Mobile Robot Localization based on Random Closed Set Model Maps

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Abstract

In this paper we present a novel approach to mobile robot navigation in natural unstructured environments. Natural scenes can be considered as random fields where a large number of individual objects of random shape appear randomly scattered in space. In this paper we use Random Closed Sets (RCS) to model the random scattering and shape of the objects observed, and base the navigation of a robot on maps of the RCS model's parameters. Contrary to the feature based approach to robot navigation, this environment representation does not require the existence of outstanding objects in the workspace, and is robust with respect to small dynamic changes. We address the problem of estimating the position of a mobile robot assuming that the (statistical) map of the environment is available a priori. We also present an adaptive guidance strategy that autonomously leads the robot to locations where the perceptual observations result in an efficient reduction of its state uncertainty. Simulations demonstrate the feasibility of our approach.

1 Introduction

There is a growing interest in the development of autonomous underwater vehicles (AUVs) for applications such as sea-floor mapping and environment monitoring. AUVs offer a better alternative to human intervention in ocean regions not easily accessible and for long missions. The majority of the navigation systems of AUVs currently in operation rely on the use of long baseline (LBL) arrays of acoustic transponders or to periodic returns to the surface for GPS fixes, increasing the effective cost of each operation. An alternative to these approaches is to infer the robot's positions from observation of its environment.

While stable localization methods have been proposed for in-door robots, navigation of robots in natural environments is still a challenge. We identify two reasons. First, we are confronted with large scale open environments, that require the ability to navigate to long distances. In the absence of external position information, if pose estimation is based only on dead-reckoning it results in an unbounded increase of the estimation error. Perceptual information can be used in order to overcome this limitation. The robot creates a map (or uses an existing one) describing the workspace. This map, if it is sufficiently rich, can be used in order to maintain bounded the uncertainty affecting the robot's position during its entire mission. The second point concerns the structure of the environment. Natural scenes are highly unstructured, lacking the geometric type of landmarks (spatially concentrated and simple to describe) on which are based most indoor maps.

It is thus important to assess the question: "What is the perceptual information that provides the best information for pose estimation ?". The most common approaches for localizing mobile robots are feature based, see e.g. [1]. Other methods use 3D elevation maps [2] (requiring non-flat sea-bottoms), or the use of mosaics [3]. All localization approaches rely on correct data association in order to match recent observations to those already contained in the map. This requires the identification of outstanding objects (that can be distinguished from neighboring ones). While in structured environments it is relatively easy to identify outstanding features, this is not always guaranteed in natural environments, specially when the field of view is limited (myopic perception) as it is the case for vision in underwater robotics. Mismatches due to unstable feature identification, or feature sparseness lead to the divergence of the navigation system.

We propose a novel environment description suitable for environments where identification of salient features is difficult or impossible. Instead of creating a detailed description of the environment as a collection of spatially registrated features, we propose a representation by statistical models that capture their local macroscopic characteristics. These characteristics can be, (i) the spatial distribution of objects, and (ii) the distribution of basic local morphological attributes, such as perimeter length or size. Such a representation has to be considered as an alternative to other mapping approaches, which, if the respective conditions are met, yield better results since they rely on more detailed information. If stable identification of details is difficult the proposed representation presents the advantage that it does not rely on precise knowledge of the position and shape of each feature, since detailed

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informations are discarded.

Another important topic on field robotics concerns the efficient use of the environment map. One of its important utilizations is for path planning. An optimal path can be chosen using a variety of criteria such as minimum path length or minimum uncertainty [4]. In this paper we present a simple 1-step ahead predictor that guides the robot to the neighboring position where the information provided by the perceptual observations is maximum.

The paper is organized as follows. In section 2 we propose the use of RCS models as suitable descriptions of unstructured environments. Section 3 gives an overview of how RCS models can be used for mobile robot navigation. Important aspects are the approximation of the non-linear filter and the guidance strategy in order to minimize the uncertainty of the robot's state. In section 4 we present preliminary simulation results that validate our approach and draw conclusions in section 5.

2 Modeling of Scattered objects as Random Closed Sets

We can identify a very *formal geometrical description* of an environment as a union of bounded sets (each one describing a single object):

$$\Xi = \bigcup_i (\Xi_i + p_i), \quad \Xi_i \in \mathcal{K}, \tag{1}$$

where $\mathcal{K}$ is the system of compact sets in $\mathbb{R}^2$ ($\Xi_i + p_i$ is the set Ξ_i translated by p_i). Additional information can be added by associating to each set a label m_i, specifying the type of object. Feature based approaches map the objects $\Xi_i + p_i$ individually, while e.g. grid-based (or mosaicing) approaches map the entire discretized workspace Ξ. In some situations (outstanding objects cannot be identified) it makes more sense to consider the field (1) as a **random pattern**. Localization is possible if distinct regions of the field, presenting different statistical characteristics, can be identified.

A way to describe random fields can be done by using the notion of **random closed sets** (RCS). Characterization of random fields as RCS is not new and has already been introduced by Kolmogoroff. We will present just the basic concepts. More theoretical and deeper studies of RCS can be found in [5, 6]. Roughly speaking a Random Closed Set can be considered as a doubly stochastic process. A first process (the *point process*) determines the sequence of locations $\{p_1, p_2, \dots\}$ of the objects. A second process (the *shape process*) determines then the morphology of the individual objects $\{\Xi_1, \Xi_2, \dots\}$ that are placed at each location (see figure 1 for examples).

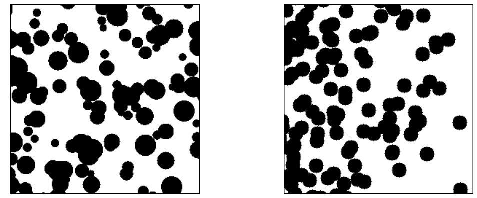

Figure 1: Random Closed Sets: left stationary, right non-stationary

We assume that the counting measure μ associated to the point process is a member of a parameterized family of distributions G_λ, where λ is the parameter that determines the distribution. The shape model constrains the set of possible elementary shapes. Similarly to the point process we consider that the measure κ associated to the shape process is a member of a parameterized family of distributions G_γ, where γ is the parameter that determines the distribution.

Different model types can be obtained (considering the point and shape process to be independent) as pairs of families (G_λ, G_γ) (for instance, for G_λ: stationary Poisson point process, regular pattern, etc., and for G_γ: discs whose radii are uniformly distributed in an interval, ellipses, etc.). The random closed set model is thus given by the model type

$$M_{i,j} = (G_\lambda^{(i)}, G_\gamma^{(j)}),$$

and a particular model $M_{i,j}(\theta) \in M_{i,j}$ is specified by the parameter vector $\theta = (\lambda, \gamma)$.

2.1 Hitting capacities

It is in general impossible to directly estimate the counting measure and the morphological characteristics of the sets. It has been established [7] that the distribution of a RCS is determined by the **hitting capacities** $T_\Xi(K)$ *for all* $K \in \mathcal{K}$. The hitting capacity is defined as the probability that the set K intersects Ξ

$$T_\Xi(K) = \mathbf{P}(\Xi \cap K \neq \emptyset). \tag{2}$$

The obvious advantage of this result is that estimates of the hitting capacities can be obtained easily from classified images, considering that the random field is locally stationary. In general we are restricted to estimates for a limited number of compact sets $K^n = \{K_1, \dots, K_n\}$ (denoted as the structuring elements). Representation of the image by hitting capacities for a finite set of structuring elements can be considered as a compression. Only basic characteristics of the scene (e.g. average size, average perimeter length of the grains and intensity measure) are kept and need consequently to be modeled. If the relation (2) is known, these characteristics can be deduced from observed hitting capacities and the scene can be partly reconstituted. Until now the family of **boolean models** (point process is a Poisson process and the grains are i.i.d. in the workspace) is the only family [8] for which $T_\Xi(K)$ can be computed analytically. Hitting capacities representing the RCS of figure 1(a) are shown in figure 2. In the sequel we write $T_\theta(K) = T_\Xi(K)$ assuming that we are able to associate Ξ to its structural parameter vector θ.

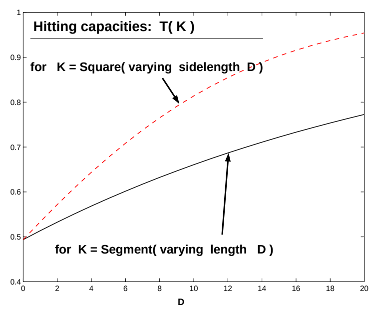

Figure 2: Hitting capacities for a set of structuring elements.

2.2 Random Closed Set maps

In general, the perceptual characteristics change throughout the workspace, induced by geophysical phenomena such as varying temperature, soil fertility, ocean current, etc. As a consequence environments have to be described by RCS models that are non-stationary. A map of the environment can be defined as a vector field, where a RCS model (of a specific type and with a specific parameter vector) is associated to each point, describing the local characteristics of the field. This field can be partitioned into disjoint areas A_k, whose boundaries indicate either an abrupt change of the model type or of the model parameter:

$$\text{Workspace} = \bigcup_{k=1}^{\infty} A_k, \quad A_k \leftrightarrow M_k(\theta) \equiv M_{i_k,j_k}(\theta), \quad (3)$$

where $M_k(\theta)$ is the RCS model associated to area A_k. To model smooth variations of the field inside each region A_k, we let the model parameter θ depend on the location x:

$$\text{Map:} \quad x \to \theta(x) = (\lambda(x), \gamma(x)); \quad x \in A_k. \quad (4)$$

The approach to navigation of robots in natural environments proposed here considers that the map given by equations (3) and (4) has been learned by (or given a priori to) the robot, i.e., the robot knows the partition $\{A_k\}$ and the continuous vector field defined by (4).

3 Robot localization based on Random Closed Set maps

We address now the problem of using the map defined in the previous section to estimate the robot location. Local observations provide useful information for localization only inside non-stationary areas of the RCS (the model parameter is a function of the location). In this case the pose error can be permanently corrected and positioning uncertainty is kept bounded. Accuracy of the localization depends on the informativeness of the field: strong variations of the field result in more accurate pose estimates since distinction between different locations inside the area is more accurate. If the areas are stationary, localization is only possible when the robot observes a boundary between adjacent areas, indicated by an abrupt change of the model parameter or of the model type. The problem is that during navigation inside stationary areas the pose error growths considerably, leading to a large uncertainty when the robot reaches the boundary, and ultimately resulting in a large set of possible true locations (large ambiguity). Most approaches to position estimation for mobile robots use Extended Kalman filters, and must thus assume that the error is small and that the observations are differentiable with respect to the robot's state. Navigation between adjacent areas requires in this case a first symbolic association step, prior to actual observations filtering. We propose a method that does not rely on this two step decomposition. We first formulate the general framework of the Bayesian approach to localization. Assume that the dynamic model of the robot's state X_k is known:

$$X_k = f(X_{k-1}, u_{k-1}) + w_{k-1}, \quad (5)$$

where $f(.,.)$ is a known (non-linear) function, $f : \mathcal{X} \times \mathcal{U} \to \mathcal{X}$, where $\mathcal{X}$ is the configuration space of the robot's state and $\mathcal{U}$ the space of control inputs u_k. The sequence w_k is assumed to be white. The optimal MMSE estimate of the robot's state X_k, given the past observations $Y^k = \{Y_1, \ldots, Y_k\}$, is given by the conditional mean

$$X_{k|k} = \int_{\mathcal{X}} X_k p(X_k|Y^k) dX_k,$$

where $p(X_k|Y^k)$ is the a posteriori density of the robot's state, given the observations. The density $p(X_k|Y^k)$ can be updated by alternating prediction and filtering steps:

$$p(X_{k-1}|Y^{k-1}) \overset{Pred.}{\to} p(X_k|Y^{k-1}) \overset{Filt.}{\to} p(X_k|Y^k).$$

The prediction step (convolution) propagates the probability distribution in the state space according to the dynamic model. If the observations Y_k are uncorrelated (the noise v_k is white), the filtering step (point-wise multiplication) computes

$$p(X_k|Y^k) \propto p(X_k|Y^{k-1}) p(Y_k|X_k).$$

The observation vector $Y_k = (D_k, Z_k)$ contains proprioceptive observations D_k (surge, heading,...) and exteroceptive observations Z_k, being estimates of the hitting capacities for a finite set of structuring elements K^n obtained directly from binary images: $Z_k \equiv \hat{T}_k = \{\hat{T}_k(K_1), \ldots, \hat{T}_k(K_n)\}$. In general given X_k, D_k and Z_k are uncorrelated, such that

$$p(Y_k|X_k) = p(D_k|X_k)p(\hat{T}_k|\theta(X_k)),$$

where we used the fact that the observations $\hat{T}_k$ depend on X_k only through the type and parameters of the RCS model at that point. We need to identify correctly the conditional density $p(\hat{T}_k|\theta(X_k))$, which we characterized for locally stationary RCS models with point processes satisfying the property of independent scattering (boolean models). An estimate $\hat{T}$ of the hitting capacities can be obtained by placing the compact set (the structuring element) K at N (sampling number) positions $\{p_i\}_{i=1}^N$ inside the observation windows $\mathcal{OW}$ and evaluate at each time the event: K hits (or not) Ξ. Under the condition that the individual events are mutually independent the probability of the number of hits k_h follows a **binomial distribution**:

$$P_{K_i}(k_h|N) = \left(\begin{array}{c} N \\ k_h \end{array} \right) T_\theta(K_i)^{k_h}(1 - T_\theta(K_i))^{N-k_h}.$$

For large sampling numbers N the binomial distribution is **well approximated by Gaussians**. We identified for boolean models an upper bound for N guaranteeing the independence of individual hits by placing the structuring elements at the nodes of a regular grid whose cell size depends on the K_i and θ. For boolean models it is also possible to correctly determine the correlations between estimates of structuring elements of distinct size, such that the full joint probability $P(T(K_1), \ldots, T(K_n)|\theta)$ can be characterized.

3.1 Optimal filtering

Direct computation of the prediction (convolution) and filtering (point-wise multiplication) steps is in practice not feasible. However if the noises of the dynamic and observation model are white zero-mean Gaussian (valid for large sampling numbers N) and if linearization of the non-linear models around the current estimate of the robot's state $X_{k|k}$ is valid the optimal filter can be approximated by the extended Kalman filter (EKF) equations. This validity of the approximation is based on the assumption that at each step the estimation error is small. To prevent filter divergence, the linearization must be a good approximation over the entire uncertainty domain. This is generally the case when permanent positioning information is available. If the robot navigates inside stationary areas the uncertainty of its state growths, such that linearization around the estimate is only valid in the vicinity of the current estimate. In [9] we approximated the optimal filter by a **Gaussian mixture model** (GMM), corresponding to a bank of EKF. The posterior density is approximated by a sum of individual terms:

$$p(X_k|Y^k) = \sum_{i=1}^{M} s_k^{(i)} p(X_k^{(i)}|Y^k); \quad \sum_{i=1}^{M} s_k^{(i)} = 1.$$

The GMM estimate is $X_{k|k} = \sum_{i=1}^M s_k^{(i)} X_{k|k}^{(i)}$. The number M of terms must be chosen such that linearization of the dynamic and observation model is valid inside the principal support of each term. It is easy to prove that repeated application of the prediction and filtering step results always in a Gaussian mixture, such that a closed form solution is available.

The scaling parameters $s_k^{(i)}$ depend strongly on the innovations: Terms for which the predicted observations correspond well to the true observations (small innovations) are reinforced, otherwise they loose importance and will not contribute significantly to the posterior density. The number of terms and the spatial distribution of the terms represent the state of ambiguity. In order to reduce the complexity of the GMM, terms whose scaling parameter decreases below a given threshold are eliminated and close terms are fused.

3.2 Navigation strategy

To choose the path resulting in an optimal reduction of positioning uncertainty we implemented a simple 1-step ahead predictor that drives the robot to a position where the information gain is optimized. It is known [10] that the estimate of the EKF, $X_{k|k}$, minimizes the mean square error for a given control input u_{k-1}, which for the GMM is given by:

$$\begin{aligned} J_k(u_{k-1}) \quad = \quad & \sum_{i=1}^{M} s_k^{(i)} \|X_{k|k}^{(i)} - K_{k|k}\|^2 + \\ & \sum_{i=1}^{M} s_k^{(i)} \int_{\mathcal{X}} \|X_{k|k}^{(i)} - X_k)\|^2 p(X_k^{(i)}|Y^k, u_{k-1}) dX_k. \end{aligned} \qquad (6)$$

We search the control input u_{k-1} that minimizes the above function:

$$u_{k-1}^* = \arg \min_{u_{k-1} \in \mathcal{U}} (J_k(u_{k-1})).$$

Since analytical computation is impossible we defined for our implementation a finite set of control inputs that guide the robot to a set of positions in the region ahead of it. For each control input we determine the resulting mean square error and choose the control input that leads to the minimum error. The computation of the mean square error depends strongly on the innovations, and thus on the perceptual observations (not know), via the scaling parameters and the estimates of each term. For each mixture term, we predict the hitting capacities assuming that the robot's nominal dynamic model is perfect, and that the observations are equal to their expected value given the predicted position.

4 Simulation results

In [11] we demonstrated navigation of a mobile robot inside non-stationary areas. The approximation of the optimal filter by an EKF was valid, since the uncertainty of the robot was maintained small during the entire trajectory and linearization around the estimated state was valid over the principal uncertainty support.

We concentrate here on navigation in workspaces, where the RCS model is widely stationary. Relocalization can in this case only be performed when the robot reaches the boundary of adjacent areas, indicated by a jump of the RCS model (either by a change of the model type or by a jump of the model parameter). Since permanent localization is not possible inside stationary areas the accumulated error of the position estimate is quite large.

We simulate navigation of an underwater robot equipped with proprioceptive sensors (compass and surge sensor) for dead-reckoning, and with a camera pointing to the sea bottom. The robot moves at a constant altitude, avoiding thus the need of rescaling of the images in order to preserve the metric. The simulated workspace is divided into two areas, inside which the RCS model is a stationary boolean model: The grains are compact discs whose radius is uniformly distributed in a known interval. The boundary between these areas indicate an abrupt change of the model parameter (the intensity of the point process). Figure 3) illustrates roughly the workspace and a small extract is shown in figure 4 (the upper left corner of A_1), representing a small part of area A_1 and A_2. The areas are hard to identify if the boundary is not indicated. A feature based approach is not suited in this case, since no salient feature can be identified.

Figure 3 indicates also the initial GMM pose estimate (by the symbol $\otimes$), along with the principal uncertainty support (the thick line). The true robot location is centered at the square, indicating the area that is actually observed by the camera. The terms of the Gaussian mixture model (which was triggered when the boundary between A_1 and A_2 crossed the principal support of uncertainty) are indicated by the small plus-signs.

A simulated ocean current ($\simeq$ 12% w.r.t. the nominal

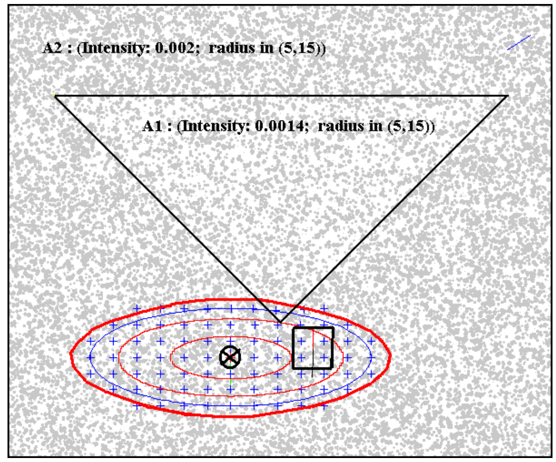

Figure 3: The simulated workspace and triggering of the GMM

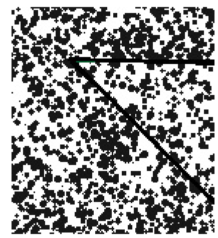

Figure 4: An extract of the above workspace, corresponding to the upper left corner of area A_1 (the triangle).

speed of the robot) produces a non observed drift between the true position and the estimate. The perceptual observations are empirical estimates of hitting capacities for two structuring elements $Z_k = \{\hat{T}(K_1), \hat{T}(K_2)\}$, whose joint distribution was approximated by a Gaussian. At each iteration the robot searches for the optimal control input u_k, restricted to those driving the robot inside a cone in front of the robot. The optimal control input drives the robot first in direction to the boundary. It is clear that the best information gain is obtained in the vicinity of the boundary, such that the robot adopts an oscillating behavior. Throughout the trajectory the weight of the terms of the GMM are transferred to those whose predicted observations coincide well with the observations. Spurious terms are eliminated and close terms are fused. At the end of the experiment a single term of the GMM remains. Figure 5 shows the trajectory of the estimated and the true position. The evolution of the mean square error, indicated in figure 6, illustrates well the gain provided by the RCS model map.

5 Conclusions

In this paper we propose a novel environment description for robot navigation, using the formalism of random closed set models. These models capture the principal

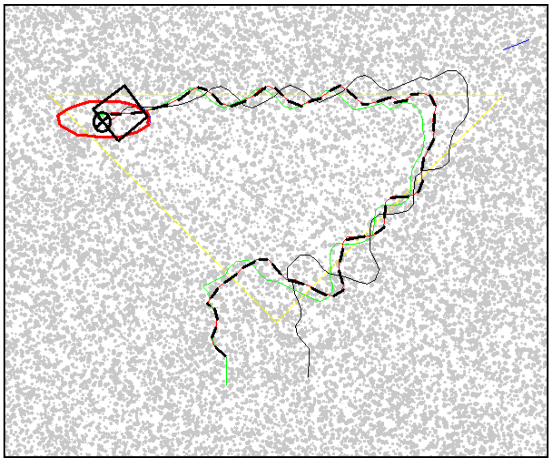

Figure 5: Estimated trajectory as a thick dash line compared with true trajectory.

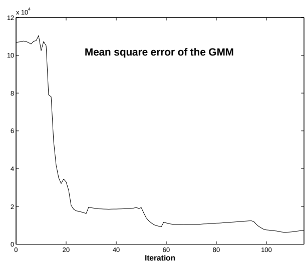

Figure 6: Mean square error $J_k(u_{k-1}^*)$.

characteristics of natural environments. The approach was motivated by the fact that identification of outstanding features, on which the majority of existing approaches to robot localization is based, is not always possible. Description by statistical models does not rely on the identification of outstanding features and knowledge of their exact location (or shapes) is not required, resulting in increased robustness with respect to small changes.

We present approximate expressions that enable definition of an approximation of the Bayesian estimator of the robot state for RCS models. We addressed the problem of ambiguity in the workspace, precluding the use of a simple EKF, when uncertainty of the estimated pose is very large. An approach that is related to multiple hypothesis, the Gaussian mixture model, is proposed and its feasibility is demonstrated by simulation results. In order to use the RCS map in an efficient way we propose an observation strategy that drives the robot to locations inside the workspace where the information provided by the perceptual observations results in the most significant reduction of uncertainty.

A series of problems must still be studied more thoroughly. In particular, it is necessary to handle more complex RCS models which are good candidates to describe real environments. Experiences with an ROV in the north of Scotland (Orkney islands) suggested sometimes a more regular spatial distribution than predicted by Poisson processes. Our present efforts focus on the crucial problem of joint mapping and localization for this kind of environment representations, where we concentrate on the autonomous segmentation of the workspace into homogeneous areas.

Acknowledgements

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References

[1] H.J. Feder, J.J. Leonard, and C.M. Smith, "Adaptive Mobile Robot Navigation and Mapping," *Int. Journal of Robotics Research*, July 1999.

[2] S. Lacroix, I.K. Jung, and A. Mallet, "Digital Elevation Map Building from Low Altitude Stereo Imagery," in *Proc. of the 9th. Int. Symposium on Intelligent Robotic Systems, Toulouse, France*, July 2001.

[3] N. Gracias, *Application of robust estimation to computer vision: Video mosaics and 3-D reconstruction*, 1997.

[4] H. Takeda, C. Facchinetti, and J.C. Latombe, "Planning the Motion of a Mobile Robot in a Sensory Uncertainty Field," *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. 16, pp. 1002–1017, October 1994.

[5] D. Stoyan, W.S. Kendall, and J. Mecke, *Stochastic Geometry and its Applications*, John Wiley & Sons, 1995.

[6] O.E. Barndorff-Nielsen, W.S. Kendall, and M.N.M. van Lieshout, *Stochastic Geometry, Likehood and Computation*, Chapman & Hall/CRC, 1999.

[7] G. Matheron, *Random Sets and Integral Geometry*, John Wiley and Sons, New York, 1975.

[8] I. Molchanov, *Limit Theorems for Unions of Random Closed Sets*, Habilitation at the Bergakademie Freiberg, Germany, 1993.

[9] M.J. Rendas and S. Rolfes, "Using Non-linear Filtering Techniques for Robot Localization in Sparse Environments Using Perception Sensors," in *Symposium on Intelligent Robotic Systems*, July 1999, pp. 429–438.

[10] A. Gelb, *Applied Optimal Estimation*, Mit Press, 1974.

[11] S. Rolfes and M.J. Rendas, "Statistical Environment Representation for navigation in Natural Environments," in *Proc. of the 9th. Int. Symposium on Intelligent Robotic Systems, Toulouse, France*, July 2001.