Interacting Conceptual Spaces

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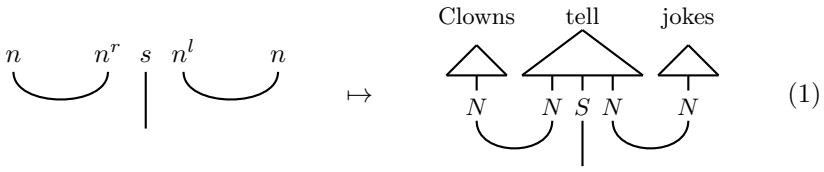
1 Programme

How should we represent concepts and how can they be composed to form new concepts, phrases and sentences? Categorical compositional distributional models [3] have provided a successful model for integrating symbolic structures with vector space representations, together with a clear and expressive graphical calculus. In [1] we extend the programme to conceptual spaces. The outline of the general programme is as follows:

1. (a) Choose a compositional structure, such as pregroup grammar.
   (b) Interpret this structure as a category, the **grammar category**.
2. (a) Craft appropriate meaning or concept spaces, such as vector spaces or conceptual spaces.
   (b) Organize these spaces into the **semantics category**, with the same abstract structure.
3. Interpret the compositional structure of the grammar category in the semantics category via a functor preserving the necessary structure.
4. Bingo! This functor maps type reductions in the grammar category onto algorithms for composing meanings in the semantics category.

The overarching idea is to take a theory of grammar such as the pregroup grammar, and then map the grammatical structures across to whichever structure is used to provide the semantics:



(1)

Detailed presentations of the ideas in this section are given in [3], and an introduction to relevant category theory is provided in [2].

We construct a new categorical setting for conceptual spaces which respects the convex structure emphasized in the literature [4, 5]. We show that this category has the necessary abstract structure required by categorical compositional models.

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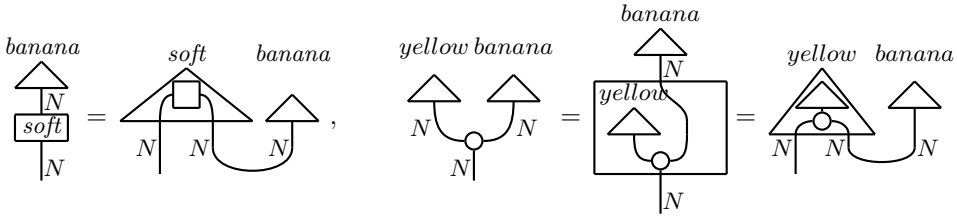
2 Concepts in Interaction

We model **conceptual spaces** as sets with an operation for forming convex combinations, referred to as **convex algebras**. Some examples of convex algebras are: real or complex vector spaces; simplices; semilattices; or trees. Convex algebras form a category **ConvexRel**, with morphisms binary relations respecting convex mixtures, called **convex relations**. **ConvexRel** has the necessary categorical structure for categorical compositional semantics. A conceptual space is an object of **ConvexRel**. We pick out a noun space N and a sentence space S.

We build adjective and verb spaces from noun and sentence spaces in accordance with the grammatical structure we have chosen. Adjectives, verbs, nouns, and sentences are represented as convex subsets of their respective spaces, with the result that adjectives are relations on the noun space, and transitive verbs are ternary relations from two copies of the noun space to the sentence space. In [1] we give detailed examples showing how words compose to form sentence meanings, and how functional words such as personal pronouns are incorporated.

Using this formalism, we are also able to analyse the internal structure of intersective adjectives as described in [6] as being formed of a 'copied' noun.



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