Path Planning in Extended Uncertain Environments

Maria João Rendas *Stefan Rolfes*

Lab. d'Informatique, Signaux et Systèmes de Sophia Antipolis (I3S)
2000 Rte des Lucioles, 06410 Biot, France
{rendas,rolfes}@i3s.unice.fr

Abstract – **This paper presents a new approach to the problem of planning the motion of a mobile robot in extended uncertain environments. All knowledge of the environment has been acquired by the robot during the current (or a previous) operation, such that the environment description reflects the accumulated error of the robot's pose during periods of dead-reckoning navigation. In this uncertain environment, the robot searches for trajectories that maximize the probability of attaining a desired target region. For that purpose we identify a discrete set of robot positions in order to construct a routing graph, whose arcs represent the probability of reaching a new position. In that way the search for an optimal trajectory is solved by searching for a minimum weight path in a routing graph. The method is based on a probabilistic model of all the errors/uncertainties affecting the reliability of the planned trajectory.**

I. INTRODUCTION

Two major approaches to the navigation of mobile robots have been proposed during the last years: *(i)* the behaviorist approach, based on chaining a series of reflexive behaviors of the robot with respect to its environment, and *(ii)* the deliberate approach, that defines the robot's trajectory as a sequence of basic purposive maneuvers, that are directly translated into a sequence of control actions. The advantages and drawbacks of each of these approaches are well known by the robotics community, the behaviorist approach been criticized by its inefficiency, and the deliberative approach by its lack of robustness, pointing the necessity of some combination of these two extreme frameworks.

In this paper, we present a navigation strategy for mobile robots who do not have access to absolute positioning information, working in a-priori unknown environments were a set of isolated objects are identified, and where the distance between objects is large compared to the range of the perception sensors, such that no continuous perceptual guidance is possible. In this context, the goal of the planning step studied in this paper is to indicate to the robot how to move from one object to another along regions were no perceptual information is available. The largest risk associated to robots operating in this kind of environments is to get lost, never being able to come back to a designated homing region (were, for instance, they must be picked-up or recharge their batteries). In this light, we adopt, for choosing the robot's trajectory, a criterion of *minimal risk*, instead of the traditional criterion of minimal length (or consumed energy).

To reduce the intractable set of possible movements we observe that (i) the uncertainty of the robots pose increases during deliberate motion with the degree of maneuvering required by the trajectory and (ii) since no absolute positioning information is available, the only possibility to minimize the uncertainty at the end of deliberate motions to minimize the uncertainty at their beginning.

Motivated by the first observation above, we restrict the robot's deliberate motions to straight line trajectories. To comply with (ii) we rely on the definition of a discrete subset of the configuration space, that we designate by "gateway points." To each object that the robot detected in the environment we associate a set of configurations, on which the accuracy of the robot's positioning with respect to the object is locally optimized.

In our approach, the robot's trajectory is thus defined as an alternation of deliberative (move in straight line from one object to another) and perceptually guided (acquire, with respect to the current object, the best position and orientation to initiate the next deliberative motion, i.e., place itself in one of the "gateway points") periods. We point out that the proposed criterion automatically yields a compromise between the deliberate and behaviorist approaches. If the positioning errors during purposive (dead-reckoning) displacements are important, a path will be found that relies heavily on the possibility of periodic repositioning us-

0-7803-4503-7/98/$10.00 1998 IEEE

ing environmental knowledge, increasing the duration of perception-based maneuvers. Otherwise, the length of straight line segments will be stretched, and a path close to the shortest path between initial and final positions will be found.

II. PROBLEM FORMULATION

Let $\mathcal{E}$ be the uncertain environment representation acquired by the robot. We assume that $\mathcal{E}$ is a collection of M objects $\mathcal{E} = \{O_1, \ldots, O_M\}$. Each object O_m is characterized by: *(i)* its position and orientation, described by the vector p_m, and *(ii)* its morphological properties (shape and, eventually, other attributes). The uncertainty associated to these entities can be described by any uncertainty characterization formalism. In our work, we assign a probability distribution to p_m and use fuzzy sets to describe object's shape [2, 5]. Based on the morphological properties of object O_m we identify the set of gateway points $\{p_{mj}\}_{j=1}^{M}$ at which the uncertainty at the beginning of the deliberate motion to object O_j is minimized.

Besides each object's position and attributes, we also assume known a measure of confidence of the relative positions and orientations of all pairs of objects. In our work, this relative uncertainty is described by conditional distributions $P(p_m|p_n)$. Note that these conditional distributions depend on the robot's trajectory during acquisition of $\mathcal{E}$, i.e., on the order by which the object's were detected. Each object defines a local frame, inside of which the robot's uncertainty is fundamentally dependent on the object's perceptual features. Moving from one frame to another is governed by the conditional distributions mentioned above.

Let $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ be a graph, whose nodes are the complete set of gateway points $\mathcal{N} = \{p_{ij}\}_{i,j=1}^{M}$, where we make $p_{ii} = p_i$, the object's positions and orientations defined previously. We call $\mathcal{G}$ the routing graph. Two sets of arcs $\mathcal{A} = \mathcal{A}_u \bigcup \mathcal{A}_d$ are defined between a subset of elements of $\mathcal{N}$. $\mathcal{A}_u$ is a set of *undirected* arcs $\{\beta_{ij}\}_{i=1,\ldots,M;j\neq i}$, linking node p_{ii} to the gateways from O_i to all other objects O_j. All these nodes have unit weight: $b(i,j) = 1$. These arcs correspond to executing a given sequence of perceptual behaviors, moving inside frame i, with reference to object O_i. The other set of *directed* arcs, $\alpha_{ij} \in \mathcal{A}_d$, corresponds to the allowed set of purposive motions, and link each gateway point p_{ij} in object O_i to the node p_{jj}. The weights of these arcs, $a(i,j)$, express the degree of confidence on passing directly from one object to another [3].

For the minimal risk criterion mentioned previously, the trajectory planning problem is equivalent to search for a minimum weight trajectory in $\mathcal{G}$ linking node p_{ii} to node p_{jj}. Let $t_{ij} = [i, t_{ij}^1, \ldots, t_{ij}^k, j]$ be one of the possible paths in $\mathcal{G}$ linking nodes p_{ii} to node p_{jj}. According to the probability algebra, the risk associated to this path is

$$C(t_{ij}) = a(i, t_{ij}^1) \cdot a(t_{ij}^1, t_{ij}^2) \cdots a(t_{ij}^k, j).$$

Once $\mathcal{G}$ is defined, any standard minimum length path algorithm can be used to find the optimal trajectory between any two pairs of objects. The routing graph $\mathcal{G}$ is updated each time the robot detects more objects, leading to an extension of the robot's autonomy (ability to reach more distant areas without getting lost) and/or safer maneuvering in the environment.

III. ARC WEIGHTS

In this section we briefly describe our approach to the determination of the weights $a(i,j)$ of each arc $\alpha_{ij} \in \mathcal{A}_d$.

We first consider that the gateway points have been determined, and address the general problem of determining the probability of reaching a given object from a given uncertain initial configuration, remitting discussion of gateway identification to subsection III.2.

This probability is formally related to a first passage time of the positioning error process. Previous studies have shown that one component of the error process during linear motions is, for platforms equipped only of proprioceptive sensors (and a compass), well approximated by a Brownian motion. This work is briefly exposed in subsection III.1. The determination of the statistics of first passage times of multivariate processes is a difficult problem, for which the majority of the analytical results concern their asymptotic behavior. We work with a one-dimensional projection of the error process, which enables the use of available results on Markov processes to determine of a lower bound on the probability of reaching a desired target object:

$$Pr(\text{reaching } R \,|\, p) \geq Pr\left(\forall_{t\in[0,T_R]}, x'(t) \in [-\ell_R, \ell_R]\right), \tag{1}$$

where x' is the positioning error in the direction orthogonal to the direction of motion, T_R is the time required to reach the center of the target with constant nominal velocity v_0, with a given probability $\gamma \simeq 1$, and $2\ell_r$ is the spatial extension of the target in the x' direction.

To compute T_R, let $P_{y'}^{\infty}$ be the asymptotic covariance of the error component along the direction of motion (see section III.1). Then, assuming that positioning errors are normally distributed,[1]

$$T_R = \frac{D}{v_0} + \frac{P_{y'}^{\infty}}{v_0}\sqrt{2}\operatorname{erf}^{-1}(2 - 2\gamma), \tag{2}$$

[1] $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$

0-7803-4503-7/98/$10.00 1998 IEEE

D being the distance from x to the center of the target. Assimilating, according to the results presented in section III.1, the error along the orthogonal direction, x', to a Brownian process, we can use available results concerning first passage and exit times to compute the lower bound in (1). Let $t^*_{\ell_R}$ be the first exit time from the interval $[-\ell_R, \ell_R]$ of the error along x' :

$$t^*_{\ell_R} = \inf_t \{ sup_{0 \leq s \leq t} |x'(s)| \geq \ell_R \} .$$

Then, it is known [1] that the distribution function of $t^*_{\ell_R}$ is given by

$$Pr(t^*_{\ell_R} \leq \tau) = 2Pr(|x'(\tau)| \geq \ell_R). \quad (3)$$

The desired bound can be computed, using the previous equation and eq. (1), by

$$Pr(\text{reaching } R \,|\, p) \geq 1 - 2Pr(|x'(T_R)| \geq \ell_R), \quad (4)$$

which can be evaluated using the statistical characterization of the error of the modified variable $x'(\cdot)$ at the final time T_R, for which closed form expression are known, see III.1 for a general expression for the covariance $P_{x'}(t|t)$.

The local positioning errors at each gateway are characterized in the local frame of the departing object, and are attached to each gateway point, as we discuss in section III.2.

III.1. Dead-reckoning periods

Advanced inertial systems presently available rely on the use of Kalman-Bucy type filtering to derive high precision estimates of the position and velocity of moving platforms. When only proprioceptive displacement measurements are available, it is well known that the uncertainty with respect to the robot's position is strictly increasing in time. The Riccati equations determine the evolution of the second-order statistical moments of the error of minimum mean square estimators, and thus quantify the minimum error increase over time that can be obtained using a given set of proprioceptive sensors.

These matrix equations are non-linear, and thus must be numerically integrated to obtain a prediction of the error at the end of a given trajectory, precluding, for long trajectories, the real-time implementation of planning approaches that consider the impact of deliberative motions on positioning uncertainty. Considering restricted types of possible motions of the robot (with constant velocity), we found *analytical expressions that set a tight upper bound on the solution of the exact Riccati equations.* Details of the work are given in [4], and we just summarize the important results. Our equations show that the error in position accumulated over linear uniform motions of a two-wheeled robot using only odometric information increases (asymptotically)[2]

- with t^3 in the direction orthogonal to the direction of motion: $P^o_{x'}(t|t) = P^o_{x'_0} + A^o_{x'} \cdot t + B^o_{x'} \cdot t^2 + C^o_{x'} \cdot t^3$;
- with t in the direction of motion: $P^o_{y'}(t|t) = P^o_{y'_0} + A^o_{y'} \cdot t$;

while, if a compass is also available, the error will

- grow only linearly in t in the direction orthogonal to the motion: $P^c_{x'}(t|t) = P^c_{x'_0} + A^c_{x'} \cdot t$;
- have an asymptotic behavior along the direction of motion: $P^c_{y'}(t|t) \to P^\infty_{y'}$.

Constants $P^o_{x'_0}$, $A^o_{x'}$, $B^o_{x'}$, $C^o_{x'}$, $P^o_{y'_0}$, $A^o_{y'}$, $P^c_{x'_0}$, $A^c_{x'}$, $P^\infty_{y'}$ on the previous expressions depend on the initial error conditions, on the dynamic characteristics of the platform, and on the precision of the sensors, see [3] for detailed expressions of these constants.

III.2. Gateway Points

Determination of the gateway points p_{ij} associated to each pair of objects is highly dependent on the particular perception sensors used by the robot, and on its holonomic constraints. We formally define the gateway points, as said previously, as the points that *allow a relative positioning of the robot with respect to the departure object that yield a lower risk of not achieving the target object*, as given by expression eq. (1). As discussed in the previous subsection, we can predict, using analytic equations, the uncertainty associated to the robot at the end of a given motion, knowing the duration of the motion and the uncertainty affecting the initial robot configuration. The initial uncertainty is, as we said previously, the sum of two components: a local error plus an uncertainty on the relative error linking the two frames.

The missing information to be able to identify the gateway points is thus a measure characterizing the local initial uncertainty, when the robot tries to *dock* into a pre-specified position and orientation, using its local perception of the object.

We assume that the estimates of the position and orientation of the robot with respect to an object O are obtained by Maximum Likelihood and use standard results on performance of parametric estimation problems to evaluate docking accuracy [4].

This approach to the definition of gateway points, although optimal, leads to a non-linear multivariate optimization problem, which is far too complex to be solved in real-time, each time a new object is found.

[2] t is the time interval since motion start.

It presents two main disadvantages: *(i)* it depends on the trajectory of the robot during docking, which further increases the set of variables with respect to which optimization must be performed; and *(ii)* the search space, for each pair of objects, consists of *all the points* in the vicinity of the departure object.

To overcome these two problems, we must

1. restrict all gateway points $p_{ij}, j \neq i$ to a discrete subset $\mathcal{C}_i$ of the neighborhood of O_i;

2. constraint the robot's maneuvers when approaching each possible gateway point in $\mathcal{C}_i$.

To define sets $\mathcal{C}_i$, we assumed that the set of admissible gateway points is first restricted to a one-dimensional variety of the configuration space, parameterized, for instance,by path length with respect to some reference point: $\mathcal{C}_i = f_i(\omega), \omega \in \Omega$. This is basically obtained by restricting the set of possible positions x to a curve containing the initial object, and imposing the departure direction $\theta_j(x)$ by directing the robot to the center of the target object. With this assumption, which is in general not too restrictive, the remaining variables, over which minimization of the risk must be performed, are ω and the docking trajectory.

By considering only polygonal objects, and robots with two driving wheels, we were able to define a convenient set of docking maneuvers and subsequently identify a small discrete set of points C_i (located near the corners of the objects) enabling the fast computation of the gate-way points whenever a new object is found. These results can be found in [4] and will be published elsewhere in the near future.

IV. EXPERIMENTAL RESULTS

We used a small circular (52mm diameter) two-wheeled mobile robot (Khepera [6]), equipped with odometric counters and infra-red proximity sensors, to demonstrate the feasibility of the navigation strategy presented in the paper. Figure 1 illustrates the uncertain environment description acquired by the mobile robot during a previous operation for the purpose of navigation. As shown in this figure the robot acquired an uncertain representation of four objects,

$$\mathcal{E} = \{O_1, O_2, O_3, O_4\},$$

where O_1 is the initial object (the homing object). Object's numbering indicates the order by which they were detected. The robot is placed in the vicinity of O_4, at the position indicated by a small circle. The two ellipsis displayed around the robot's position indicate the uncertainty affecting the robot's state before (large ellipsis) and after (small ellipsis) repositioning in the local frame of O_4.

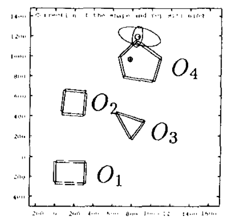

Figure 1: Learned environment representation.

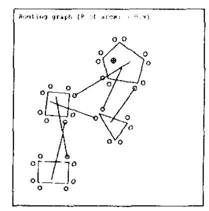

Figure 2: Routing graph.

In all figures shown below, the robot position is indicated by $\bigcirc$, along with the uncertainty ellipsis affecting its state knowledge. The current routing graph $\mathcal{G}$, is shown in figure 2. Gateway points are represented by $\circ$, and are in this case (polygonal shapes) located at the corners of the objects. Only the directed arcs $\alpha_{ij} \in \mathcal{A}_d$ with a weight larger than 0.9 are indicated, being represented by line segments with source at the gateway point, and directed at the target object. We point out that arcs connecting objects O_1 and O_3 do exist, but with a very small weight. This routing graph shows that the only object that can be directly reached from O_1 with a success probability of at least 0.9 is O_2; that from O_2 the robot can reach O_1 and O_4; from O_3 it can go to either to O_4 or O_2, and from O_4 to O_3.

In a first step, illustrated in figure 3, we let the robot return to the homing object O_1. The optimal path $t_{41} = [4\,3\,2\,1]$ chosen by the robot is indicated in plot (1) and consists of the consecutive trajectory segments $t(4,3)$, $t(3,2)$ and $t(2,1)$. Planned paths are represented by their routing arcs. For each routing arc, which necessarily starts at a gateway point ($\circ$), we indicate the nominal straight line motion, and two other line segments indicating the angular cone inside which the robot is expected to evolve. The actions that correspond to the segment $t(4,3)$ can be summarized as follows: *(i)* execution of perception-based motions to dock into gateway p_{43} – in order to minimize uncertainty at

0-7803-4503-7/98/$10.00 1998 IEEE

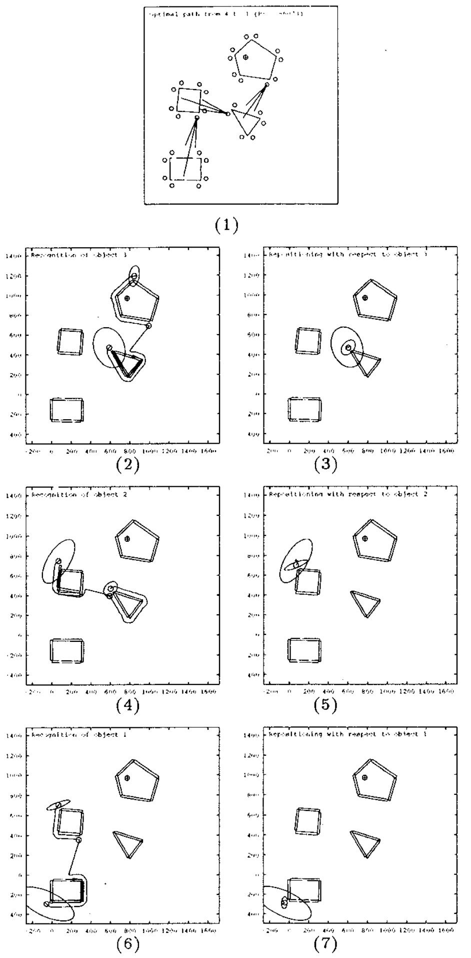

Figure 3: Returning to O_1.

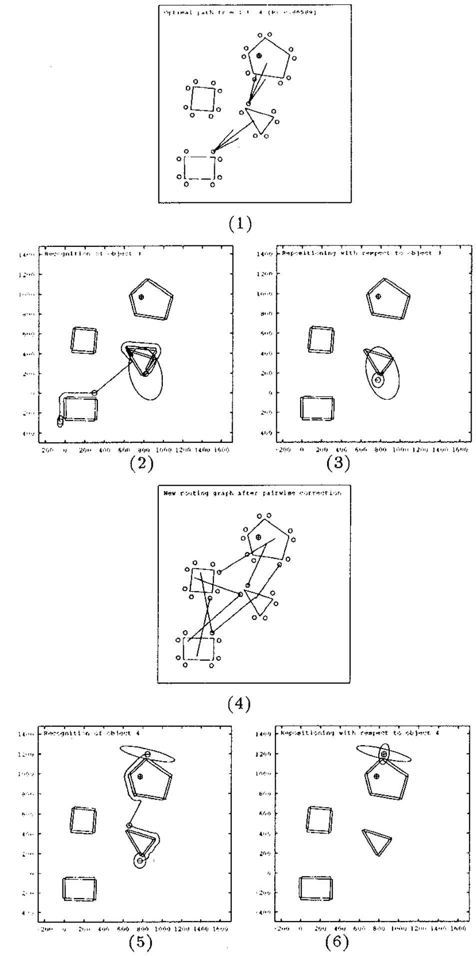

Figure 4: Moving to O_4.

the beginning of the deliberate motion to object O_3 – *(ii)* moving in straight line motion in the direction of O_3, and *(iii)* observation of object O_3 in order to reposition the robot with respect to O_3 – acquire its position in the local frame defined by this object. These actions are shown in plot (2) of figure 3. Plot (3) shows the final state of the robot at the end of execution of this segment. Two ellipsis are again sown in this figure: the large one expressing state uncertainty before repositioning in the local frame of O_3 and the smaller state uncertainty after repositioning. The robot repeats the same actions for the trajectory segments $t(3,2)$, see plots (4),(5) and $t(2,1)$, see plots (6),(7).

After having reached the homing object O_1 the robot moves back to object O_4, see figure 4. The optimal path, that consists of the trajectory segments $t(1,3)$ and $t(3,4)$ is displayed in plot (1). Comparison of this graph with the one shown in plot (1) of figure 3 shows clearly that the arcs of the routing graph are not symmetric. This is explained by the fact that the probabilities of reaching a given target object O_m from a given current object O_n depend, not only on their relative conditional distributions $P(p_m|p_n)$, but also on the morphological properties of the objects, in particular the spatial extension of the target object and docking accuracy at the departing gateway.

Plots (2) through (6) in figure 4 detail the way back to O_4. After having performed the trajectory segment $t(1,3)$, see plots (2) and (3), the robot updated the routing graph $\mathcal{G}$, since it acquired a more accurate knowledge of the relative positions between objects O_1 and O_3, expressed by a new conditional distribution $P(p_3|p_1)$

0-7803-4503-7/98/$10.00 1998 IEEE

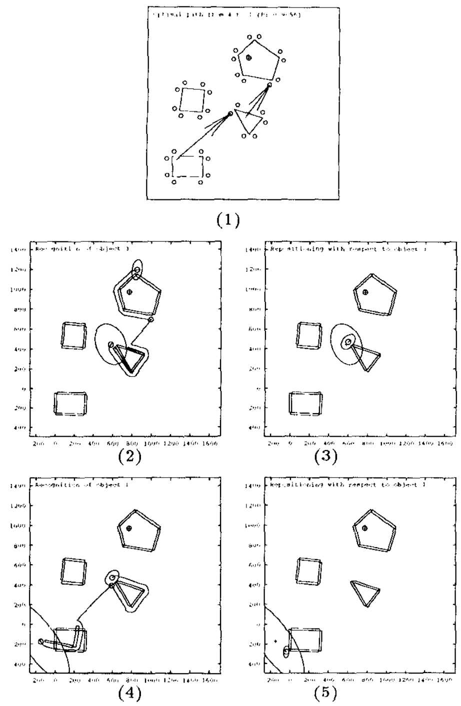

Figure 5: Final homing.

with reduced uncertainty. Note that the previous conditional distribution models the uncertainty accumulated over the longer trajectory $O_1 \rightarrow O_2 \rightarrow O_3$. The updated conditional distribution leads to a larger probability of reaching object O_3. This is reflected in the updated routing graph shown in plot (4) containing new routing arcs between O_1 and O_3, which allow for shorter planned trajectories. The robot continued, see plots (5) and (6) and finally reached object O_4.

The last phase of the exploration takes the robot to the homing object O_1, and is represented in figure 5. The optimal path, indicated in plot (1) differs from the path found previously, see plot (1) of figure 3. Since a more accurate knowledge of the relative positions of O_3 and O_1 is now available, a path closer to the shortest path between O_4 and O_1 has been found. Finally, the robot executed the planned path to O_1, see plots (2) through (5), finishing its mission after having repositioned itself in the framework of O_1, see plot (5).

V. CONCLUSIONS

We presented a probabilistic framework for planning the navigation of mobile robots in uncertain sparse environments. Our work assumes that the robot has no access to its global position, and that the environment consists of a set of objects whose separation is large compared to the range of the perception sensors. Our approach is based on maximizing the probability of successfull transition between any pairs of known environmental components, by requiring, if necessary, that the robot uses other objects as intermediate landmarks, optimally combining periods of deliberative motions and perceptual behaviors. The feasibility of the approach was demonstrated in real robotic experiments, during which the mobile robot autonomously planned its trajectory on the basis of a learned uncertain environment description.

VI. ACKNOWLEDGMENTS

This work has been partially funded by the European Union through the ESPRIT/LTR project NARVAL.

VII. REFERENCES

[1] Leo Breiman. Probability. Classics in Applied Mathematics Series, SIAM, Philadelphia, 1992.

[2] M. João Rendas and Stefan Rolfes. Propagating Multiple Hypothesis for Unknown Environment Learning. Int. Symp. on Intelligent Robotic Systems, Lisbon, Portugal, July 1996.

[3] M. João Rendas and Isabel Lourtie. Prediction of Positioning Uncertainty Accumulated Over Linear Motion Using Dead-Reckoning. CNRS, Laboratoire I3S (96-42), September 1996.

[4] M. João Rendas and Isabel Lourtie. *Navigation and Mapping*. Deliverable D3, Project ESPRIT/LTR 20185 - NARVAL, February 1997.

[5] Stefan Rolfes and M. João Rendas. Shape Recognition: a Fuzzy Approach, "*Int. Conf. on Robotics and Automation*, ICRA'98, May 1998. Leuven, Belgium.

[6] K-Team. Khepera: Users manual, Laboratoire de microinformatique, Swiss Federal Institute of Technology (EPFL), INF - Ecublens, 1015 Lausanne/Switzerland