Introduction to Finite Dynamical Systems

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Instructions. You have to make groups of two peoples. Each group chooses a conjecture, and to groups cannot choose the same conjecture. Each group then have to give a talk of 30min about the conjecture, as interesting as possible (the date will be during the second half of January, to be fixed precisely later). Positive aspects of the presentation should be: a good presentation of the conjecture and its context, related articles find by yourself, partial positive results obtained by computer of by hand, and (of course) a complete positive or negative solution!

1 Optimality of a lower bound

Given a directed graph G, we denote by $\nu(G)$ the maximum integer k such that G has k pairwise vertex-disjoint cycles, and by $\max(G)$ be the maximum number of fixed points in a Boolean network with G as interaction graph.

Conjecture 1. *For every integer $k \geq 0$, there is a graph G with $\nu(G) = k$ and $\max(G) = k + 1$.*

2 Feedback vertex set versus error correcting codes

Given a directed graph G, we denote by $\tau(G)$ the minimum size of a feedback vertex set of G, and by $g(G)$ the minimum length of a cycle in G (with $g(G) = n + 1$ if G is acyclic, where n is the number of vertices in G). Given integer n, d, we denote by $A(n, d)$ the maximum size of a set $X \subseteq \{0, 1\}^n$ such that the Hamming distance of any two distinct elements in X is at least d.

Conjecture 2. *For every n-vertex directed graph G,*

$$2^{\tau(G)} \leq A(n, g(G)).$$

3 Loopless simple signed digraphs

Let D be a signed directed graph. D is simple if it has no both a positive and a negative arc from one vertex to another. A loop is a cycle of length one. We denote by $\max(D)$ be the maximum number of fixed points in a Boolean network with D as signed interaction graph.

Conjecture 3. *For every n-vertex simple signed graph D without loop,*

$$\max(D) \leq \binom{n}{\lfloor n/2 \rfloor}.$$

4 Disjoint cycles and maximum number of fixed points

Given a signed directed graph D, we denote by $\nu^+(D)$ the maximum integer k such that G has k pairwise vertex-disjoint positive cycles, and $\max(D)$ is the maximum number of fixed points in a Boolean network with D as signed interaction graph.

Conjecture 4. *There is a function $h : \mathbb{N} \to \mathbb{N}$ such that, for every signed directed graph D,*

$$\max(D) \leq h(\nu^+(D)).$$

5 Unique fixed point problem

Conjecture 5. *It is NP-hard to decide, given a directed graph G, if there is a Boolean network with a unique fixed point and with G as interaction graph.*