TER Proposal - Master 2 Informatique - 2019-2020

FPT algorithm for the interaction graph consistency problem

Adrien Richard[1]

[1]I3S, UMR CNRS 7271 & Universit´ Côte D'Azur

September 26, 2019

A *Boolean network* with n components is a discrete dynamical systems described by the successive iterations of a function $f : \{0,1\}^n \to \{0,1\}^n$. The signed interaction graph of such a function f is defined as follows: the vertices are the n components, denoted from 1 to n, and there is a positive (resp. negative) arc from one component j to a component i (eventually $j = i$) if there exists $x, y \in \{0,1\}^n$ that only differ in $x_j < y_j$ such that $f_i(y) - f_i(x)$ is positive (resp. negative). Note that we can have both a positive and a negative arc from one vertex to another; and if this never happens, then we say that the signed interaction graph is *simple*.

Boolean networks are very classical models in biology for the dynamics of gene networks. In this context, the signed interaction graph of the gene network is very often well approximated from experimental data, while very few information about the dynamics are available. Hence, from a modeling point of view, we have a signed interaction graph G (with vertices indexed from 1 to n) and the few observations concerning the dynamics can be expressed by a *partial Boolean network*, that is, a function $h : X \to \{0,1\}^n$ where $X \subseteq \{0,1\}^n$. The Boolean network $f : \{0,1\}^n \to \{0,1\}^n$ modeling the global dynamics of the gene network must have G as interaction graph and it must *extend* the partial Boolean network h, that is, $f(x) = h(x)$ for all $x \in X$.

A natural question is: given G and $h : X \to \{0,1\}^n$, does such a f exists? (If it does not exists then there are some inconsistency between the data, and either G or h must be modified.)

We recently proved that, if G is simple, there is an $O(|X|^2 n^2)$-time algorithm for this decision problem [1]. Thus, if $|X|$ is bounded, the algorithm is polynomial in the number of components: this is a Fixed Parameter Tractable algorithm.

The aim of this TER is:

- to implement an FPT-algorithm, based on the one mentioned above, which *returns* f if it exists and 'no' otherwise (the present algorithm only answers 'yes' or 'no').
- to test this algorithm on real data.
- to extend this algorithm to no-simple signed interaction graphs.

References

[1] Florian Bridoux, Nicolas Durbec, Kévin Perrot, and Adrien Richard. Complexity of Maximum Fixed Point Problem in Boolean Networks. In *Proceedings of CiE'2019*, volume 11558 of *LNCS*, pages 132–143, 2019.