Master Mathmods, march 12, 2012

Trees

Exploring the definition of a tree

Definition 1 *A tree T is a finite non-empty set of nodes $T = \{r\} \cup T_1 \cup \ldots \cup T_n$ such that:*

(1) r is a distinguished node, the root

(2) the remaining nodes are partitioned into $n \geq 0$ subsets, each denoting a tree.

(1) From the above definition, describe the minimal tree.

(2) If we denote a tree by $T = \{r, T_1, T_2, \ldots, T_n\}$, draw the tree corresponding to

$$T_3 = \{D, \{E, \{F\}\}, \{G, \{H, \{I\}\}, \{J, \{K\}, \{L\}\}, \{M\}\}\}$$

Do you think it remains a tree according to def. 1 if we add $T_1 = \{A\}$ and $T_2 = \{B, \{C\}\}$ to the set of nodes?

Expression trees

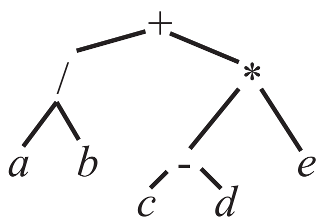

Figure 1: An expression tree.

(1) From figure 1, give the expression corresponding to the different tree traversals.

(2) Which is the most suitable for printing the expression?

(3) An improvement would be the following while visiting an internal node:

- print a left parenthesis, and then;
- traverse the left subtree, and then;
- print the root, and then;
- traverse the right subtree, and then;
- print a right parenthesis.

Apply this improvement to the above tree.

Trees in arrays

When the number of the nodes is known in advance, one can use an array of records where left and right nodes are indexes in array $(i, 2.i, 2, i+1)$.

(1) Give the array representation of the previous expression tree.

(2) Write the algorithms for traversals by replacing the trees by arrays.

(3) Try to find the time complexity of your algorithms.

(4) Implement in `Ruby` the array creation and the inorder traversal.