

MAP estimation for curve modeling with free-knot splines.

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Abstract: In the context of curve modeling, splines have been widely used and studied ([1],[2],[3],[4]). Main arguments for using spline functions as function approximators are their ability to fit complex forms with arbitrary good accuracy, and the existence of basis functions, the B-splines, with attractive numerical properties. Our use of splines is motivated by the desire to find a parsimonious parametric description of a curve $s(t)$ of which we observe noisy samples:

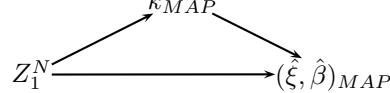
$$Z_n = s(t_n) + \epsilon(t_n) \simeq f(t_n; \hat{\theta}(Z_1^N)) + \varepsilon(t_n), \quad n = 1, \dots, N, \quad t_n \in [0, 1].$$

In this equation, $f(t; \theta)$ is a spline function, and $\varepsilon(t)$ represents observation noise ($\epsilon(t)$) and modeling errors. The approximating spline is an element of

$$\mathcal{G} = \cup_{k \in \{k_{min}, k_{max}\}} \$_k^m,$$

where, inspired by [1], $\$_k^m$ is the space of splines of degree m with k knots. Vector $\theta = (k, \xi, \beta)$ identifies $f(\cdot; \theta)$ as a member of \mathcal{G} : the number of knots, k , identifies $\$_k^m$, the knot vector ξ indicates that $f \in \$_{k,\xi}^m$, and β are the B-Spline.

We use a Bayesian approach and $\hat{\theta}(Z_1^N)$ are the MAP estimates in \mathcal{G} for a postulated prior. While MAP estimation of $s(t)$ has been addressed by other authors, leading to a model $\hat{s}(t) \notin \mathcal{G}$, the problem of building a parametric model for s in \mathcal{G} has received much less attention. An exception is [5], where in a slightly different context, MAP estimation in “union models” is also addressed. We depart in a significant way from the work of these authors, in the crucial issue of defining MAP estimates for spaces with this composite structure. A careful interpretation of the notation $p(\theta|Z_1^N)$ – that we prefer to decompose as $p(\xi, \beta|k, Z_1^N)P(k|Z_1^N)$ – is required: for different values of k , densities $p(\xi, \beta|k, Z)$ are defined with respect to distinct measures, and their direct comparison is meaningless, invalidating direct maximization of the numbers $p(\theta|Z_1^N)$. We advocate that MAP estimation in models with the union structure of \mathcal{G} must proceed in a stepwise manner:



Our implementation is inspired of BARS [4], using MC methods to sample from the joint posterior on (k, ξ) . By marginalizing over ξ , we approximate $P(k|Z_1^N)$. Optimization is based on a simulated annealing chain over ξ for k fixed at \hat{k}_{MAP} ($\hat{\beta}_{MAP}$ is determined analytically). We note that we also estimate the noise variance. In the experiments presented, the approximation accuracy (MSE) of our MAP model $f(t; \hat{\theta}_{MAP})$ is comparable to that of BARS’s estimates \hat{s}_{BARS} , with the advantage of providing a compact data description with $2\hat{k}_{MAP} + 1$ parameters.

References

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