

## Sensitivity of Range Localization in a Multipath Environment

José M. F. Moura  
M. João D. Rendas

Dept. of Electr. and Computer Eng., Carnegie Mellon University, Pittsburgh, PA 15213, USA

### Abstract

In [2], we developed an algebraic procedure that determines the ray structure for a multi-linear environment. The present paper studies the sensitivity of range localization to several features, focusing on features recovered by temporal processing and develop a metric that plays the role of an ambiguity function now generalized to the multipath environment. Contrasting with the matched wavefield approaches that treat the channel as a black box, our modeling strategy effectively provides a manageable tool to understand the role that different features can play in localization. Contour plots illustrate the sensitivity of ranging to the features and metric adopted.

## 1 Introduction

The present paper studies the sensitivity of range localization to several features. We focus here on features recovered by temporal processing. We use for the channel the parametrized model of [2], which assumes a one dimensional multilinear velocity profile and a horizontally homogeneous transmission channel. This is an idealized version of more sophisticated existing models, our approach sitting in between full fledged field matching techniques [1] and ranging methods with only reflected paths [4]. Contrasting with the matched wavefield approaches that treat the channel as a black box and so obtain insight only through complex and expensive simulation studies, our modeling strategy effectively provides a manageable tool to understand the role that different features can play in localization. Besides considering individual features, the paper further reports on a metric that is adapted to the characteristics of the other two blocks with which it interacts: the propagation parameters estimator and the propagation model itself. In this model fitting, the

metric plays the role of an ambiguity function now generalized to the multipath environment. Contour plots illustrate the sensitivity of ranging to the features and metric adopted.

Usual mechanisms in source location are based on the spatial variation of the incoming waveform which is determined by processing the spatial correlation of the observations. We take here a different approach having in mind a two-step localization procedure. First, the channel transfer function is estimated or some of its relevant parameters, e.g., the set of inter-path delays. Then, a "pattern recognition" type technique matches the estimated impulse response against the predicted transfer function.

The paper is organized as follows: in the next section we present the propagation model used, and display typical channel impulse responses; in the third section, we pursue the identification of features that can be used for localization, by looking at their patterns of variation, and by analyzing their discriminatory power with respect to the source position; finally, in the last section, we define a distance in the set of possible impulse responses, and study the ambiguity surfaces intrinsic to the channel.

## 2 Model

Throughout this study, we use the algorithm detailed in [2] which determines the ray structure for a general multilinear velocity profile. Here, we take the simplest approximation, that of a bilinear profile. For simplicity, we also ignore in the sequel the bottom reflected paths. The following assumptions are in force: planar medium boundaries; the signal frequency is large enough for the ray-acoustics approximation to be valid; the velocity profile can be approximated by a bilinear variation; the frequency variation of the impulse response is negligible.

For this scenario, we are able to determine analytically the set of launching angles of the rays that join a given source/receiver pair (see [2]), and con-

\*On leave from CAPS, Instituto Superior Técnico, Lisboa, Portugal, on a fellowship from INVOTAN (Portugal).

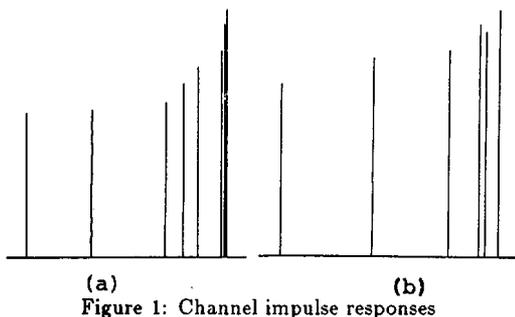


Figure 1: Channel impulse responses

sequently the delay and attenuation along each one. The ocean transfer function model obtained is the combination of delay/attenuation blocks, each one corresponding to an existing ray. We distinguish the SOFAR rays (that are completely refracted, never undergoing any type of interaction with the medium boundaries) and the rays that reflect only at the surface (where the reflection coefficient is near one in magnitude). As mentioned above, bottom reflected paths are here ignored.

This approximate model does not exhibit the whole complexity of real propagation environments. This is consistent with the goal of the paper of not pursuing the development of accurate channel models, rather the study of the general characteristics of its ambiguity structure and its impact in ranging when multipath is present.

A useful representation of the ray configuration is in Fig. 1, that illustrates two point-to-point channel impulse responses. Each vertical bar represents a ray arrival, its amplitude being proportional to its strength. The horizontal coordinate is time delay, so that if a ray appears on the left of another ray it has a smaller propagation time.

The pattern of the point-to-point impulse response depends strongly on the position of the two points considered. Fig. 2, represents a "waterfall" display of the impulse response along closely equispaced points over a horizontal (range) (Fig. 2-a) and a vertical (depth) (Fig. 2-b) line. In Fig. 2-a, the range varies between 49 km and 50 km, while in Fig. 2-b the depth is between 600 m and 1500 m.

From Fig. 2-a, we conclude that the shape of the pattern is relatively insensitive to short range variations, whereas it varies significantly with depth. This relative insensitivity along range of the shape of the impulse response helps to understand the difficulties associated with range estimation. The figure also displays the ray combination and separation that is observed in experiments conducted in the ocean [3]. This ray creation and annihilation accounted for by the model poses difficulties to traditional processing schemes that ignore the multi-

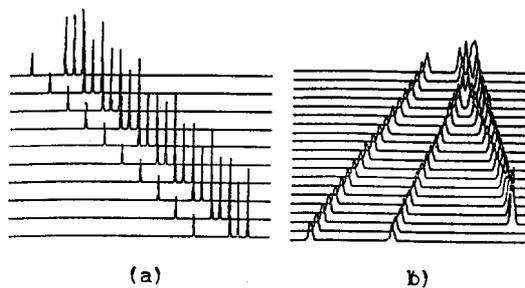


Figure 2: Range and depth dependence.

path propagation phenomena. On the other hand, its very existence helps the inversion of the channel, and so the solution of the localization problem.

Conventional array processing is based on the determination, from the wavefield observed at the output of an array of sensors, of the angles of arrivals of the incoming replicas. Subsequent estimation of the source localization is based on geometric considerations. The angles of arrival are calculated using knowledge of the array shape and of their relation with the inter-sensor delays. The variation of the delays between points in consecutive horizontal lines of Fig. 2-b gives the inter-sensor delay for sensors located in those points. Plots of this type serve to test the planar wave front assumption. This hypothesis to hold requires persistence of the rays and a constant slope for the corresponding lines. For either of the two most left rays of Fig. 2-b, the plane wave assumption is in fact validated. However, the equal slope of the rays makes them indistinguishable by spatial processing means. In order not to lose the information provided by these two arrivals, it becomes necessary to utilize temporal methods that can separate these replicas in the time domain.

In summary, we may say that the total number of rays increases with the distance from the duct revealing the familiar concentration of energy along depth and, albeit not so pronounced, it also increases with range. The latter does not imply an increase of the received energy, since the attenuation of each individual ray also decreases with range.

### 3 Feature Extraction

The ray structure presents a rich diversity that can be explored to perform the inversion of the channel, i.e., the localization of the source with respect to the receiver. We consider now characteristic parameters of the ideal impulse response predicted by the model, in order to study their usefulness for localization: total number of rays, time spreading of the returns, time spreading of the SOFAR rays, maximum inter-path delay. Other possible features not

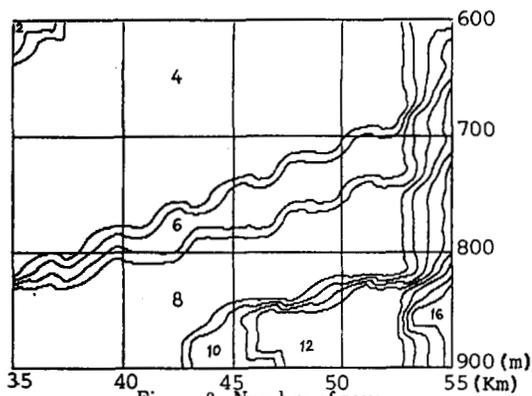


Figure 3: Number of rays

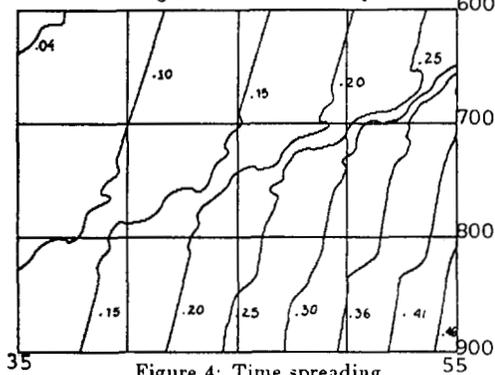


Figure 4: Time spreading

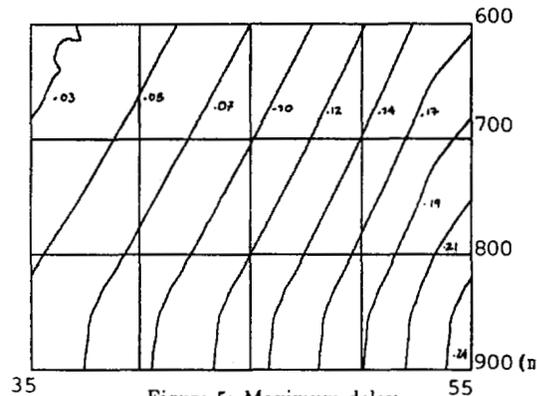


Figure 5: Maximum delay

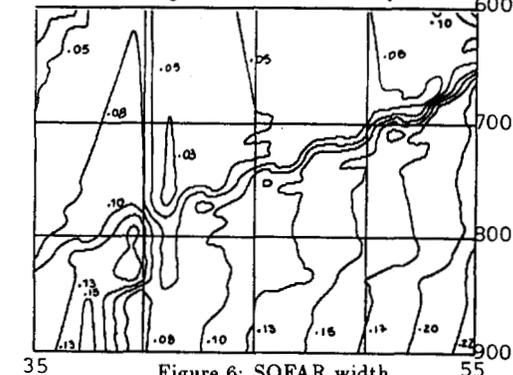


Figure 6: SOFAR width

presented here include the relative strengths of the arrivals, the number of refracted rays, their ratio, the angles of arrival. Given the vertical inhomogeneity of the medium, the detailed variations of the features studied depend on several factors like the particular source/receiver position, or the velocity profile considered. The conclusions reported here correspond to general trends.

A feature is useful for range estimation if its contour plots exhibit a predominantly vertical layering, while it is suited for depth estimation if a strong horizontal orientation is revealed by its contour plots.

Unless otherwise stated, the following plots are for a nominal configuration. Depths are measured downward from the surface, while ranges are with respect to the location of the source. The duct depth is at 914 m, the source is fixed at the depth of 850 m, and the receiver scans a rectangle grid of (70 km, 90 km) by (600 m, 900 m). Fig. 3 shows contour plots of the number of rays as the source traverses the grid. Aside the abrupt changes in the number of rays, mainly due to the neglecting of the bottom reflected rays, we distinguish an oblique variation of the number of rays received, showing the already mentioned increase with depth and range.

The time spreading of the propagation delays, i.e., the difference between the maximum and the min-

imum ray time delays, is shown in Fig. 4. We see that this parameter is strongly dependent on range, being a good candidate for range estimation. The first arrival is usually a surface reflected ray, while the more delayed ray is the one that remains closer to the duct. If we make the correspondence of these rays to the usual direct and surface reflected rays, this parameter translates into the time delay between the direct and the surface reflected rays, commonly used for ranging purposes (see [4]). A similar but clearer pattern is obtained when the total time spreading is substituted by the maximum interpath delay, see Fig. 5.

Another feature that exhibits an interesting variation is the time spreading of the SOFAR channel, see Fig. 6. The contour plots indicate a marked dependence in range of this parameter. Given the oblique nature of the level lines, this feature may be used, in conjunction with for example an integrated measure, to resolve ambiguities (see next section).

## 4 Generalized Ambiguity

While in the previous section, we looked at the role that individual features may play in the location problem, here we look at how a measure that in-

tegrates the effect of different features can be used to distinguish between two point-to-point transfer functions. The measure we chose is remnant of the ambiguity function of the radar/sonar signal processing literature. The ambiguity function plays a significant role since it helps to determine the performance limits of the localization procedure (large or decision errors, as well as local mean square errors, see [5]). It measures the angle between the reference field and the scanning field. Intuitively, this represents the similarity or dissimilarity of the impulse responses.

The probing signal that we use is a Gaussian function. The variance parameter  $\sigma$  subsums the delay discrimination ability of the receiver. Large values of  $\sigma$  combine rays, while small values of  $\sigma$  will resolve the individual returns. The proposed measure is the generalized cosine

$$J = \frac{\langle h_0(\tau - d), h_s(\tau) \rangle_{G, \tau}}{\|h_0(\tau)\|_{G, \tau} \|h_s(\tau)\|_{G, \tau}}$$

where the subindex 0 identifies the reference impulse response,  $s$  the scanning one,  $G$  is the Gauss kernel referred to above, and  $d$  is chosen as the delay that aligns the most delayed rays in both  $h_0$  and  $h_s$ . The numerator computes the inner product of the two impulse responses, the denominator is a normalization factor. Because the Gauss functions and the channel responses are positive  $0 \leq J \leq 1$ . The alignment operation reflects the inability of estimating absolute time delays. Unfortunately, it contributes to a squeeze of the range of variation of  $J$  explaining the absence of low level lines in the contour plots. We have tried other measures, but, at present,  $J$  seems to be a reasonable compromise.

Fig. 7 shows two contour plots of  $J$  for a "short" range and a "long" range (see grid dimensions on the plots). These plots were obtained with a relatively large value of  $\sigma$  (implying a resolution of the order of milliseconds). They exhibit a rather complicated irregular structure with a main lobe and important secondary lobes. The width of these lobes may be sharpened by decreasing  $\sigma$ . In our experiments, we have determined that the value of  $\sigma$ , i.e., the required ray resolution, has a threshold behavior, which is independent of the particular source/receiver configuration. For the cases we have studied, the threshold value of  $\sigma$  is within the delay resolution limits of present technology.

These plots show the high degree of sensitivity of ambiguity measures. We feel that mechanisms that couple to the ambiguity function type analysis the information provided by individual features drawn from temporal processing as here analysed and from spatial processing (e.g., angles of arrival) will lead to improved performance.

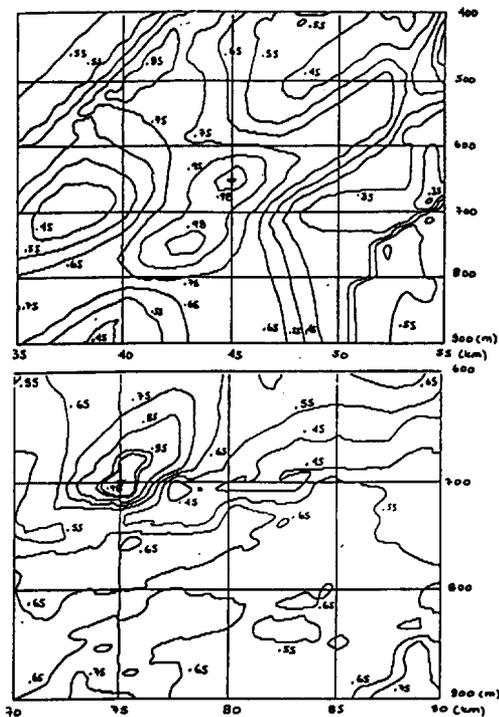


Figure 7: Generalized ambiguity contour plots

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