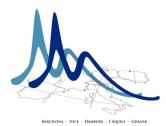
Master Mathmods, may 14, 2012



## NP and intractability

## 1 Back to TSP

Assuming that the HC is an NP-complete problem, prove that TSP is NP-complete. HAMILTONIAN CIRCUIT (HC): INSTANCE : A graph G = (V, E)

QUESTION : Does G contain a simple circuit that includes all the vertices of G?

## TRAVELING SALESMAN (TSP):

INSTANCE : A finite set  $C = \{c_1, c_2, \dots, c_m\}$  of cities, a distance  $d(c_i, c_j) \in \mathbb{Z}^+$  for each pair of cities

 $c_i, c_j \in C$ , and a bound  $B \in \mathbb{Z}^+$ . QUESTION : Is there a tour of all the cities having total length no more than B, that is, an ordering  $\langle c_{\pi(1)}, c_{\pi(2)}, \ldots, c_{\pi(m)} \rangle$  of C such that:

$$\left(\sum_{i=1}^{m-1} d(c_{\pi(i)}, c_{\pi(i+1)})\right) + d(c_{\pi(m)}, c_{\pi(1)}) \leqslant B?$$

## 2 More difficult: subset sum [from last year's exam]

Now, can you find a polytime transformation from SUBSET SUM to PARTITION? SUBSET SUM

INSTANCE : A finite set A of positive integers, a positive integer B

QUESTION : Is there a  $A' \subseteq A$  such that the sum of the elements in A' is exactly B?

PARTITION

INSTANCE : A finite multiset A of positive integers

QUESTION : Are there 2 sets B and C :  $B \cup C = A, B \cap C = \emptyset$  and  $\sum_{b \in B} b = \sum_{c \in C} c$ ?