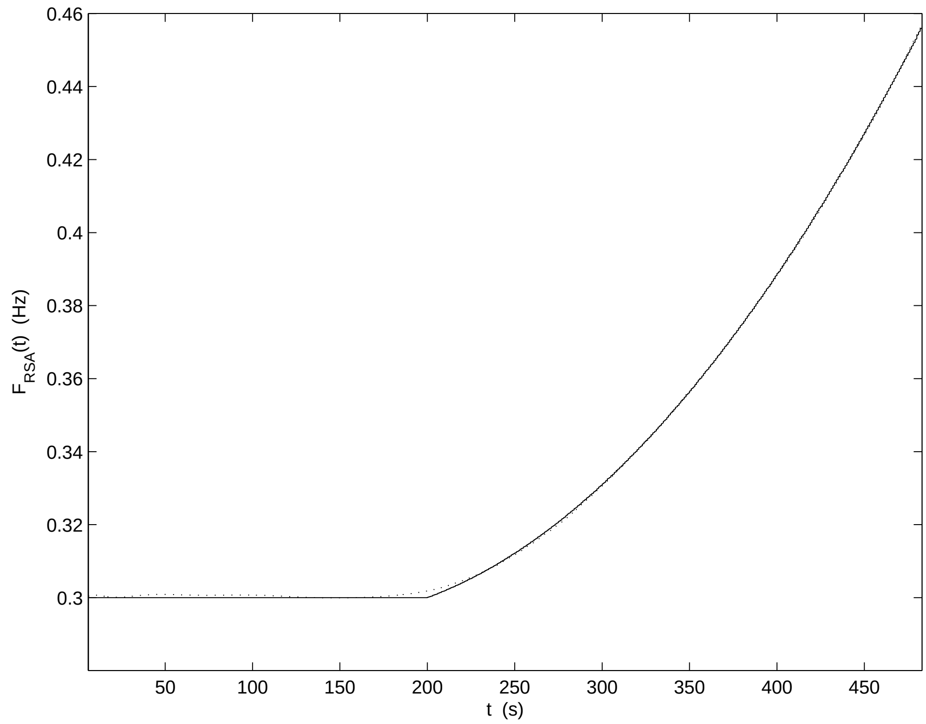

Fig. 5. The real (solid line) and the estimated (dotted line) simulated instantaneous respiratory frequency F_{RSA}

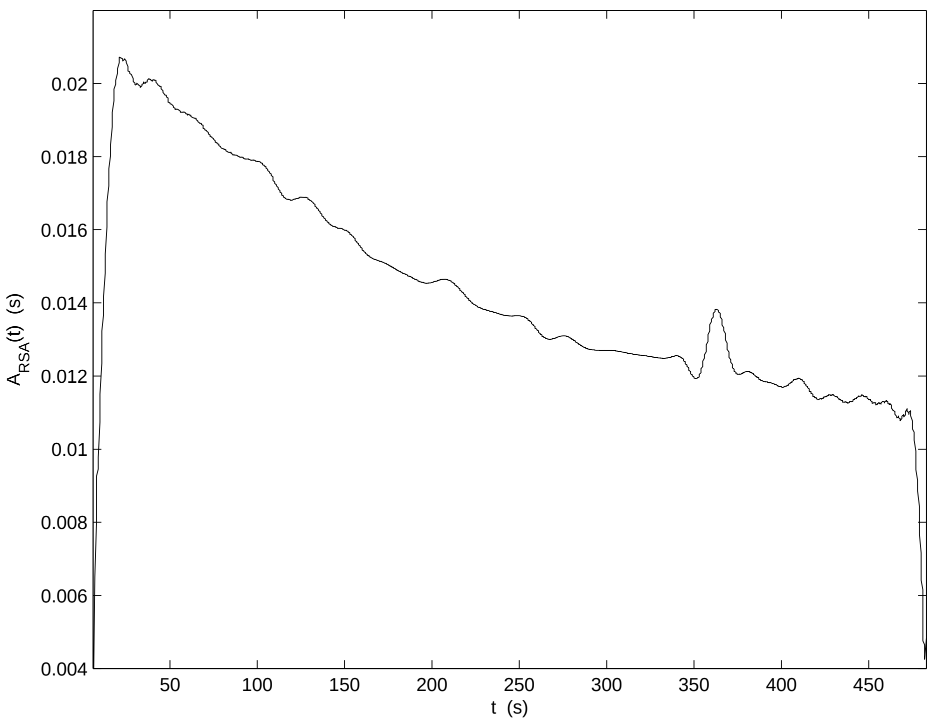

Fig. 6. The amplitude A_{RSA} of the variability signal $m(k)$ for the first simulation

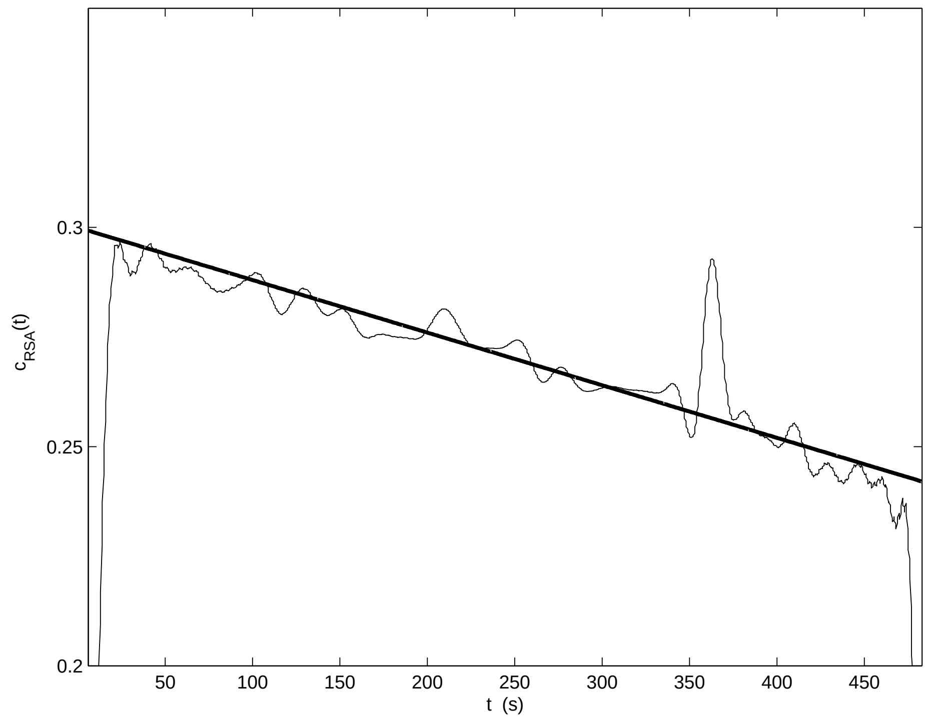

Fig. 7. The real (thick line) and the estimated (thin line) amplitude c_{RSA} of the first simulated respiration. Note the difference of shape with A_{RSA} in fig. 6

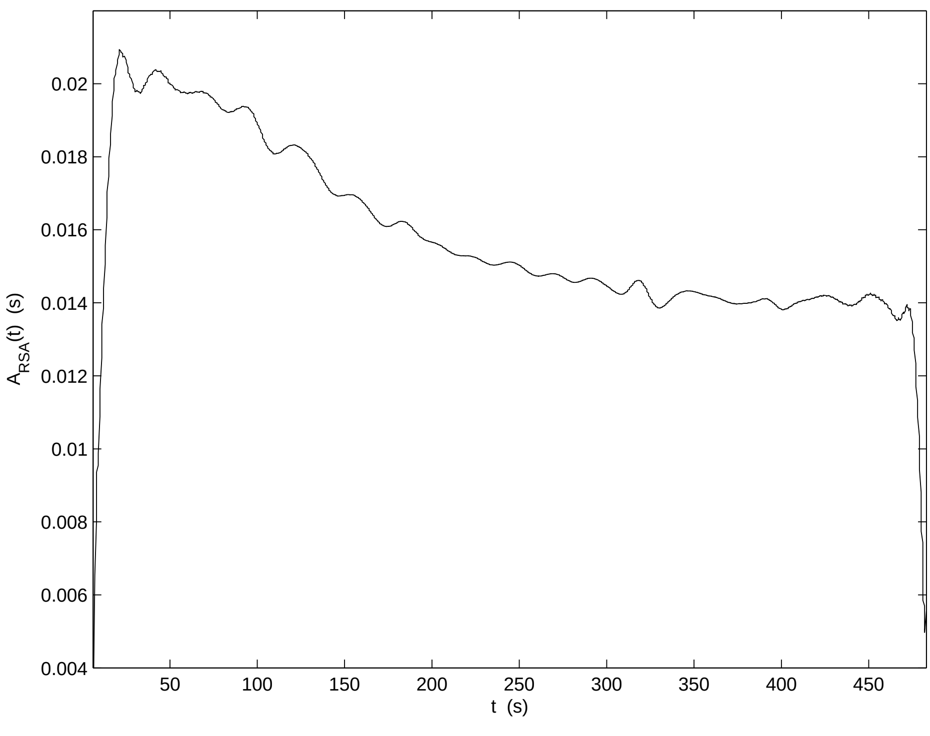

Fig. 8. The amplitude A_{RSA} of the variability signal $m(k)$ for the second simulation

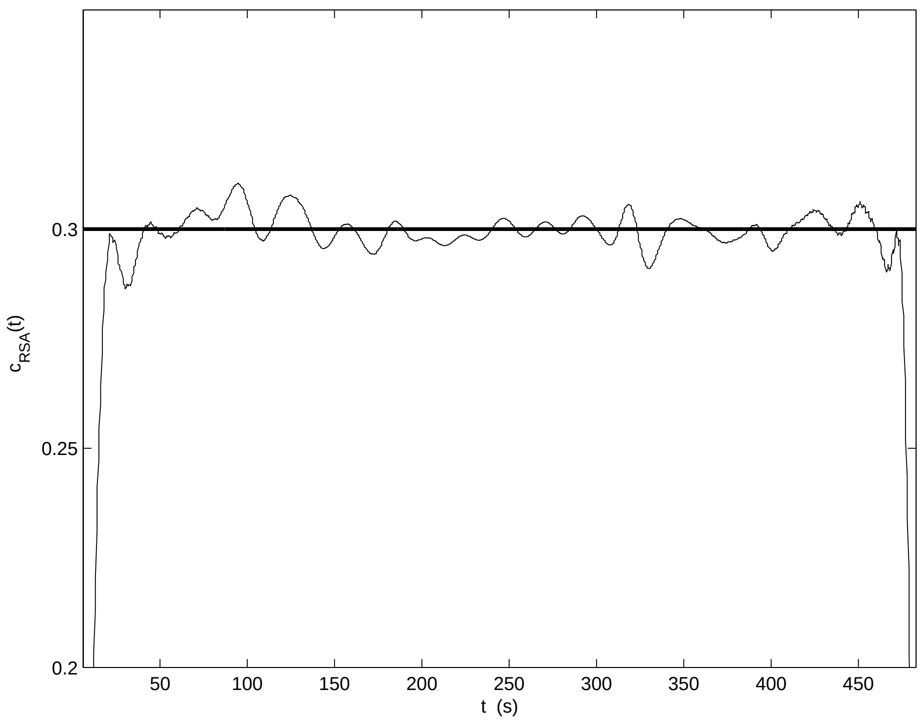

Fig. 9. The real (thick line) and the estimated (thin line) amplitude c_{RSA} of the second simulated respiration. Note the difference of shape with A_{RSA} in fig. 8