Barcelona - Nice - Hamburg - L'Aquila - Gdansk

Master Mathmods, may 14, 2012

NP and intractability

1 Back to TSP

Assuming that the HC is an NP-complete problem, prove that TSP is NP-complete.
HAMILTONIAN CIRCUIT (HC):
INSTANCE : A graph $G = (V, E)$

QUESTION : Does G contain a simple circuit that includes all the vertices of G?

TRAVELING SALESMAN (TSP):
INSTANCE : A finite set $C = \{c_1, c_2, \ldots, c_m\}$ of cities, a distance $d(c_i, c_j) \in \mathbb{Z}^+$ for each pair of cities $c_i, c_j \in C$, and a bound $B \in \mathbb{Z}^+$.
QUESTION : Is there a tour of all the cities having total length no more than B, that is, an ordering $\langle c_{\pi(1)}, c_{\pi(2)}, \ldots, c_{\pi(m)} \rangle$ of C such that:

$$\left(\sum_{i=1}^{m-1} d(c_{\pi(i)}, c_{\pi(i+1)}) \right) + d(c_{\pi(m)}, c_{\pi(1)}) \leqslant B?$$

2 More difficult: subset sum [from last year's exam]

Now, can you find a polytime transformation from SUBSET SUM to PARTITION?
SUBSET SUM
INSTANCE : A finite set A of positive integers, a positive integer B

QUESTION : Is there a $A' \subseteq A$ such that the sum of the elements in A' is exactly B?

PARTITION
INSTANCE : A finite multiset A of positive integers

QUESTION : Are there 2 sets B and C : $B \cup C = A, B \cap C = \varnothing$ and $\sum_{b \in B} b = \sum_{c \in C} c$?