Proposition de stage M2

Titre : **Asynchronous convergence of monotone Boolean networks**

Encadrant : **Adrien Richard** (`http://www.i3s.unice.fr/~richard/`)

Laboratoire : **I3S, CNRS & Université de Nice-Sophia Antipolis**

Boolean networks are finite dynamical systems that consist in a finite number of binary variables evolving, in a discrete time, through mutual interactions and according to a fixed low. They are use in many disciplines for their ability to model non-linear behaviors. Most famous applications are in biology, where there are classical models for the dynamics of neural and gene networks.

This training course consists in studying the asynchronous dynamics of monotone Boolean networks, which present an interesting intermediate level of complexity, and which has interesting applications in the context of biological networks. It has been proved in [1] that, given a monotone Boolean network and an initial configuration x, the set R of configurations reachable from x by some asynchronous updating has the following properties :

1. R contains a fixed point reachable from x by updating to most one time each component ;
2. R contains a unique maximal fixed point and a unique minimal fixed point, both reachable from x by updating at most two times each component.

These results suggest the following natural question, which is the core of the subject : *Is every fixed point of R reachable from x by updating a polynomial number of times each component ?*

The study could start with AND-OR nets, i.e. monotonous Boolean networks in which each local transition function is either a disjonction or a conjonction of positive literals. These networks can thus be represented by a directed graph with two types of vertices (and-vertices and or-vertices), and the use of concepts from graph theory could allows rapid and informative progress on the main question, which is difficult in the general case.

[1] T. Melliti, D. Regnault, A. Richard and S. Sené. On the convergence of Boolean automata networks without negative cycles. In J. Kari, M. Kutrib, A. Malcher, Cellular Automata and Discrete Complex Systems (Proceeding of AUTOMATA 2013), Springer Berlin Heidelberg, LNCS 8155, 124-138. *PDF version* : `http://www.i3s.unice.fr/~richard/`