Performance Analysis of Ocean Tomography Systems

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En tomographie océanique on utilise des moyens acoustiques pour estimer les paramètres caractéristiques du milieu sous-marin, comme le profil de température et les vitesses des courants. Ces techniques sont basées sur la dépendance entre les transformations subies par le signal acoustique pendant la propagation dans le milieu et les paramètres qui doivent être estimés. Un des problèmes en tomographie acoustique est de défnir le système de mesure (taille, geométrie et position des antennes, forme du signal, etc.) de façon à ce qu'il identifie les paramètres voulus avec une précision donnée (biais et variance). Dans cet exposé, on étudie ce problème en utilisant un outil d'analyse global mis au point originellement pour l'analyse des performances en localisation de sources.

In ocean tomography acoustical means are used to infer characteristic parameters of the underwater medium such as temperature profile and current velocity. These techniques are based on the functional dependency between the transformation suffered by acoustical signals when traveling in the ocean and the parameters that must be estimated. One of the problems in acoustic tomography is to design the measuring system (size, geometry, and location of the antennas, signal shape, etc.) such that it will identify the desired parameters within a given accuracy (biases and variance). In this paper, we study this issue, using a global analysis tool originally designed for performance analysis of source location problems.

1 Introduction

Ocean tomography uses acoustic signals to infer parameters of the underwater channel such as the sound velocity profile (SVP), temperature field distributions, or current velocities. These physical parameters are usually assumed known in other applications of underwater acoustics, as in sonar systems, for instance, for localization and tracking of targets. Modern developments in source localization couple to the signal processing algorithms complex propagation models. The accuracy of these techniques depends on the refinement of the propagation model and on the degree to which the physical parameters are precisely measured. Ocean tomography solves a problem which in a sense is the inverse of the localization problem. Its goal is to determine the physical parameters assuming known the relative position between the source/ receiver pairs. Due to several effects like for example ocean currents or errors in instalation, this position cannot be known precisely. For the sake of being concrete, we carry out the analysis of tomographic systems assuming in this paper that, besides measurement noises, the uncertainty in the tomography system relates to the errors in source position.

Several different approaches can be used to deal with this type of uncertainty: 1) Use of a nominal known value for the position parameter, i.e., to assume perfect knowledge of the position; 2) Admit that the position parameter has a known statistical distribution. This has the effect of blurring the description of the observed data; 3) Try to improve the description of the model, i.e, to attempt to estimate jointly the ocean parameters of interest and the model parameters.

Each of these three approaches has a distinct impact on the overall performance of the system and corresponds to receivers of distinct complexity: both the quality and the complexity of the estimates increases, in general, from 1) to 3). The relative payoffs depend on the actual role played by the imprecisely known parameter, i.e., on the sensitivity of the model to this parameter, on the amount of à priori uncertainty, and on the degree to which the unknown position and the tomographic parameters are jointly observable.

The paper presents a tool devoted to the analysis of the performance of ocean tomography systems by applying to this study a generalization of the ambiguity function introduced in [3] for localization systems. The ambiguity function is a global analysis tool that accounts for large errors rather than the local errors taken into consideration by the

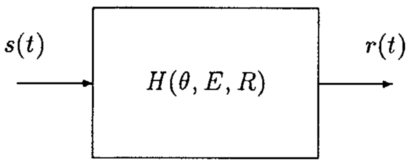

Figure 1: Inverse problems in underwater signal processing.

Cramér-Rao bounds. Based on the information provided by the geometric properties of the probabilistic manifold that describes the observed data, this function describes the statistical observability of the parameters being estimated, incorporating in an integrated manner the invertibility of the transmission operator, the effect of the noise, and the impact of uncertainties. As we will see, the ambiguity index introduced in [3] is defined directly in terms of the Kullback directed divergence between the conditional distributions associated with a parametrized family modeling the observed data and can thus be directly applied to many estimation problems, in particular to ocean tomography.

In the next section we formulate the problem. Then, we introduce the ambiguity function. In the last section, using a very simple case study, we demonstrate the application of the ambiguity function to the analysis of ocean tomography.

2 Problem Formulation

As brifely discussed in the Introduction, ocean tomography and sonar localization share a very important characteristic, namely they utilize the same probabilistic model to describe the observed data, only its parametrization is different. The diagram of Figure 1 illustrates this point. In the diagram, $s(t)$ denotes a vector of source signals which corresponds to multiple sources in source location problems or to several emitters in tomography; $r(t)$ is the signal observed by an array of sensors; E and R are the parameters describing the geometry and location of the emitting and receiving antennas, respectively; θ describes the physical medium parameters; and $H(\theta, E, R)$ is the matrix transmission operator that combines the directional characteristics of the emitters and of the receivers with the propagation effects of the channel. The transmission operator $H(\theta, E, R)$ is parametrized by the emitter and the receivers' location parameters E and R, respectively, as well as by the channel physical parameters. With ocean tomography, we are interested in estimating θ assuming that E and R are known. With localization problems, it is θ and either E or R that are assumed to be known while it is R or E that are to be estimated. It is in this sense that tomography and localization are said to be inverse problems.

As mentioned above, we carry out the global analysis by applying the definition of ambiguity function as in [3]. To demonstrate the validity of the approach, the paper considers the simple case of an horizontally stratified medium, with perfectly flat boundaries. The ocean is divided in two horizontal layers where the velocity gradient is constant: in the upper layer, sound speed decreases linearly with depth, in the lower layer there is a positive constant gradient (ducted propagation). This simple bilinear model has the advantage of preserving a certain degree of analyticity, exibiting, at the same time, the multipath propagation which is commonly present in vast ocean areas, with a number of distinct rays between any two given points being present. In [3], we studied the performance of location systems using this tool, showing the potential advantage of explicitly modeling the temporal (inter-path) delay structure of the observations.

We assume that the bottom depth, the sound speed at the surface, and the gradient of the sound speed profile (SVP) at the lower layer are known and that it is the duct's depth and the gradient in the upper layer that are to be estimated. We consider that the emitting and receiving antennas are fixed (i,e,, their position do not change with time). The emitter is a point source radiating a wideband pseudo-random signal with known power density, and that the receiving antenna consists of several sensors arranged in a uniformly spaced vertical linear array.

For this simple scenario, the following studies are pursued:

Observability study: is it possible to identify the actual values of the physical parameters (e.g., upper layer SVP gradient and duct's depth).

Incorrect prior knowledge: This is a sensitivity issue. How does erroneous information regarding position parameters affect the ambiguity structure associated with the physical parameters.

These studies are designed to illustrate the relevance of the global analysis tool in [3] in analyzing the expected performance of a tomography system. In particular, how it may be used to assess the impact that wrong modeling assumptions have in the ability of the tomographic system in estimating the physical parameters of interest.

3 Ambiguity Function

Consider a family $\mathcal{G}_\alpha$ of density functions, indexed by a parameter $\alpha \in A$:

$$\mathcal{G}_\alpha \triangleq \{p(x|\alpha), \quad \alpha \in A\}.$$

The Kullback-Leibler number (also called Kullback directed divergence or cross-entropy) between two members of $\mathcal{G}_\alpha$ is [1]:

$$I(\alpha_1, \alpha_2) \triangleq \mathrm{E}_{\alpha_1}\left\{\ln \frac{p(x|\alpha_1)}{p(x|\alpha_2)}\right\}.$$

In this equation, E_{α_1} is expectation with respect to the probability density function $p(x|\alpha_1)$. This functional was introduced by Kullback [1] in the framework of information theory. Although it has some distance-like properties, it is not, in fact, a distance. As it can be easily seen, it is not symmetric and it does not satisfy, in general, the triangular inequality. However, $I(\alpha_1, \alpha_2) \geq 0$, with equality iff $\alpha_1 = \alpha_2$. Note that

$$I(\alpha_1, \alpha_2) = \mathrm{E}_{\alpha_1}\{\ln p(x|\alpha_1) - \ln p(x|\alpha_2)\},$$

i.e., $I(\cdot,\cdot)$ is the mean value of the difference between the values of the log-likelihood function for two points in the parameter space, for observations x, conditioned on one of those points. The value of $I(\cdot,\cdot)$ depends, naturally, on the size of the observation interval. Here, we consider only the asymptotic case of very long observation interval.

Heuristically, $I(\alpha_1, \alpha_2)$ is a measure of the resemblance, or proximity, of the two models described by $p(x|\alpha_1)$ and $p(x|\alpha_2)$. The values of α_2 that yield small values of $I(\alpha_1, \alpha_2)$ indicate possible erroneous estimates of α when the true value of the parameter is α_1.

Based on these arguments, ambiguity between two points (α_1, α_2) in the parameter space is defined as

$$\mathcal{A}(\alpha_1, \alpha_2) \triangleq \frac{I_{MAX}(\alpha_1) - I(\alpha_1, \alpha_2)}{I_{MAX}(\alpha_1)} \quad (1)$$

where $I_{MAX}(\alpha_1)$ denotes an upper bound on the value of $I(\alpha_1, \alpha_2)$ over $\alpha_2 \in A$. Since $I(\cdot,\cdot)$ is not symmetric, $\mathcal{A}(\alpha_1, \alpha_2)$ is not, in general, a symmetric function of its two arguments.

Consider that the observations' power spectrum is described by

$$R_\theta(\omega) = \mathcal{S}(\omega) h_\theta(\omega) h_\theta(\omega)^H + \sigma^2(\omega) I_K$$

where we assume that the observation noise is spatially incoherent, with known power density $\sigma^2(\omega)$. In the previous equation, $\mathcal{S}(\omega)$ is the *unknown* source spectral density and $h_\theta(\omega)$ is the resultant vector, that describes the coherent combination of the steering vectors corresponding to the P replicas received, see [2, 3] for further details.

The resultant vector can be decomposed as

$$h_\theta(\omega) = D(\theta) b(\theta)$$

where the $K \times P$ matrix $D(\theta)$ describes the spatial structure of the individual replicas, depending only on the intersensor delays for each received path, and $b(\theta)$ is a P dimensional vector that depends only on their temporal alignment.

Using the relation

$$\mathcal{R}_\theta(\omega)^{-1} = \frac{1}{\sigma^2(\omega)} \left(I - \frac{\mathcal{S}(\omega)}{E_\theta(\omega)} h_\theta(\omega) h_\theta(\omega)^H \right)$$

where the scalar $E_\theta(\omega)$ is defined by

$$E_\theta(\omega) = \sigma^2(\omega) + \mathcal{S}(\omega) \|h_\theta(\omega)\|^2,$$

leads to

$$\begin{aligned} \bar{I}(\theta_0 : \theta) &= \frac{1}{2} \int \left[\frac{\mathcal{S}(\omega)}{\sigma^2(\omega)} \|h_{\theta_0}(\omega)\|^2 - \frac{\mathcal{S}(\omega)}{E_\theta(\omega)} \|h_\theta(\omega)\|^2 \right. \\ &- \frac{\mathcal{S}(\omega)^2}{\sigma^2(\omega) E_\theta(\omega)} |h_{\theta_0}(\omega)^H h_\theta(\omega)|^2 \\ &+ \left. \ln \frac{E_\theta(\omega)}{E_{\theta_0}(\omega)} \right] d\omega. \end{aligned}$$

Ambiguity is computed using this equation in the general definition given before, see [4] for a complete discussion.

4 Case Studies

We present in this section ambiguity plots for the estimation of suface layer velocity gradient and duct depth in a deep ocean area, considering a bilinear approximation to a velocity profile typical of the North Atlantic with the following nominal values: gradient above duct (g_0): -0.0035 s^{-1}, gradient below duct (g_1): .013 s^{-1}, duct depht: 950 m, and sound speed at the surface: 1500 m^{-1}.

In all scenarios studied, the receiving antenna is a uniform vertical linear array, with inter-element spacing described by the parameter d, the number of sensors K, and the depth y_0 of the top-most element. The source signal has a flat spectrum in the bandwidth of the receiving system, and the signal to noise (Gaussian, white) ratio is described by the parameter SNR (ratio between the power density of the signal at the source to the noise density at the receiver).

The first two plots show the ambiguity surfaces in the ideal case, where perfect knowledge of all the modeling parameters is asumed, except of those being estimated. They illustrate the impact of the antenna placement on the ability to estimate the desired parameters. In both case, but in particular when the antenna is placed at 100 meters (Fig. 2), where the main lobe is clearly defined, the ambiguity surfaces reveal a strong correlation between duct depth and upper layer gradient. Comparing plots 2 and 3, wee see that important secondary lobes are formed when the antenna is located well inside the Sofar channel, but that the width of the main lobe is wider for the antenna at 100 meters. This kind of trade-off between local and global properties is well known in active sonar systems, and demonstrates that the role of the radiated signals in those systems is here also played by source/antenna placements.

The second group of plots, Fig. 4 through Fig. 6 show the sensitivity of the tomography systems to uncertainty on the parameters that are treated as being known (source position, ocean bottom, deep layer gradient). Comparing these plots to the corresponding ideal one Fig. 3, we see that utilization of nominal erroneous parameter values results in the introduction of biases (the ambiguity curves no longer peak at the right values) and/or lead to deformation of the original structure.

Fig. 4 shows that an error of 10 meters in source immersion has a drastic effect on the ability to estimate the two

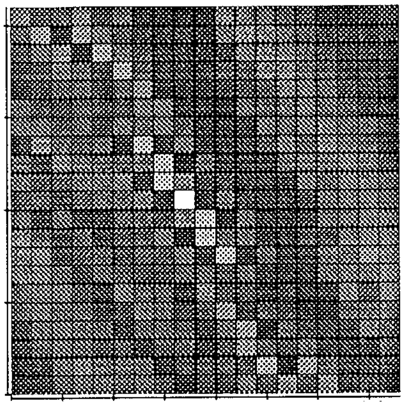

Figure 2: Antenna immersion 100 m.

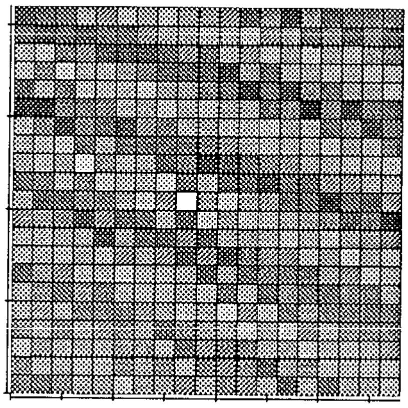

Figure 3: Antenna immersion 610 m.

parameters of interest, leading to a complete breakdown of the structure of the surface.

In Fig. 5 we see that an error of 50 meters on bottom depth has no significant influence on system's performance.

The last figure (6) corresponds to a mismatch in the velocity gradient on the lower layer. In this case, the structure is roughly preserved, but a large bias in both upper layer gradient and duct depth appears.

References

[1] Solomon Kullback. *Information Theory and Statistics.* Peter Smith, 1978.

[2] M. João D. Rendas and José M. F. Moura. Cramer-Rao bounds for source location systems in multipath environments. *IEEE Trans. on Acoustic, Speech and Signal Processing*, Vol. 39, No. 12:2593–2610, Dec. 1991.

[3] M. João D. Rendas and José M. F. Moura. Ambiguity analysis in source location. *Submitted for publication*, May, 1991.

[4] Maria João Rendas. *Erro e Ambiguidade em Sistemas de Localização.* PhD thesis, Instituto Superior Técnico, Lisboa, 1991.

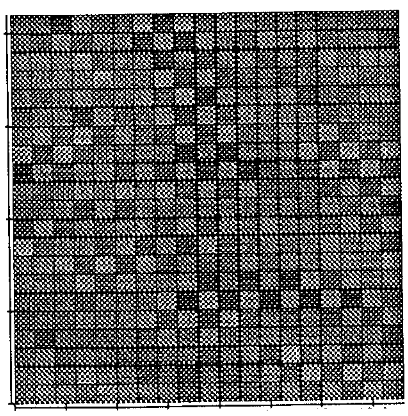

Figure 4: Wrong value of source immersion (600 m).

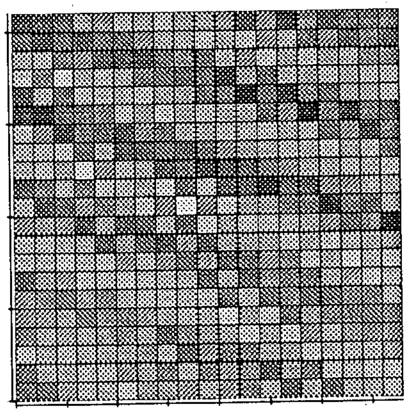

Figure 5: Wrong value of bottom layer gradient (g_1 = .014 s^{-1}.

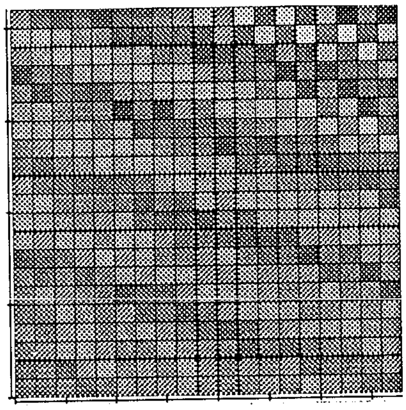

Figure 6: Wrong value of ocean bottom (4050 m).