Statistical environment representation to support navigation of mobile robots in unstructured environments

Stefan Rolfes[1] rolfes@i3s.unice.fr

Maria-João Rendas[1] rendas@i3s.unice.fr

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Abstract

In this paper we present a novel approach to mobile robot navigation based on environment representation by statistical models. Natural unstructured scenes are interpreted as realisations of *Random Closed Sets* (RCS), whose global characteristics are mapped. Contrary to the feature based approach, this environment representation does not require the existence of outstanding objects in the workspace, and is robust with respect to small dynamic changes. We address the relocalisation problem assuming that a statistical model, serving as a map of the environment, is available a priori. Simulation results demonstrate the feasibility of our approach.

1 introduction

Mobile robot navigation, which is still an open issue in the context of structured indoor environments, is even more challenging when applied to unstructured and large scale outdoor environments. Whereas land robots can use GPS as a support for relocalisation, an autonomous vehicle operating underwater (AUV) has to strongly rely on its perceptual capabilities in order to estimate its position.

The main challenge of navigation in large underwater areas is the safety of the robot: if the robot looses track of its current position autonomous recovery can no longer be guaranteed. To overcome the problems associated with search and recovery, the vehicle needs to be able to return to a *homing area* (e.g. the vicinity of the boat from which it was launched), where it can be picked up after completion of the mission. If the positioning error is too large, the robot will miss the homing area and get lost.

Navigation of AUV's requires thus that the estimation error be maintained below appropriate bounds. This cannot be guaranteed if the vehicle relies only on proprioceptive (inertial) sensors. Perceptual data must be used to find natural landmarks that can be periodically used for relocalising the robot.

The most common approach for relocalisation of mobile robots is feature based, see e.g. [1, 2]. Features, described by low dimensional parameter vectors, are stored in an internal map. Relocalisation is subsequently done by estimating the rigid motion that matches recently observed features to those already contained in the map.

While the feature based approach yields good results for structured indoor environments, where dense maps of easily identified features can be obtained, it is less suited to unstructured environments where unstable feature identification leads to frequent mismatch or to absence of feature matches.

Instead of creating a detailed description of the environment as a collection of features, describing salient characteristics of the environment, we propose here a novel representation. Using statistical models, our description captures the local macroscopic characteristics of unstructured environments. Such characteristics can be (i) the number of objects per unit area, (ii) their spatial distribution, or (iii) the distribution of basic local morphological attributes, such as shape, color or size. The advantage of this representation is that it does not rely on individual features. Mismatch problems are thus eliminated, and the representation is robust to perturbations (small displacement of objects).

The paper is organized as follows. In section 2 we propose to model the robot's environment as a realization of a *Random Closed Set* model. In section 3

[1]I3S CNRS URA 1376, Université de Nice Sophia Antipolis; 2000, Route des lucioles; Les Algorithmes; Bat. Euclide B; 06903 Sophia Antipolis - Cedex; France.

we discuss how random closed set models can be used for navigation. We assume that such a description is available and do not address here the problem of joint mapping and localization. In section 4 we present preliminary simulation results that validate our approach. Finally, some conclusions are drawn in section 5.

2 Modelisation of scattered objects as random closed sets

We admit that a convenient segmentation of the perceptual data collected by the robot (images, sonar scans, etc.) induces a series of patches on the robot's workspace. This is the case for alga, coral (figure 1 shows the patchy nature of 'maerl', found in Orkney, Scotland), or rocky sea beds. This type of environments can be represented as a union of closed sets

$$\Xi = \bigcup_{i=1}^{\infty} (\Xi_i + p_i), \tag{1}$$

where Ξ_i represent the morphology of the individual patches and p_i their spatial location. Based on this assumption we use *Random Closed Sets* (RCS), see [3],[4] in order to describe basic characteristics of natural scenes by statistical models. The theory of random closed sets has been frequently used in biological and physical contexts, in order to analyze natural patterns. One of its earliest applications was the modelisation of the crystallization in metals by Kolmogorov in 1937 [5]. A random closed set Ξ (as given by equation (1)) is a doubly stochastic process, also called germ-grain model. A first random point process describes the spatial location of objects (germs), denoted by p_i in the above equation, at which realizations of a second stochastic process (grains) determine the local morphology of the field, i.e. the characteristics of the sets Ξ_i. The distribution of the germs can in particular be clustered, structured or uniformly distributed, see figure 2.

We assume that the counting measure μ associated to the point process (germ model) is a member of a parameterized family of distributions G_p:

$$\mu \in G_p = \{\mu_\lambda, \lambda \in \Lambda\},$$

where Λ is a compact set. The vector λ is the collection of parameters that determine the statistical distribution of the locations p_i. The shape process (grain model) constrains the possible elementary shapes (e.g. to discs of random radius, lines of random orientation

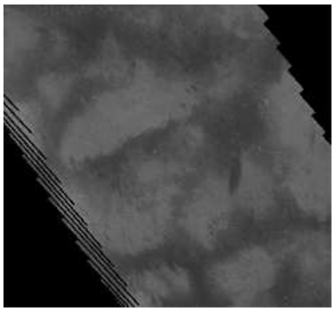
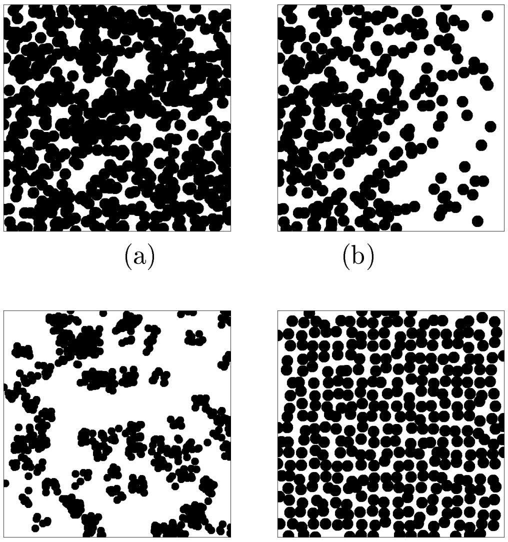


Figure 1: A mosaic of the coral 'Maerl' observed in Orkney, Scotland [Courtesy of IST, Lisbon].

or to mixtures of them). Similarly to the germ process we consider that the distribution of the shapes can be parameterized by a finite number of parameters γ, such that



(a) (b)

(c) (d)

Figure 2: Examples of RCS models. (a) isotropic boolean model (uniformly distribution of the patches), (b) anisotropic boolean model, the number of patches depends on the location, (c) a clustered distribution of the patches and (d) the patches exhibit a regular structure

$$\nu \in G_{\Xi_0} = \{\nu_\gamma, \gamma \in \Gamma\},$$

where ν is a probability measure over the space of possible shapes and Γ is a compact set. Different model types can be obtained by considering distinct families G_p, G_{Ξ_0} (for instance, homogeneous Poisson point process, disc of random radius, etc.). The random closed set model is thus given by the model type (G_p, G_{Ξ_0})

and the corresponding model parameters $\theta = (\lambda, \gamma)$. The aim of the theory of random closed sets is to determine θ and the model type (G_p, G_{Ξ_0}), such that the analyzed scene is a typical realization of the random closed set. Methods of parameter estimation for selected families of random closed set models can be found in the literature, see [3, 6].

It is often difficult to obtain direct estimates of the count measures from the images, since the elementary Ξ_i (the simple elementary shapes) cannot always be separated, as illustrated in figure 2. We exploit here an important property of random closed sets [7], stating that the distribution of any general random compact set is uniquely determined by the hitting capacity for each compact set K,

$$T_\Xi(K) = \mathbf{P}(\Xi \cap K \neq \emptyset); \quad \forall K \in \mathcal{K}, \qquad (2)$$

where $\mathcal{K}$ is the family of all compact sets. These hitting capacities can be directly estimated from images of natural scenes, using empirical estimates. The important result is that knowledge of the hitting capacities is equivalent to the knowledge of the model parameter θ.

For some random closed set models analytical forms of equation (2) can be found. One of the most studied random closed set models is the **boolean model**. The germ process is a Poisson point process, determined by the intensity parameter λ, and the grains are considered to be members of parameterizable families of compact sets. The hitting capacity for boolean models is given by [3]

$$T_\Xi(K) = 1 - exp(-\lambda \mathbf{E}(\nu_d(\Xi_0 \oplus \check{K}))), \qquad (3)$$

where $\nu_d(K)$ is the Lebesgue measure of the compact set K, $\oplus$ is the Minkowski-addition ($(A \oplus B) = \{a + b; \forall a \in A, b \in B\}$), $\mathbf{E}()$ is the statistical expectation operator and $\check{K} = \{-x; x \in K\}$. For other model types, such as clustered models, or regular structures, analytical forms of equation (2) are not available. We concentrate here on modelisation of natural scenes by boolean models. Ongoing work concerns characterization of other types of random closed set models (illustrated in figure 2), in particular clustered models, which seem good candidates to describe natural scenes.

In general, the perceptual characteristics change throughout the workspace, induced by varying temperature, soil fertility, ocean current, etc. If these variations are abrupt, we can partition the workspace into disjoint areas A_i (see figure 3), whose macroscopic characteristics are described by different types of statistical models $M_i(\theta)$:

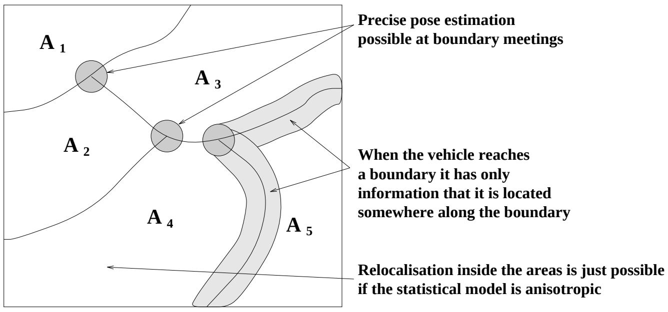


Figure 3: Segmentation of the workspace into areas where the characteristics of the environment are described by different random closed set models.

$$\text{Workspace} = \bigcup_{i=1}^{\infty} A_i, \quad A_i \leftrightarrow M_i(\theta) = (\mu_\lambda, \nu_\gamma),$$

with $\mu_\lambda \in G_p^i$ and $\nu_\gamma \in G_\Xi^i$. To model smooth variations of the field inside each region A_i, we let the model parameter θ depend on the location x

$$\text{Map:} \quad x \to \theta(x) = (\lambda(x), \gamma(x)); \quad x \in A_i. \qquad (4)$$

The approach to navigation of robots in natural environments proposed here considers that the map given in the previous equation has been learned by the robot, i.e., the partition $\{A_i\}$ and the vector field defined by (4).

3 Use of RCS models for mobile robot navigation

We address now the problem of using the map defined in the previous section to estimate the robot location. Consider first local navigation inside each region A_i.

If $\theta(x)$ is known for all $x \in A_i$, the inverse map $\theta^{-1} : \theta \to \mathcal{B} \cap A_i$, where $\mathcal{B}$ is the field of all Borel sets of the ambient space, can be computed. If $\theta^{-1} = A_i$ the model M_i is *isotropic* and *relocalisation* inside A_i is not possible. If, however, $\theta^{-1} = B \subset A_i$ (a strict subset), the model is *anisotropic* and the vehicle's pose, after observation of θ, can be restricted to locations in B. An example of an isotropic scene is given in figure 2(a), and of an anisotropic scene in 2(b).

In the homogeneous case, relocalisation is only possible on the boundaries between adjacent areas, indicated by an abrupt change of the model. In fact each area A_i can be further segmented into subareas, where the model is either isotropic or anisotropic.As long as the vehicle moves inside isotropic areas relocalisation is only possible when it crosses a boundary where either the model type or the model parameter changes. If the robot crosses the boundary of two areas that are described by different model types, hypothesis testing must be performed, in order to be able to detect the model change. This can be done using available tools for model selection, such as generalized maximum Likelihood, or the MDL or AIC criteria [?].

Once the model type has been identified, the best estimate of the vehicle's location for that model must be determined. Here we assume that the characteristics of the entire workspace are represented by the same RCS model, being anisotropic in some areas and isotropic in others. The restriction to a single model type removes thus the necessity of addressing the hypothesis testing problem.

We use a Bayesian approach in order to solve the localization problem. An optimal estimate of the robot's state X_k is obtained, given the past observations $Y^k = \{Y_1, \dots, Y_k\}$, as the MMSE estimate

$$\hat{X}_k = \int_{\mathcal{X}} X_k p(X_k|Y^k) dX_k,$$

where $p(X_k|Y^k)$ is the a posteriori density of the robot's state, given the observations. The observations $Y_i = (D_i, Z_i)$ contain proprioceptive observations D_i (velocity, heading,...) - allowing an estimate of the displacement of the vehicle - and exterioceptive observations Z_i, obtained using perceptual sensors (vision, sonar,...).

The a posteriori density $p(X_k|Y^k)$ can be updated by alternating prediction and filtering steps:

$$p(X_{k-1}|Y^{k-1}) \stackrel{Pred.}{\rightarrow} p(X_k|Y^{k-1}) \stackrel{Filt.}{\rightarrow} p(X_k|Y^k).$$

The prediction step is obtained from the state space model $X_k = f(X_{k-1}) + w_k$. Under the hypothesis that Y_k is a memoryless observation we obtain $p(X_k|Y^k)$ as

$$p(X_k|Y^k) \propto p(X_k|Y^{k-1}) p(Y_k|X_k).$$

Since $p(X_k|Y^{k-1})$ is already given by the prediction step we concentrate on $p(Y_k|X_k)$. For our approach the perceptual observations Z_k are estimated hitting capacities $\hat{T}_k = \{T_k(K_1), \dots, T_k(K_n)\}$. If the proprioceptive and perceptual observations are uncorrelated $p(Y_k|X_k)$ can be factorized

$$p(Y_k|X_k) = p(D_k|X_k) p(\hat{T}_k|X_k),$$

where $p(\hat{T}_k|X_k) = p(\hat{T}_k|\theta(X_k))$, which stems from the fact that the hitting capacities are determined by the model parameter at location X_k, given by the map. In order to solve the localisation problem we need to identify $p(\hat{T}_k|\theta(X_k))$.

An approximation by Gaussian densities, $p(\hat{T}_k|\theta(X_k)) \simeq \mathcal{N}(\hat{T}_k, \Sigma_T)$, allow the use of an Extended Kalman filter, in order to perform the prediction and filtering steps mentionned above. In general the probability density $p(\hat{T}_k|\theta(X_k))$ is not Gaussian. For the case of boolean models we obtained analytical expressions which could be well approximated by Gaussians.

4 Results

We present in this section results that demonstrate the feasibility of mobile robot navigation based on environment descriptions by random closed set models. We simulated the navigation of an underwater robot equipped with proprioceptive sensors (compass, gyroscope, speed sensor) and a camera pointing at the sea bottom. The robot moves in an environment that was obtained by sampling from a partly anisotropic *boolean model*. The patches that cover the bottom are compact discs whose radii are uniformly distributed in an interval, $r \in [r_1, r_2]$. See figure 6, where details of the obtained field are shown. We let the intensity λ, of the germ process (Poisson point process), depend on the location x, while the distribution of the radii (the parameters r_1 and r_2) does not change throughout the workspace. The map of the environment is thus given by

$$\theta(x) = (\lambda(x), r_1, r_2), \quad x \in [-440, 1500] \times [-600, 1400].$$

The variation of λ is illustrated in figure 4. The realization of the environment (the locations of the compact discs) is shown in figure 5 along with the initial location of the obot and the initial pose estimate (coinciding). The small square indicates the area that is locally observed by the camera (the corresponding image is shown in figure 6(a)).

We point out that a feature based approach could not be applied in this case: no single salient perceptual feature can be easily identified in the environment, and representation of the individual elements would impose unrealistic memory requirements, while being, at the same time, highly sensitive to small perturbations.

A simulated (non-modeled) ocean current perturbates the nominal trajectory of the robot, resulting in an important drift between the true and the estimated dead-reckoning position. Throughout the trajectory

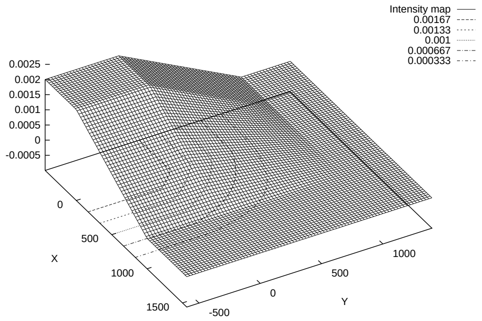


Figure 4: $\lambda(x)$ indicates variation of the model parameter. The workspace is divided in two isotropic areas (no variation of λ), divided by an anisotropic region.

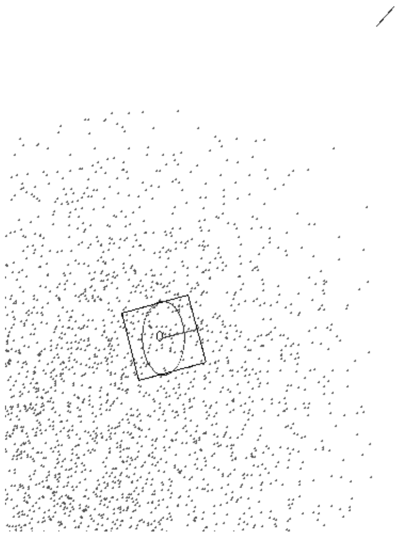


Figure 5: The environment and the initial position of the estimated and true robot position.

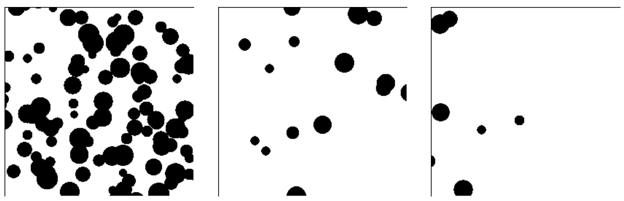


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Figure 6: Local realisations of the environments at different positions along the trajectory.

images of the bottom are acquired at regular time intervals. From these images we estimated the hitting capacities for three compact sets (squares of different side length 'd'), that are used as the only perceptual observations, $Z_i = \{\hat{T}_i(K_1), \hat{T}_i(K_2), \hat{T}_i(K_3)\}$. For boolean models we obtained analytical expressions for the probability density $p(Z_i|\theta(X_k))$ (see previous section) and approximations by Gaussian densities were satisfying. An approximation of the optimal nonlinear filter by an Extended Kalman Filter, in order to update the a posteriori density $p(X_k|Y^k)$, is thus justified.

Since the images that are taken at consecutive positions overlap strongly (reobservation of 90% of the bottom), an appropriate sampling strategy (for estimation of the hitting capacities) is required to guarantee uncorrelated observations. In order to obtain a non-biased estimate of the robot's position we use, at each time instant, only a small fraction of the image, which was sampled at random (using 10% of the pixels).

The estimated trajectory (dotted line) is indicated in figure 7 along along with the trajectory of the simulated robot. The results show clearly, that the use of hitting capacities as perceptual observations allows us to maintain the pose error small and the uncertainty bounded (when the robot is navigating in anisotropic areas). This can be better appreciated in figures 8 and 9, showing the pose error (distance of the estimated position to the simulated position) and the evolution of the uncertainty of the pose estimate.

During some periods the robot passes from anisotropic areas to isotropic area, inside which perceptual observations cannot be exploited for pose estimation. These periods are reflected on the evolution of the uncertainty shown in figure 9: each time that the robot navigates in an isotropic area (e.g. from iteration 270 to iteration 330) the uncertainty grows linearly. Pose correction is performed when the robot reenters an anisotropic area, resulting in a sudden decrease of the error.

The impact of the perceptual observations on the resulting pose estimate is made clear when we compare the estimated trajectory of figure 7 with the estimated trajectory shown in figure 10, where pose estimation was based merely on proprioceptive data.

5 Conclusions

In this paper we proposed a novel environment description, using the formalism of random closed set models. These models capture the principal charac-

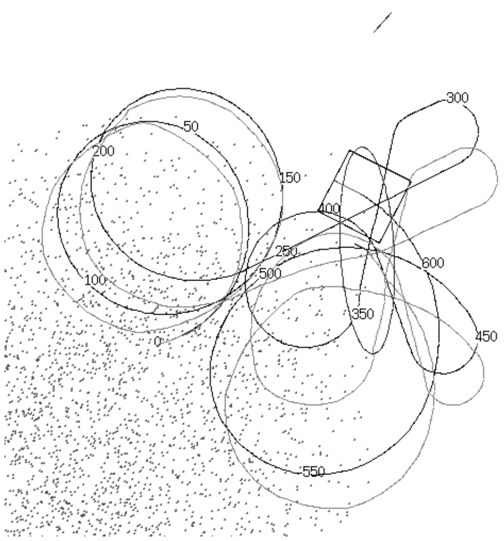

Figure 7: The true trajectory (the numbers indicate the iteration) is indicated along with the estimated trajectory. Pose estimation was performed using proprioceptive sensors and hitting capacities as perceptual observations.

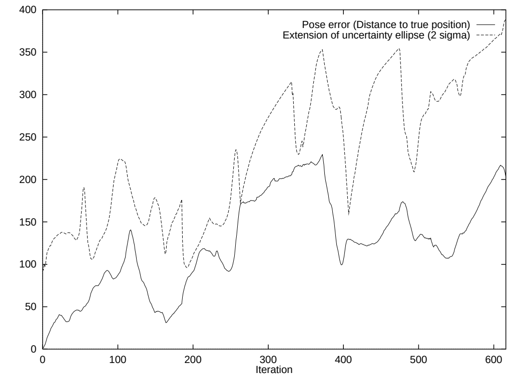

Figure 8: Evolution of the pose error

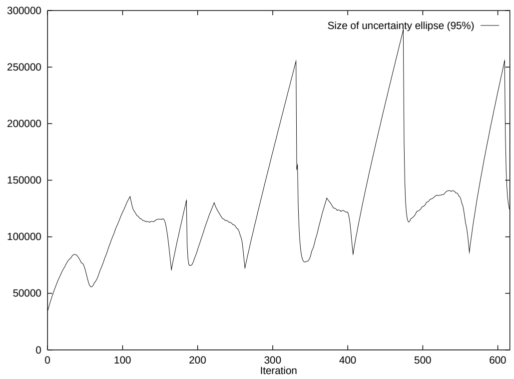

Figure 9: Evolution of the uncertainty, increasing strongly while the robot navigates in isotropic areas

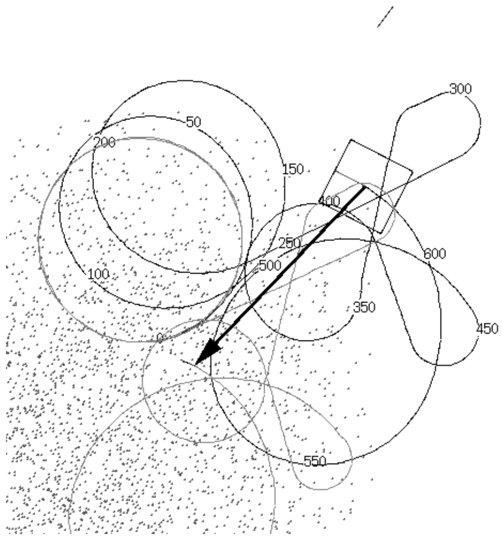

Figure 10: Robot's trajectory is estimated using only proprioceptive data. The position error at the end of the trajectory is indicated by the thick arrow.

teristics of an environment and do not rely on individual feature description. Knowledge of exact location (or shapes) of features is not required, resulting in increased robustness w.r.t. small changes.

The feasibility of the approach demonstrated by simulation results, in which the environments are simulated as simple boolean models.

A series of open problems must still be studied more thoroughly. In particular, we need to address the problem of correlated observations due to overlaps of successive images. Here we minimized this problem by using only small portions of the images. This was shown to yield good results, but an optimal solution is yet to be obtained.

Another open issue concerns the use of more complex RCS models, in order to describe clustered or regular environments. Analytical expressions need to be found for these models and the associated hypothesis testing formulated in order to discriminate between different types of regions. Finally the problem of joint mapping and localization for this kind of environment representations must be addressed in order to realize fully autonomous progression of a robot in a priori unknown environments. The robot must be able to simultaneously estimate the map (the model type along with the model parameter) and its position, using the autonomously created map.

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