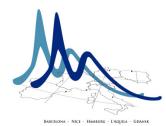
Master Mathmods, march 4, 2012



## **Complexity and recursivity**

# Order of magnitude

Prove the following:

- (1) If  $\alpha, \beta \geq 1$  are two real-valued constants, then  $\log_{\alpha}(n) = \Theta(\log_{\beta}(n))$
- (2) u = v + O(f) iff v = u + O(f)

## **Iterative complexity**

Estimate the complexity of the following programs. We assume instructions I1, I2, I3 in O(1).

```
i,j = 1,1
                                      while i < n
for i in 1...-1
                                         i+=1
   for j in i+1...n
                                          Ι1
      for k in i..j
                                          while j < n and Condition
         Ι1
                                             j+=1
      end
                                             Ι2
   end
                                          end
end
                                          IЗ
                                      end
```

## **Recursive complexity**

We consider the next two algorithms' complexity for computing the product of two large integers. Let U and V two integers of size 2n in basis  $\beta$  such that  $U = A\beta^n + B$  and  $V = C\beta^n + D$ .

- We first use the equality (Aβ + B)(Cβ + D) = A.Cβ + β(A.D + B.C) + B.D. Instead of multiplying two integers of size 2n, we have to compute four products of integers of size n, three shifts (multiplication by β) and three additions. We assume that the complexity of the last two arithmetic operations is O(n) and that T(1) = k. First write the recurrence relation between T(2.n), the time complexity for multiplying two integers of size n and T(n). Solve this recurrence relation.
- Same question when using the equality  $(A\beta + B)(C\beta + D) = A.C\beta 2 + ((A B).(D C) + A.C + B.D)\beta + B.D$ . First, count the number of basic arithmetical operations and assume also T(1) = k. The latter equality is known as Toom-Cook multiplication

#### Limits of the recurrence-solving theorem

Show that the theorem for solving recurrence relations does not apply to  $T(n) = 2T(n/2) + n \cdot \log(n)$ . Do you know any other means for solving this recurrence?